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Surface adsorption of branched polymers: Mapping onto the Yang-Lee edge singularity and exact results for three dimensions

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Starting from the general epidemic process, a stochastic multiparticle process which exhibits a critical point near an absorbing state and leads to big clusters belonging to the branched polymer universality class, we exploit supersymmetry to map the statistics of branched polymers in half-infinite systems onto the problem of the Yang-Lee edge singularity. Using this connection, we exactly calculate the scaling behavior and various scaling functions for the crossover between the nonadsorbed and adsorbed state in three dimensions.

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I. INTRODUCTION

The theory of critical phenomena near surfaces has been of interest for more than twenty years (for reviews, see Refs. [1] and [2]). In particular, the behavior and the adsorption of linear polymers at a wall have been studied by scaling, Monte Carlo simulations, and renormalization-group methods [3] (for a recent review, see Ref. [4]). Also the behavior of polymer networks or branched polymers with fixed topology and of star polymers has been analyzed extensively [5].

In contrast to the great number of all these studies, only a little work has been done on the corresponding problem of randomly branched polymers or lattice trees. Using Monte Carlo methods Lam and Binder [6] calculated the specific heat of branched polymers modeled as site lattice animals near a hard wall in the presence of an attractive force between the monomers and the wall. They found an adsorption transition and determined the crossover exponent. De'Bell and co-workers [7] analyzed lattice trees attached to a surface by exact enumeration data. They estimated the location of the adsorption transition for various two- and three-dimensional lattices and showed that the so-called ordinary behavior is determined by the bulk exponent. In particular they found for the number of surface-rooted trees

$$\mathcal{T}_N(E)=\sum_{N_1\geq 1}\mathcal{T}_{N,N_1}e^{EN_1}\sim\lambda^N N^{-\theta(E)}\,,$$
$$\theta(E)=\theta\quad\text{if } E<E_c\quad(N\gg 1)\,. \tag{1}$$

Here $\mathcal{T}_{N,N_1}$ is the number of different trees with N_1 sites out of N at the surface and θ is the known bulk exponent which determines the subleading asymptotic behavior of large (bulk) animal numbers. For $E>E_c$, $\theta(E)$ is the bulk exponent of a $(d-1)$-dimensional system. Thus E_c is the critical energy for adsorption of trees at the surface.

Here and in a subsequent paper we start an investigation of the adsorption transition of randomly branched polymers at an impenetrable surface by analytical field-theoretic methods. A short communication of exact results in $d=3$ was already given in [8]. We think that this analysis, apart from its theoretical interest, is experimentally relevant. Suppose the branched polymers are diluted in a good solvent; we find the universal properties of such solutions near a wall. This is of interest for the understanding of surface properties of physical systems like rubbers and for the influence of surfaces on the sol-gel transition and the collapse transition of branched polymers.

The field theory of branched polymers is usually constructed either by generalization of the de Gennes Φ^4 theory for linear polymers allowing for multifunctional units besides the monomers [9] or by development of an asymmetric Potts model which generates lattice animals [10]. Both methods lead to the same effective Hamiltonian. Parisi and Sourlas [11] have later shown that the $n\rightarrow 0$ limit inherent in the theory generate the same perturbation expansion as the field theory of the Yang-Lee edge singularity in an imaginary random field. The dimensional reduction which follows from the hidden supersymmetry of this model is the origin of the remarkable connection between the branched polymer problem in d dimensions and the nonrandom Yang-Lee problem in $d-2$ dimensions. The predictions of dimensional reduction for the critical exponents in two and three dimensions agree quite well with the results of numerical studies. Thus one can hope that studying the Yang-Lee edge of a spin system in d dimensions with a surface will yield information about the statistics of branched polymers in $d+2$ dimensions as well. But the arguments of Parisi and Sourlas are heavily based on perturbation theory for systems in an infinite medium without surfaces. Thus we prefer a derivation which avoids such an argumentation.

Ten years ago Grassberger [12] introduced a kinetic model for the so-called general epidemic process (GEP) which belongs to the universality class of dynamic percolation. The GEP is a local stochastic multiparticle process which describes the temporal evolution of a density of infected individuals $n(\mathbf{x},t)$. It is characterized by the

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following features.

(i) There is an absorbing stationary state $n(\mathbf{x},t)=0$, corresponding to the situation where the epidemic has become extinct.

(ii) The disease spreads (diffusively) in the available environment.

(iii) Infected individuals become immune to the disease after their illness. Thus the net infection rate locally depends on the number of infected individuals and of immune individuals, introducing a memory term into the process.

(iv) Microscopic degrees of freedom are subsumed in the form of a local Langevin noise term which, however, must respect the absorbing state. Hence it correlations have to vanish for $n(\mathbf{x},t)=0$.

The field theory of the GEP was developed in [13,14]. It was shown that, after the epidemic has become extinct, the distribution of immune individuals proportional to $\int_{-\infty}^{\infty} dt\, n(\mathbf{x},t)$ defines static percolation clusters. These clusters percolate if the infection rate exceeds a critical value. In [13] one of us showed that the statistics of large clusters below the percolation point is indeed reducible to the problem of the Yang-Lee edge singularity in a random imaginary field. This demonstrates that the statistics of these clusters and of branched polymers or lattice animals belong to the same universality class. Therefore we take the field-theoretic model of the GEP in a semi-infinite geometry, originally formulated in a paper of Janssen *et al.* [15] and used to calculate surface properties of percolation, as a suitable start point for our investigation of surface properties of branched polymers.

II. MODEL BUILDING AND BASIC QUANTITIES

Let us consider a Langevin equation for the local density $n(\mathbf{x},t)$ of the infected individuals which incorporates the characteristics (i)–(iv) in a simple, self-contained form (up to irrelevant terms in the renormalization-group sense):

$$\partial_t n(\mathbf{x},t)=\lambda[\Delta-r-m(\mathbf{x},t)]n(\mathbf{x},t)+q(\mathbf{x},t)+q_r(\mathbf{x},t)\ . \qquad (2)$$

The first term in the bracket on the right-hand side models the diffusive spreading (ii) of the disease. The next two terms in the bracket represent the net infection according to (iii). Here $m(\mathbf{x},t)=\lambda w\int_{-\infty}^{t} dt' n(\mathbf{x},t')$ is proportional to the density of immune individuals at time t at the point $\mathbf{x}$. Besides an external source $q(\mathbf{x},t)$ of the epidemic, $q_r(\mathbf{x},t)$ is a Gaussian random source, with correlations subject to (iv)

$$\langle q_r(\mathbf{x},t)q_r(\mathbf{x}',t')\rangle=\lambda\gamma n(\mathbf{x},t)\delta(\mathbf{x}-\mathbf{x}')\delta(t-t')\ . \qquad (3)$$

The parameter r in (2) measures the distance to the critical point. In terms of percolation probabilities it can be interpreted as $r\sim p_c-p$ near this point up to fluctuation corrections. We shall study the GEP in a semi-infinite geometry $\{\mathbf{x}=(\mathbf{y},z)|\ \mathbf{y}\in\mathbb{R}^{d-1},\ 0\le z<\infty\}$. Thus we have to endow our stochastic process with appropriate boundary conditions.

To study the stochastic properties of the GEP and especially the statistics of large clusters of immune individuals, it is advantageous to recast the Langevin equation (2) in conjunction with (3) as a dynamic functional [16,17].

$$\mathcal{J}[\tilde{n},n]=\int dt\, d^dx\, \tilde{n}(\mathbf{x},t)\left\{\partial_t+\lambda[r-\Delta+m(\mathbf{x},t)]-\lambda\frac{\gamma}{2}\tilde{n}(\mathbf{x},t)\right\}n(\mathbf{x},t)+\int dt\, d^{d-1}y\, \lambda c\tilde{n}_0(\mathbf{y},t)n_0(\mathbf{y},t)\ , \qquad (4)$$

where $n_0(\mathbf{y},t)=n(\mathbf{x},z=0,t)$, etc. Within this formalism all correlation and response functions can be expressed as functional averages with weight $\exp(-\mathcal{J})$. In (4) we have added the only relevant surface interaction term which respects all the conditions (i)–(iv) together with the causality requirement [15]. The Laplacian Δ is defined as $-\overleftarrow{\nabla}\cdot\overrightarrow{\nabla}$, where the arrows show the direction of differentiation, and we have neglected redundant surface terms.

In the following we use the notation

$$m(\mathbf{x})=m(\mathbf{x},t\to\infty)=\lambda w\int_{-\infty}^{\infty} dt\, n(\mathbf{x},t)$$

for the density of immune individuals after the epidemic has become extinct. From now on the term immune individuals is called "monomers" in polymer language. Recalling that with

$$(q,\tilde{n})=\int dt\, d^dx\, q(\mathbf{x},t)\tilde{n}(\mathbf{x},t)$$

the expression $\int\mathcal{D}\tilde{n}\exp[-\mathcal{J}+(q,\tilde{n})]$ defines the joint probability for a specific realization of a path $n(t)$, we find that

$$\begin{aligned}\mathcal{W}_N(z_R)&=\langle\tilde{n}(\mathbf{x}_R,0)\rangle_N\\&=\int\mathcal{D}(\tilde{n},n)\,\delta\left[N-\int d^dx\, m(\mathbf{x})\right]\tilde{n}(\mathbf{x}_R,0)e^{-\mathcal{J}}\end{aligned} \qquad (5)$$

measures the probability for the generation of a single cluster of N monomers from a weak pointlike source q as a root of an epidemic at $\mathbf{x}_R=(\mathbf{y}_R,z_R)$. In the bulk limit we have $\mathcal{W}_N(z_R\to\infty)\sim Np_0^N\mathcal{A}_N$ where $\mathcal{A}_N\sim\lambda_0^N N^{-\theta}$ are the animal numbers which count the different configurations of the large N-sited clusters. The constants λ_0 and p_0 are nonuniversal (we combine the two into one $\lambda=p_0\lambda_0$). The factor N arises because, in the bulk limit, each point of a given cluster may be the root; thus by translational symmetry the same configuration can be generated N times.

To find out the modifications of the properties of branched polymers on their approach to the adsorptive surface we introduce several quantities of interest. We consider clusters rooted in the point $\mathbf{x}_R$ made of N monomers with N_α monomers in a layer parallel to the surface at a distance z_α. The corresponding animal numbers are given by

$$N_R\mathcal{A}_N(N_1|z_1,\ldots;N_R|z_R)=\int\mathcal{D}(\tilde{n},n)\,\delta\left[N-\int d^dx\,m(\mathbf{x})\right]\times\delta(N_1-m^{(1)}(z_1))\cdots\times\delta(N_R-m^{(1)}(z_R))\tilde{n}(\mathbf{x}_R,0)e^{-\mathcal{J}}\ , \tag{6}$$

where we have defined layer densities $m^{(1)}(z)=\int d^{d-1}y\,m(\mathbf{y},z)$. The factor N_R in (6) arises from the restricted translational symmetry: each point in the layer at z_R may be the root of the same configuration. The definition (6) leads directly to the sum rules

$$\int_0^\infty dN_1\mathcal{A}_N(N_1|z_1,N_2|z_2,\ldots;N_R|z_R)=\mathcal{A}_N(N_2|z_2,\ldots;N_R|z_R)\ , \tag{7}$$

$$\int_0^\infty dz_1\int_0^\infty dN_1N_1\mathcal{A}_N(N_1|z_1,N_2|z_2,\ldots;N_R|z_R)=N\mathcal{A}_N(N_2|z_2,\ldots;N_R|z_R)\ , \tag{8}$$

and

$$\int_0^\infty dN_R\mathcal{A}_N(N_1|z_1,\ldots;N_R|z_R)=\mathcal{W}_N(N_1|z_1,\ldots;z_R) \tag{9}$$

as the generalization of (5). Later we will show that the layer animal numbers are fully symmetric in all their arguments, as one feels intuitively from their definition.

Having defined layer animal numbers we now weight monomers at the surface by a Boltzmann factor $e^{-\omega_0}$. Here ω_0 represents a surface adsorption energy $-E$ (divided by temperature). Consequently we introduce weighted layer animal numbers

$$\mathcal{A}_N(N_1|z_1,\ldots;N_R|z_R;\omega_0)=\int_0^\infty dN_0e^{-\omega_0N_0}\mathcal{A}_N(N_0|0,N_1|z_1,\ldots;N_R|z_R)\ . \tag{10}$$

We expect a transition of the behavior of the animals near the surface from a nonadsorbed to an adsorbed phase at a critical value $\omega_{0c}=-E_c$ of the adsorption energy.

Using these weighted animal numbers we define moments

$$Z_N^{(1)}(z_1,\ldots,z_R)=\int_0^\infty dN_1N_1\cdots\int_0^\infty dN_RN_R\times\mathcal{A}_N(N_1|z_1,\ldots;N_R|z_R;\omega_0)\ . \tag{11}$$

These moments are layer-integrated restricted correlation functions

$$Z_N(\mathbf{x}_1,\ldots,\mathbf{x}_R)=\langle m(\mathbf{x}_1)\cdots\tilde{n}(\mathbf{x}_R,0)\rangle_N=\int\mathcal{D}(\tilde{n},n)\,\delta\left[N-\int d^dx\,m(\mathbf{x})\right]\times m(\mathbf{x}_1)\cdots\tilde{n}(\mathbf{x}_R,0)e^{-\mathcal{J}-\omega_0m^{(1)}(0)}\ . \tag{12}$$

The restricted correlation functions are (linked) cumulants. Indeed, if one separates a group of arguments of $Z_N(\mathbf{x}_1,\ldots,\mathbf{x}_R)$ to infinity, we get $Z_N=0$ by reason of the connectedness of the animals.

Various mean values are given by these moments. Especially for the mean monomer density at the point $\mathbf{x}$ in the (surface-weighted) clusters with root at $\mathbf{x}_R$ we obtain

$$\rho_N(\mathbf{x}|\mathbf{x}_R)=Z_N(\mathbf{x},\mathbf{x}_R)/Z_N(\mathbf{x}_R)\ . \tag{13}$$

With the help of (8), (10), and (12) the obvious sum rule $\int d^dx\,\rho_N(\mathbf{x}|\mathbf{x}_R)=N$ follows.

The radius of gyration is a characteristic length of an animal. Here we have to distinguish between a radius perpendicular to the surface and a parallel one. They are defined by

$$\overline{R_\perp^2}(\mathbf{x}_R)_N=\int d^dx(z-z_R)^2\rho_N(\mathbf{x}|\mathbf{x}_R)/N\ , \tag{14a}$$

$$\overline{R_\parallel^2}(\mathbf{x}_R)_N=\int d^dx(\mathbf{y}-\mathbf{y}_R)^2\rho_N(\mathbf{x}|\mathbf{x}_R)/N\ . \tag{14b}$$

Also the surface induces an eccentricity of the root. This eccentricity which defines the position of the center of mass as $(z_R+\overline{R}_\perp)$ is then given by

$$\overline{R}_\perp(\mathbf{x}_R)_N=\int d^dx(z-z_R)\rho_N(\mathbf{x}|\mathbf{x}_R)/N\ . \tag{15}$$

Of course many other quantities which characterize the shape can be defined, but we will not consider them in this paper.

Now we introduce the Laplace transforms of the moments

$$G(\mathbf{x}_1,\ldots,\mathbf{x}_R;\omega,\omega_0)=\int_0^\infty dN\,e^{-\omega N}Z_N(\mathbf{x}_1,\ldots,\mathbf{x}_R;\omega_0)=\int\mathcal{D}(\tilde{n},n)m(\mathbf{x}_1)\cdots\tilde{n}(\mathbf{x}_R,0)\exp\left[-\mathcal{J}-\omega_0m^{(1)}(0)-\omega\int dz\,m^{(1)}(z)\right]\ . \tag{16}$$

We will show that the cumulants $G(\mathbf{x}_1,\ldots,\mathbf{x}_R)$ are directly related to the correlations of the Yang-Lee edge problem in $d-2$ spatial dimensions. We note that the nonuniversal factor λ^N in the animal numbers at this stage can be included in a redefinition of the Laplace variable ω, which plays the role of a chemical potential in a grand canonical description, by a simple shift $\ln\lambda$.

III. MAPPING ONTO THE YANG-LEE PROBLEM

Exploiting the supersymmetry we will show in this section that also in our half-infinite problem the cumulants defined in (16) are nothing else than the cumulants of an Ising spin system near the Yang-Lee edge singularity. Consider the generating functional

$$\mathcal{Z}([j,q],\omega,\omega_0) = \int \mathcal{D}(\tilde{n},n) e^{-\mathcal{J}([\tilde{n},n],\omega,\omega_0)+(j,n)+(q,\tilde{n})} \tag{17}$$

with

$$\mathcal{J}([\tilde{n},n],\omega,\omega_0)=\mathcal{J}[\tilde{n},n]+\omega\int d^dx\, m(\mathbf{x}) +\omega_0\int d^{d-1}y\, m_0(\mathbf{y}) \,. \tag{18}$$

Here $\mathcal{J}[n,\tilde{n}]$ is the dynamic functional defined in (4). Specializing the sources to $j=j(\mathbf{x})$ independent of time t and $q=q(\mathbf{x})\delta(t)$ we obtain the cumulants defined in (16) by suitable functional derivatives with respect to the sources. Following [13] we perform a shift and a duality transform of our dynamic fields n and $\tilde{n}$,

$$\tilde{n}(\mathbf{x},t)=M-i\alpha s(\mathbf{x},-t) \,, \tag{19a}$$

$$n(\mathbf{x},t)=i\alpha^{-1}\tilde{s}(\mathbf{x},-t) \,. \tag{19b}$$

The definition of $M=M(\omega)$ will be given shortly. Note that the integration path of the response variables $\tilde{n}$ and $\tilde{s}$ in the complex plane always goes from $-i\infty$ to $i\infty$. Inserting the transformation (19) into the dynamic functional $\mathcal{J}$ (18) we obtain

$$\begin{aligned}\mathcal{J}'[\tilde{s},s]:&=\mathcal{J}([\tilde{n},n],\omega,\omega_0)\\ &=\int d^dx\left[\int dt\,\tilde{s}(\mathbf{x},t)\left\{\partial_t+\lambda\left[\tau-\Delta+\frac{i}{2}gs(\mathbf{x},t)+i\lambda f\int_t^\infty dt'\tilde{s}(\mathbf{x},t')\right]\right\}s(\mathbf{x},t)\right.\\ &\qquad\left.-\left\{\lambda\int dt\,\tilde{s}(\mathbf{x},t)\right\}^2-i\lambda H\int dt\,\tilde{s}(\mathbf{x},t)\right]+\int d^{d-1}y\,\lambda\int dt\,\tilde{s}_0(\mathbf{y},t)[cs_0(\mathbf{y},t)-iH_0] \,.\end{aligned} \tag{20}$$

The new parameters are given by $\tau=r-\gamma M$, $2\alpha^2=wM$, $g=\alpha\gamma$, and $f=\alpha^{-1}w$, and the new external fields linear in ω and ω_0 are

$$-H=\alpha^{-1}[\omega+(r-\gamma M/2)M] \,, \quad -H_0=\alpha^{-1}[\omega_0+cM] \,. \tag{21}$$

Now the parameter M is determined by the condition $\lim_{z\to\infty}\langle s(\mathbf{x},t)\rangle=0$, where the mean value has to be calculated with the weight $\exp(-\mathcal{J}')$. In a graphical perturbation expansion this condition is equivalent to the compensation of the tadpole graphs by H in the bulk limit. At the tree order this yields $H=0$ leading to $\gamma M=r-\sqrt{r^2+2\gamma\omega}$, $\tau=\sqrt{r^2+2\gamma\omega}$. From (5), (12), (16), and (19a) we find that $M(\omega)$ is the Laplace transform of $N\mathcal{A}_N$. Thus the square root singularity of M at $\omega_c=-\frac{1}{2}r^2/\gamma$ results in the mean field expression $\mathcal{A}_N\sim N^{-5/2}e^{N\omega_c}$. Corrections due to fluctuations are calculated in perturbation theory near the critical value $\tau\approx 0$, $M\approx r/\gamma\neq 0$. Thus our scaling parameter α has a genuinely nonzero value as long as we are right below the percolation threshold.

Introducing an external length scale μ^{-1}, we easily determine the canonical scaling behavior $s\sim\mu^{(d-4)/2}$, $\tilde{s}\sim\mu^{(d+4)/2}$, $g\sim\mu^{(8-d)/2}$, $f\sim\mu^{(4-d)/2}$ from (20). The scaling of the coupling constant g shows that the critical dimension is 8 and that near this dimension f is irrelevant for critical behavior. In fact, it seems that f is irrelevant in the neighborhood of the nontrivial fixed point of the theory up to $d=2$, as is suggested by comparison between the theoretical and numerical results for the bulk properties of the animals [11]. Now neglecting the irrelevant terms of (20), we get the relevant dynamic functional which describes the critical behavior near the animal line $\tau=\tau_c(r,\omega)=0$,

$$\begin{aligned}\mathcal{J}_{\text{rel}}=\lambda\int dt&\left\{\int d^dx\,\tilde{s}\left[\lambda^{-1}\dot{s}+\left[\tau-\Delta+i\frac{g}{2}s\right]s-iH\right]\right.\\ &\left.+\int d^{d-1}y\,\tilde{s}_0[cs_0-iH_0\right\}\\ &-\int d^dx\left[\lambda\int dt\,\tilde{s}\right]^2 \,.\end{aligned} \tag{22}$$

It is the animal line which will be mapped to the Yang-Lee edge.

It is well known [17] that the form of the dynamic functional depends on the way time is discretized in the definition of the functional integral. The cumbersome point is the discontinuity of the propagator $\langle s(t)\tilde{s}(t')\rangle\sim\Theta(t-t')$. Up to now we have assumed a so-called prepoint discretization corresponding to the definition of the step function $\Theta(t)=0$ if $t\leq 0$. In a diagrammatic perturbation expansion this definition deletes all diagrams with closed propagator loops and ultimately leads to causality. But independent of any definition of the step function unwanted diagrams with propagator loops can be eliminated by the introduction of anticommuting "fermionic" Grassmann fields ("ghost fields") with appropriate couplings to the "bosonic" pair of fields $s,\tilde{s}$. By the fermionic (-1) rule for each ghost loop all unwanted diagrams are compensated by diagrams which possess corresponding closed loops of Grassman fields. A closer look at the diagrams shows that this mechanism will only work if the dangerous loops are isolated from each other. Diagrams with propagator loops which have some parts in common are not fully compensated. Fortunately, our functional $\mathcal{J}_{\rm rel}$ (22) contains only quadratic terms $\sim\tilde{s}^2$ with field independent factors besides the linear terms of the response field, so that such digrams do not arise. Thus we complete it by a pair of ghost fields $\bar{\psi}$, ψ and in this way avoid any consideration of time discretizations and time ordering in the following. The completed dynamic functional for our animal problem now reads

$$\hat{\mathcal{J}}=\mathcal{J}_{\rm rel}+\int dt\left[\int d^dx\,\bar{\psi}[\dot{\psi}+\lambda(\tau-\Delta+igs)\psi]\right.$$
$$\left.+\int d^{d-1}y\,\lambda c\bar{\psi}_0\psi_0\right] . \tag{23}$$

As a consequence of the purely static noise term $\sim(\int dt\,\tilde{s})^2$ of $\hat{\mathcal{J}}$, the correlator $\langle s(t,\mathbf{x})s(t',\mathbf{x}')\rangle=C(\mathbf{x},\mathbf{x}')$ of the perturbation theory is independent of time. Thus, after Fourier transformation, only zero frequencies "flow" in the correlator lines of the diagrams. Therefore is we now cut all correlator lines each diagram breaks down to a collection of tree diagrams with propagator lines (which cannot build closed loops alone) in which consequently no internal frequencies appear. Thus setting the external frequencies to zero our diagrams are now constructed only from the zero-frequency part of $\hat{\mathcal{J}}$. This static or "frozen" part $\mathcal{J}_{\rm fr}$ can be set up by specializing the fields to $s(t,\mathbf{x})\to\varphi(\mathbf{x})$ and $\lambda\int dt\,\tilde{s}(t,\mathbf{x})\to\tilde{\varphi}(\mathbf{x})$ with the same procedure for the Grassmann fields. This construction leads to the frozen functional

$$\hat{\mathcal{J}}_{\rm fr}=\int d^dx\left\{\tilde{\varphi}\left[\tau-\Delta+\frac{i}{2}g\varphi\right]\varphi-iH\tilde{\varphi}\right.$$
$$\left.+\bar{\psi}(\tau-\Delta+ig\varphi)\psi-\tilde{\varphi}^2\right\}$$
$$+\int d^{d-1}y\left\{c\tilde{\varphi}_0\varphi_0-iH_0\tilde{\varphi}_0+c\bar{\psi}_0\psi_0\right\} . \tag{24}$$

Of course we can also obtain this functional following the line of arguments given by Parisi and Sourlas [11] as demonstrated in [13] by one of us. First of all one recognizes that the relevant dynamic functional (22) represents a stochastic differential equation with a quenched noise. The static part of this equation can then be written again in the form of a functional integral. The Jacobian in this integral is exponentiated with the help of Grassmann fields yielding the frozen functional (24). But we prefer the introduction of the fermions as ghost fields, thus clarifying their role as a tool to delete unwanted diagrams in the sense of McKane [18].

The generating functional $\mathcal{Z}([j,q],\omega,\omega_0)$ (17) can now be calculated with $\hat{\mathcal{J}}$ (23). Especially from the properties of dynamic functionals we recognize that $\mathcal{Z}([0,0],\omega,\omega_0)=1$. Thus the cumulants are calculable as

$$G(\mathbf{x}_1,\dots,\mathbf{x}_R;\omega,\omega_0)=\left.\frac{\delta\cdots\delta\ln\mathcal{Z}([j,q],\omega,\omega_0)}{\delta j(\mathbf{x}_1)\cdots\delta q(\mathbf{x}_R)}\right|_{j,q=0} \tag{25}$$

after replacing the sources by their time independent counterparts as remarked after (18). Using the substitutions (19a) and (19b) we obtain the relations

$$\begin{aligned}G(\mathbf{x}_1,\dots,\mathbf{x}_n,\mathbf{x}_R;\omega,\omega_0)&=\langle m(\mathbf{x}_1)\cdots m(\mathbf{x}_n)\tilde{n}(\mathbf{x}_R,0)\rangle^{\rm cum}\\&=\int dt_1\cdots\int dt_n\langle\tilde{s}(\mathbf{x}_1,t_1)\cdots\tilde{s}(\mathbf{x}_n,t_n)s(\mathbf{x}_R,0)\rangle^{\rm cum}\\&=\langle\tilde{\varphi}(\mathbf{x}_1)\cdots\tilde{\varphi}(\mathbf{x}_n)\varphi(\mathbf{x}_R)\rangle^{\rm cum} ,\end{aligned} \tag{26}$$

where the last cumulant has to be calculated with the frozen functional $\hat{\mathcal{J}}_{\rm fr}$ since it is a zero-frequency correlation.

Now we show that the frozen functional $\mathcal{J}_{\rm fr}$ (24) exhibits a restricted supersymmetry (the full homogeneity and isotropy in the superspace is broken by the surface). We introduce anticoming Grassmann coordinates $\theta,\bar{\theta}(\bar{\bar{\theta}}=-\theta)$ with the usual integration rules $\int d\,\theta\,\theta=\int d\bar{\theta}\,\bar{\theta}=1$, $\int d\theta=\int d\bar{\theta}=0$, supercoordinates $X=(\mathbf{x},\bar{\theta},\theta)$, $[\bar{X}=(\mathbf{x},-\theta,\bar{\theta})]$ with a metric in the $(d+2)$-dimensional superspace given by $\bar{X}\cdot X=\mathbf{x}^2+2\bar{\theta}\theta$, and a corresponding superspace Laplacian $\Delta_{\rm ss}=\Delta+2\partial_{\bar{\theta}}\partial_\theta$. Introducing a superfield

$$\Phi(X)=\varphi(\mathbf{x})+\psi(\mathbf{x})\bar{\theta}+\theta\bar{\psi}(\mathbf{x})+\theta\bar{\theta}\tilde{\varphi}(\mathbf{x}) \tag{27}$$

the frozen dynamic functional can be written as

$$\begin{aligned}\hat{\mathcal{J}}_{\rm fr}&=\mathcal{H}_{\rm ss}\\&=\int d^dx\,d\bar{\theta}\,d\theta\left\{\frac{1}{2}\Phi\left[\tau-\Delta_{\rm ss}+\frac{ig}{6}\Phi\right]\Phi-iH\Phi\right\}\\&\quad+\int d^{d-1}y\,d\bar{\theta}\,d\theta\left\{\frac{c}{2}\Phi_0^2-iH_0\Phi_0\right\} .\end{aligned} \tag{28}$$

This Hamiltonian $\mathcal{H}_{\rm ss}$ explicitly shows the restricted supersymmetry (invariance against super-rotations and -translations) in the $[(d-1)+2]$-dimensional subsuperspace $\{(\mathbf{y},\bar{\theta},\theta|\ \mathbf{y}\in\mathbb{R}^{d-1}\}$. Of course the surface breaks the full supersymmetry in the full superspace $\{\mathbf{x}=(\mathbf{y},z,\bar{\theta},\theta)|\ \mathbf{y}\in\mathbb{R}^{d-1},\ 0\le z<\infty\}$. But the restricted supersymmetry suffices for Cardy's nonperturbative proof of dimensional reduction [19] as long as $d\ge3$. This proof consists of the construction of a Hamiltonian interpolating between the supersymmetric one and the scalar one reduced by two dimensions and shows the independence of the cumulants from the interpolation parameter. In fact, this proof requires supersymmetry only in a (2 + 2)-dimensional subspace with coordinates $Y_\parallel=(\mathbf{y}_\parallel,\bar\theta,\theta)$, $\mathbf{y}_\parallel\in\mathbb{R}^2$. Labeling the complement of $Y_\parallel$ by $\mathbf{x}_\perp=(\mathbf{y}_\perp,z)$, $\mathbf{y}_\perp\in\mathbb{R}^{d-3}$, $0\le z<\infty$, the dimensional reduction reads for the correlation functions

$$\left\langle \prod_\alpha \Phi(\mathbf{x}_{\perp\alpha},Y_{\parallel\alpha}=0)\right\rangle_{\mathrm{ss}}=\left\langle \prod_\alpha \varphi(\mathbf{x}_{\perp\alpha})\right\rangle , \tag{29}$$

where the mean values $\langle\ \rangle_{\mathrm{ss}}$ are calculated with the Hamiltonian $\mathcal{H}_{\mathrm{ss}}$ and $\langle\ \rangle$ with the corresponding scalar Hamiltonian of the theory in $d-2$ dimensions

$$\begin{aligned}\mathcal{H}=2\pi\int d^{d-2}x\left\{\frac{1}{2}\varphi[\tau-\Delta+\frac{ig}{6}\varphi]\varphi-iH\varphi\right\}\\ +2\pi\int d^{d-3}y\left\{\frac{c}{2}\varphi_0^2-iH_0\varphi_0\right\} .\end{aligned} \tag{30}$$

The overall factor 2π can be removed by a simple scale transformation $\varphi\to\sqrt{2\pi}\varphi$, $H\to\sqrt{2\pi}H$, $g\to g/\sqrt{2\pi}$.

To relate the cumulants (26) to quantities of the reduced theory we again use the supersymmetry. Consider the expression

$$\begin{aligned}&C_{M,N}(\mathbf{x}_{1\perp},\ldots\mathbf{x}_{M+N,\perp};Y_\parallel)\\ &\quad=\left\langle\prod_{\alpha=1}^{M-1}\Phi(\mathbf{x}_{\alpha\perp},Y_{\alpha\parallel}=0)\Phi(\mathbf{x}_{M,\perp},Y_\parallel)\right.\\ &\qquad\left.\times\prod_{\beta=M+1}^{M+N}\int d^2y'_\parallel d\bar\theta' d\theta'\Phi(\mathbf{x}_{\beta\perp},Y'_\parallel)\right\rangle_{\mathrm{ss}}^{\mathrm{cum}} .\end{aligned} \tag{31}$$

Supersymmetry demands that $C_{M,N}$ is a function of the $\{\mathbf{x}_\perp\}$ and $\bar Y_\parallel\cdot Y_\parallel=\mathbf{y}_\parallel^2+2\bar\theta\theta$ only. It follows that

$$\begin{aligned}&C_{M,N}(\{\mathbf{x}_\perp\},\bar Y_\parallel\cdot Y_\parallel)\\ &\quad=C_{M,N}(\{\mathbf{x}_\perp\},y_\parallel^2)+2\frac{\partial C_{M,N}(\{\mathbf{x}_\perp\},\mathbf{y}_\parallel^2)}{\partial y_\parallel^2}\bar\theta\theta\end{aligned} \tag{32}$$

and

$$\begin{aligned}\int d^2y_\parallel d\bar\theta d\theta\, C_{M,N}(\{\mathbf{x}_\perp\},\bar Y_\parallel\cdot Y_\parallel)&=-2\int d^2y_\parallel\frac{\partial C_{M,N}(\{\mathbf{x}_\perp\},\mathbf{y}_\parallel^2)}{\partial y_\parallel^2}\\ &=-2\pi\int_0^\infty d\mathbf{y}_\parallel^2\frac{\partial C_{M,N}(\{\mathbf{x}_\perp\},\mathbf{y}_\parallel^2)}{\partial y_\parallel^2}=2\pi C_{M,N}(\{\mathbf{x}_\perp\},0)\end{aligned} \tag{33}$$

[$C_{M,N}(\{\mathbf{x}_\perp\},\infty)=0$ by cluster decomposition of the cumulants]. This yields

$$2\pi C_{M,N}(\{\mathbf{x}_\perp\},0)=C_{M-1,N+1}(\{\mathbf{x}_\perp\},0) . \tag{34}$$

Using the representation (27) of the superfield Φ, from the last equation recursively follows

$$\begin{aligned}&\left\langle\prod_{\alpha=1}^{M}\varphi(\mathbf{x}_{\alpha\perp},\mathbf{y}_{\alpha\parallel}=0)\prod_{\beta=M+1}^{M+N}\int d^2y_\parallel\tilde\varphi(\mathbf{x}_{\beta\perp},y_\parallel)\right\rangle_{\mathrm{ss}}^{\mathrm{cum}}\\ &\quad=(2\pi)^{-N}\left\langle\prod_{\alpha=1}^{M+N}\varphi(\mathbf{x}_{\alpha\perp},\mathbf{y}_{\alpha\parallel}=0)\right\rangle_{\mathrm{ss}}^{\mathrm{cum}}\\ &\quad=(2\pi)^{-N}\left\langle\prod_{\alpha=1}^{M+N}\varphi(\mathbf{x}_{\alpha\perp})\right\rangle^{\mathrm{cum}} ,\end{aligned} \tag{35}$$

where the last expression is defined in the dimensionally reduced scalar theory. Finally from (26) we obtain for the Laplace transformed "layer"-density correlations of the branched polymers rooted at the point $\mathbf{x}_0=(\mathbf{x}_{0_\perp},\mathbf{y}_{0_\parallel}=0)$

$$\begin{aligned}&G^{(1)}(\mathbf{x}_{1_\perp},\ldots,\mathbf{x}_{n_\perp},\mathbf{x}_0;\omega,\omega_0)\\ &\quad=\left\langle\tilde n(\mathbf{x}_0,0)\prod_{\alpha=1}^{n}\int d^2y_\parallel m(\mathbf{x}_{\alpha\perp},\mathbf{y}_\parallel)\right\rangle^{\mathrm{cum}}\\ &\quad\sim\left\langle\prod_{\alpha=0}^{n}\varphi(\mathbf{x}_{\alpha\perp})\right\rangle^{\mathrm{cum}} .\end{aligned} \tag{36}$$

The last cumulant has to be calculated with the Hamiltonian $\mathcal{H}$ (30) and shows the full symmetry in the arguments $\mathbf{x}_{0_\perp},\ldots,\mathbf{x}_{n_\perp}$.

The Hamiltonian $\mathcal{H}$ describes the critical properties of an Ising order parameter in a half-infinite system at the Yang-Lee edge singularity [20]. As usual the form of this Hamiltonian is derived from a Landau-Ginzburg-Wilson functional with surface terms and imaginary bulk and surface fields by shifting the order parameter by an imaginary part and reducing to relevant terms only.

IV. RESULTS FROM THE ONE-DIMENSIONAL ISING MODEL

In the last section we have shown that the problem of d-dimensional branched polymers near an adsorptive wall is mapped to the problem of a half-infinite Ising magnet in imaginary bulk and surface fields at the Yang-Lee singularity in $d-2$ dimensions. Therefore, if we are interested in the three-dimensional polymer problem, we will have to consider the one-dimensional Ising chain with one open end in imaginary fields. This is of course textbook matter, and here we will only report the main results for completeness.

The probability distribution of a chain of $N+1$ spins $\sigma_i=\pm1$, $i=0,\ldots,N$, is defined by

$$\rho(\{\sigma\})=\frac{1}{Z}e^{iH_0\sigma_0}T(\sigma_0,\sigma_1)\cdots T(\sigma_{N-1},\sigma_N)e^{iH_N\sigma_N} \tag{37}$$

with the transfer matrix

$$T(\sigma,\sigma')=e^{K\sigma\sigma'+iH(\sigma+\sigma')/2}$$
$$=\langle\sigma|+\rangle\omega_+\langle+|\sigma'\rangle+\langle\sigma|-\rangle\omega_-\langle-|\sigma'\rangle\ . \tag{38}$$

The eigenvalues are given by

$$\omega_\pm=e^K(\cos H\pm\sqrt{e^{-4K}-\sin^2 H}\ ) \tag{39}$$

and the eigenvector of the eigenvalue ω_+ is

$$\langle\sigma|+\rangle=\sqrt{(1+iM\sigma)/2}\ , \tag{40}$$

where iM denotes the bulk magnetization

$$M=-i\langle+|\hat{\sigma}|+\rangle=-\frac{\partial\ln\omega_+}{\partial H}=\frac{\sin H}{\sqrt{e^{-4K}-\sin^2 H}}$$
$$=1/\sqrt{h} \tag{41}$$

and $\hat{\sigma}$ denotes the matrix $[\begin{smallmatrix}1 & 0\\ 0 & -1\end{smallmatrix}]$. For $N\gg 1$ the partition function is given by

$$Z=\omega_+^N\langle A|+\rangle\langle+|E\rangle\ , \tag{42}$$

where $\langle A|$ and $|E\rangle$ are defined by

$$\langle A|\sigma\rangle=e^{iH_0\sigma}\ ,\quad\langle\sigma|E\rangle=e^{iH_N\sigma}\ . \tag{43}$$

For the magnetization iM_0 at the surface we find

$$M_0=-i\frac{\langle A|\hat{\sigma}|+\rangle}{\langle A|+\rangle}$$
$$=-i\frac{e^{2iH}+i[(1+i\sqrt{h}\ )/(1-i\sqrt{h}\ )]^{1/2}}{e^{2iH}-i[(1+i\sqrt{h}\ )/(1-i\sqrt{h}\ )]^{1/2}}$$
$$=\frac{(\sqrt{h}+\sqrt{h+1})t+1}{(\sqrt{h}+\sqrt{h+1})-t} \tag{44}$$

with $t=\tan H_0$.

Equations (41) and (44) show the one-dimensional bulk Yang-Lee edge singularity for $h=(e^{2K}\sin H)^{-2}-1=0$ and the zero-dimensional surface Yang-Lee edge singularity at the surface transition line $t=\sqrt{h}+\sqrt{h+1}$. The surface-bulk multicritical point is given by the parameter values $h=0$, $t=1$, and the crossover exponent is $\phi=\frac{1}{2}$.

We find the mean values

$$\langle\sigma_k\rangle=\langle A|T^k\hat{\sigma}|+\rangle/(\omega_+^k\langle A|+\rangle)$$
$$=i[1-(\omega_-/\omega_+)^k]M+i(\omega_-/\omega_+)^k M_0\ , \tag{45a}$$

$$\langle\sigma_j\sigma_k\rangle=\langle A|T^j\hat{\sigma}T^{k-j}\hat{\sigma}|+\rangle/(\omega_+^k\langle A|+\rangle)$$
$$=-[1-(\omega_-/\omega_+)^{k-j}]M_jM+(\omega_-/\omega_+)^{k-j} \tag{45b}$$

(with $j\le k$), from which we get the correlation function

$$\langle\sigma_j\sigma_k\rangle^{\text{cum}}=\langle\sigma_j\sigma_k\rangle-\langle\sigma_j\rangle\langle\sigma_k\rangle$$
$$=(\omega_-/\omega_+)^{k-j}(1+M_j^2)\ . \tag{46}$$

With the lattice constant denoted by a, the correlation length is given by $\xi=a[\ln(\omega_+/\omega_-)]^{-1}$. It diverges at the Yang-Lee edge as $1/\sqrt{h}$. The critical properties are universal at the entire singularity edge and therefore independent of the coupling constant K. Thus we simplify all expressions without losing their universal properties by choosing the "low temperature" region $K\gg 1$ keeping $\xi_0=ae^{2K}$ finite. Choosing now ξ_0 as a unit for the length scale, we have a suitable device for performing the continuum limit $a\to 0$. In this limit we find for the correlation length

$$\xi=\xi_0\sqrt{(1+h)/4h} \tag{47}$$

and the relation between H and h follows as $h\approx 2(1-e^{2K}H)$. From (44) and (46) we get the magnetization profile and the correlation function

$$M(z)=(1-e^{-z/\xi})M+e^{-z/\xi}M_0 \tag{48a}$$

$$C(z,z')=e^{-|z-z'|/\xi}[1+M(\min(z,z'))^2]\ , \tag{48b}$$

where z denotes the distance from the surface.

The dissipation-fluctuation theorem

$$\frac{\partial}{\partial H}\langle\sigma_j\rangle=i\left|\langle\sigma_0\sigma_j\rangle/2+\sum_{k=1}^{\infty}\langle\sigma_k\sigma_j\rangle\right| \tag{49}$$

in the continuum limit reads

$$\frac{\partial M(z)}{\partial h}=-\frac{1}{2\xi_0}\int_0^\infty dz'C(z',z)\ . \tag{50}$$

V. SURFACE ADSORPTION OF BRANCHED POLYMERS IN THREE DIMENSIONS

Now we are in the position to calculate exactly many properties of the adsorption transition of branched polymers to a two-dimensional surface of a three-dimensional bulk system. If $d=3$, as a consequence of (35) we find for the layer-integrated correlation functions

$$G^{(1)}(z_1,\ldots,z_n,z_0;\omega,\omega_0)\sim C(z_1,\ldots,z_n,z_0;h,h_0)\ , \tag{51}$$

where $C(\cdots)$ are the correlation functions of the one-dimensional Ising model near the Yang-Lee edge. Therefore the moments defined by (11) are given, as a consequence of (16), by the inverse Laplace transforms $\int_{-i\infty}^{i\infty}d\omega\, e^{N\omega}C(\cdots)$. The integration path has to be chosen so that the singularities of $C(\cdots)$ are to its left side. Equation (21) tells us that ω is a linear function of the magnetic field H which itself is linearly related to h. Thus by a suitable redefinition of the scale of N we get

$$Z_N^{(1)}(z)=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}dh\, e^{hN}M(z)\ , \tag{52a}$$

$$2Z_N^{(1)}(z,z')=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}dh\, e^{hN}C(z,z')\ . \tag{52b}$$

From the magnetization profile and the correlation function (48) we now find the fundamental results

$$Z_N^{(1)}(z)=\frac{1}{\sqrt{\pi N}}\{1+e^{-\zeta^2}[1-2\sqrt{\pi}\Gamma e^{(\Gamma+\zeta)^2}\mathrm{erfc}(\Gamma+\zeta)]\}\ , \tag{53a}$$

$$\begin{aligned}2Z_N^{(1)}(z,z')={}&\mathrm{erfc}(\zeta-\zeta')-2\,\mathrm{erfc}(\zeta)+\mathrm{erfc}(\zeta+\zeta')+4e^{(\Gamma+\zeta)^2-\zeta^2}\mathrm{erfc}(\Gamma+\zeta)\\&+8\Gamma e^{-(\zeta+\zeta')^2}[(\Gamma+\zeta+\zeta')e^{(\Gamma+\zeta+\zeta')^2}\mathrm{erfc}(\Gamma+\zeta+\zeta')-1/\sqrt{\pi}]\ .\end{aligned} \tag{53b}$$

Here the last equation for $Z_N^{(1)}(z,z')=Z_N^{(1)}(z',z)$ is written down for $z\geq z'$ and erfc denotes the complement of the error function: $\mathrm{erfc}(x)=(2/\sqrt{\pi})\int_x^{\infty}\exp(-y^2)dy$. If z is measured in units of ξ_0, the variables ζ and Γ are defined by

$$\zeta=z/\sqrt{N}\ ,\quad \Gamma=(1-t)\sqrt{N}\sim(E_c-E)\sqrt{N} \tag{54}$$

near the critical value E_c of the adsorption energy. From the definition of the crossover exponent ϕ, $\Gamma\sim(E_c-E)N^{\phi}$, we read off

$$\phi=\tfrac{1}{2}\ . \tag{55}$$

The expressions (53) have the same structure as the corresponding quantities in the problem of the surface adsorption of ideal linear polymers as presented in the work of Eisenriegler, Kremer, and Binder [3]. Thus we follow their discussion.

Up to the nonuniversal exponential growth factor λ^N, $Z_N^{(1)}(z)$ is proportional to the (Boltzmann weighted) number of polymer configurations with one monomer rooted in the layer z. For $z\rightarrow\infty$ this quantity changes over to the animal number $\mathcal{A}_N\sim N^{-\theta}\lambda^N$ in the bulk without a wall, and with the exponent $\theta=\frac{3}{2}$ independent of E

$$Z_N^{(1)}(z)\rightarrow Z_N^{\mathrm{bulk}}=1/\sqrt{\pi N}\sim N\lambda^{-N}\mathcal{A}_N\ . \tag{56}$$

In the general case we have to distinguish between the three cases nonadsorbed ($E<E_c$), critical ($E=E_c$), and adsorbed ($E>E_c$). Because $N\gg1$, these cases correspond to the limiting values of the scaling variable $\Gamma\gg1$, $\Gamma=0$, and $-\Gamma\gg1$. We find

$$Z_N^{(1)}(z)=\frac{1}{\sqrt{\pi N}}\begin{cases}(1-e^{-\zeta^2})+\Gamma^{-1}[2\zeta+\Gamma^{-1}]e^{-\zeta^2}\ , & E<E_c\ ,\\(1+e^{-\zeta^2})\ , & E=E_c\ ,\\1+4\sqrt{\pi}|\Gamma|e^{|\Gamma|(|\Gamma|-2\zeta)}\ , & E>E_c\ .\end{cases} \tag{57}$$

For $E<E_c$, $Z_N^{(1)}(z)$ approaches the bulk value $1/\sqrt{\pi N}$ for z large compared to the correlation length $\xi\sim\sqrt{N}$. However, this is not the case for $E>E_c$, where

$$Z_N^{(1)}(z)/Z_N^{\mathrm{bulk}}-1=4\sqrt{\pi}|\Gamma|\exp[(z_0-z)/\Delta z_0]\ , \tag{58}$$

which defines two more length scales

$$\begin{aligned}&\Delta z_0=\tfrac{1}{2}(t-1)^{-1}\sim(E-E_c)^{-1}\ ,\\&z_0=\frac{N}{2}(t-1)\sim N(E-E_c)\end{aligned} \tag{59}$$

with the orders of magnitude $\Delta z_0\ll\xi\ll z_0$. If one moves the root towards the surface, at the characteristic distance $z=z_0$ the physical properties rapidly change within a small interval of width Δz_0 around z_0 from bulk to adsorbed behavior.

The (Boltzmann weighted) number of polymer configurations rooted in the surface follows from the asymptotic form of $Z_N^{(1)}(0)$. These animal numbers for surface-rooted branched polymers can be defined in analogy to the bulk animals by $\mathcal{A}_N(E)\sim\lambda^N Z_N^{(1)}(0)/N$. It follows in the three cases that

$$\mathcal{A}_N(E)\sim\begin{cases}N^{-5/2}\lambda^N\ , & E<E_c\ ,\\N^{-3/2}\lambda^N\ , & E=E_c\ ,\\N^{-1}\lambda'^N\ , & E>E_c\ .\end{cases} \tag{60}$$

In the nonadsorbed case $E<E_c$, we thus find $\mathcal{A}_N(E)\sim N^{-\theta_1}\lambda^N$ with an exponent $\theta_1=\frac{5}{2}=\theta+1$. The last relation between the bulk and the surface exponents was found for all dimensions by De'Bell and co-workers [7] for lattice trees by rigorous arguments. In the critical case $E=E_c$, the animal numbers are $\mathcal{A}_N(E)\sim N^{-\theta_a}\lambda^N$ with $\theta_a=\frac{3}{2}$. In general dimensions below 8, the exponent θ_a depends on the bulk exponent θ and the crossover exponent ϕ. We have derived the relation

$$\theta_a=\frac{d-3}{d-2}(\theta-1)+(2-\phi) \tag{61}$$

using field-theoretic renormalization group methods [21]. In the adsorbed case $E>E_c$, we find $\mathcal{A}_N(E)\sim\mathcal{A}'_N\sim N^{-\theta'}\lambda'^N$ with $\theta'=1$, and a modified growth factor $\lambda'=\lambda e^{(t-1)^2}$. This result shows that the adsorbed branched polymers are indeed quasi-two-dimensional animals. In general θ' is the exponent in $d-1$ bulk dimensions.

We now consider the monomer density $\rho_N(z|z')$ of branched polymers in the layer z that are rooted in the layer z'. In general, we have the relation

$$\int_0^{\infty}dz\,Z_N^{(1)}(z,z')=NZ_N^{(1)}(z')\ ,$$

which follows from the dissipation-fluctuation theorem (50) of the Ising model by inverse Laplace transformation. Therefore the monomer density is correctly normalized by the definition

$$\rho_N(z|z')=Z_N^{(1)}(z,z')/Z_N^{(1)}(z')\ . \tag{62}$$

For $N\gg1$ we find the asymptotic forms of (53b)

$$2Z_N^{(1)}(z,z')=\mathrm{erfc}(\zeta-\zeta')-2\,\mathrm{erfc}(\zeta)+\mathrm{erfc}(\zeta+\zeta')+\begin{cases}4\Gamma^{-1}\{(1-\zeta/\Gamma-\Gamma^{-2}/2)e^{-\zeta^2}-[1-2(\zeta+\zeta')/\Gamma-3\Gamma^{-2}/2]e^{-(\zeta+\zeta')^2}\}/\sqrt{\pi}\,, & E<E_c\,,\\ 4\,\mathrm{erfc}(\zeta)\,, & E=E_c\,,\\ 16\Gamma^2e^{|\Gamma|(|\Gamma|-2\zeta-2\zeta')}\,, & E>E_c\,,\end{cases}\tag{63}$$

if $z>z'$. In particular (57), (62), and (63) yield the asymptotic forms of the monomer density of a surface-rooted polymer

$$\rho_N(z|0)=\sqrt{N}\begin{cases}2e^{-\zeta^2}(\zeta+\Gamma^{-1})\,, & E<E_c\,,\\ \sqrt{\pi}\,\mathrm{erfc}(\zeta)\,, & E=E_c\,,\\ 2|\Gamma|e^{-2|\Gamma|\zeta}\,, & E>E_c\,.\end{cases}\tag{64}$$

The monomer density of the surface-rooted polymer at the wall is generally given by

$$\rho_N(0|0)=\sqrt{N}\,\frac{\sqrt{\pi}e^{\Gamma^2}\mathrm{erfc}(\Gamma)(2\Gamma^2+1)-2\Gamma}{1-\sqrt{\pi}\Gamma e^{\Gamma^2}\mathrm{erfc}(\Gamma)}\,.\tag{65}$$

The limits of this crossover scaling expression for $N\gg1$ are

$$\rho_N(0|0)=\begin{cases}2/(1-t)\,, & E<E_c\,,\\ \sqrt{\pi N}\,, & E=E_c,\\ 2(t-1)N\,, & E>E_c\,.\end{cases}\tag{66}$$

Note from (64) that in the adsorbed case $E>E_c$ the thickness of the polymer at the surface is given by the length $\Delta z_0\sim(E-E_c)^{-1}$. In this case the monomer density is generally given by

$$\rho_N(z|z')=\sqrt{\pi N}\,/2\,\frac{\mathrm{erfc}(|z-z'|/\xi)+16\Gamma^2e^{(z_0-z-z')/\Delta z_0}}{1+4\sqrt{\pi}|\Gamma|e^{(z_0-z')/\Delta z_0}}\,.\tag{67}$$

The monomer distribution of the branched polymer then shows the following behavior: if one moves the root towards the surface, at the distance z_0 the distribution rapidly changes from the symmetric bulk form $\sim\sqrt{N}\,\mathrm{erfc}(|z-z'|/\xi)$ to the practically root independent adsorbed form $\sim N\exp(-z/\Delta z_0)$ within the small interval Δz_0. In the last case, the root is connected to the adsorbed monomers only by a very thin monomer bridge (see Figs. 1, 2, and 3).

Unfortunately, the results from the one-dimensional Yang-Lee problem do not give us any information on the shape of the branched polymer parallel to the surface. But with the help of the formulas (62) and (63), we calculate the characteristic perpendicular lengths $\overline{R_\perp^2}$, $\overline{R}_\perp$ defined in (14) and (15). We find

$$\overline{R_\perp^2}(z)_N=Z_N^{(1)}(z)^{-1}\sqrt{N}\left\{\frac{1}{3\sqrt{\pi}}+\frac{1}{\sqrt{\pi}}(\tfrac{8}{3}\zeta^2-\tfrac{1}{3}+4\Gamma^{-1}\zeta+2\Gamma^{-2})e^{-\zeta^2}-(\zeta+\Gamma^{-1})^3\mathrm{erfc}(\zeta)\right.$$
$$\left.-[\tfrac{4}{3}\zeta^3+\Gamma\zeta^2(2+\Gamma^{-2})+\zeta(2-\Gamma^{-2})+\Gamma^{-1}-\Gamma^{-3}]e^{\Gamma^2+2\Gamma\zeta}\mathrm{erfc}(\Gamma+\zeta)\right\}\,,\tag{68a}$$

$$\overline{R}_\perp(z)_N=[2Z_N^{(1)}(z)]^{-1}\left\{(2\zeta^2+4\Gamma\zeta+2-\Gamma^{-2})e^{\Gamma^2+2\Gamma\zeta}\mathrm{erfc}(\Gamma+\zeta)-\frac{2}{\sqrt{\pi}}(2\zeta+\Gamma^{-1})e^{-\zeta^2}+(\zeta+\Gamma^{-1})^2\mathrm{erfc}(\zeta)\right\}\,.\tag{68b}$$

We first discuss surface-rooted polymers. For $z=0$, (68a) and (68b) reduce to

$$\overline{R_\perp^2}(0)_N=\frac{N}{\Gamma^2}\,\frac{1-\frac{1}{2}\sqrt{\pi}\Gamma^{-1}-\frac{1}{2}\sqrt{\pi}(\Gamma-\Gamma^{-1})e^{\Gamma^2}\mathrm{erfc}(\Gamma)}{1-\sqrt{\pi}\Gamma e^{\Gamma^2}\mathrm{erfc}(\Gamma)}\,,\tag{69a}$$

$$\overline{R}_\perp(0)_N=\frac{\sqrt{\pi N}}{4\Gamma^2}\,\frac{1-2\Gamma/\sqrt{\pi}+(2\Gamma^2-1)e^{\Gamma^2}\mathrm{erfc}(\Gamma)}{1-\sqrt{\pi}\Gamma e^{\Gamma^2}\mathrm{erfc}(\Gamma)}\,,\tag{69b}$$

with the limiting behavior

$$\overline{R_\perp^2}(0)_N=N\begin{cases}1\,, & E<E_c\,,\\ \frac{1}{3}\,, & E=E_c\,,\\ \Gamma^{-2}/2\,, & E>E_c\,,\end{cases}\tag{70a}$$

$$\overline{R}_\perp(0)_N=\sqrt{\pi N}\,/2\begin{cases}1\,, & E<E_c\,,\\ \frac{1}{2}\,, & E=E_c\,,\\ |\Gamma|^{-1}/\sqrt{\pi}\,, & E>E_c\,.\end{cases}\tag{70b}$$

In all three cases, the eccentricity $\overline{R}_\perp(0)_N$ is clearly greater than zero because the surface-rooted polymers can only grow in the bulk direction. Whereas for $E\leq E_c$ the perpendicular thickness is of order $\xi\sim\sqrt{N}$, for

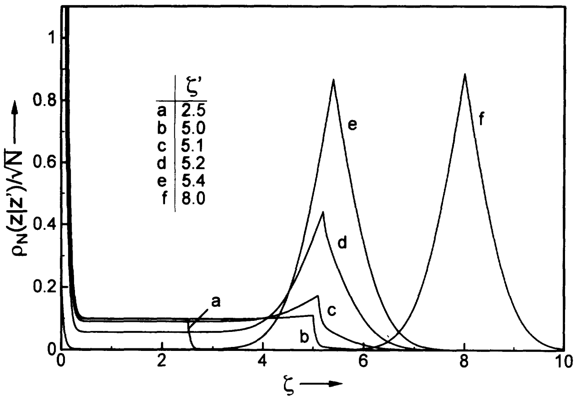

FIG. 1. Plot of the normalized monomer density profile $\rho_N(z|z')/\sqrt{N}$ against the scaled variable $\zeta = z/\sqrt{N}$ in the case of an attractive ($\Gamma = -10$) surface situated at $z=0$ [Eqs. (53) and (62)]. The various drawings show the changeover from nonadsorbed to adsorbed behavior within a small interval if the root ζ' is moving towards the surface.

$E \geq E_c$ this length is suddenly reduced to Δz_0.

We now discuss how the bulk values $\overline{R_\perp^2}(\infty)_N = N/3$, $\overline{R}_\perp(\infty)_N = 0$ approach the limits (70a) and (70b) at the surface. For the nonadsorbing case $E < E_c$, (68a) and (68b) yield

$$\overline{R_\perp^2}(z)_N/\overline{R_\perp^2}(\infty)_N = 1 + \frac{\zeta^2}{e^{\zeta^2}-1}[2 - 3\sqrt{\pi}\zeta e^{\zeta^2}\operatorname{erfc}(\zeta)] \quad (71)$$

with the limiting behavior for $\zeta \ll 1$, $\zeta \gg 1$, respectively,

$$3(1-\sqrt{\pi}\zeta)\,, \quad (1-\zeta^2 e^{-\zeta^2})\,, \quad (72)$$

and

$$2\overline{R}_\perp(z)_N/\sqrt{N} = \sqrt{\pi}\frac{\zeta^2\operatorname{erfc}(\zeta)}{1-e^{-\zeta^2}} \quad (73)$$

with the limiting behavior

$$(\sqrt{\pi}-2\zeta)\,, \quad \zeta e^{-\zeta^2}\,. \quad (74)$$

At the threshold $E = E_c$ we obtain roughly the same behavior as for $E < E_c$,

$$\overline{R_\perp^2}(z)_N/\overline{R_\perp^2}(\infty)_N = 1 + \frac{\zeta}{e^{\zeta^2}+1}[4\zeta - 3\sqrt{\pi}(1+\zeta^2)e^{\zeta^2}\operatorname{erfc}(\zeta)] \quad (75)$$

with the limiting behavior

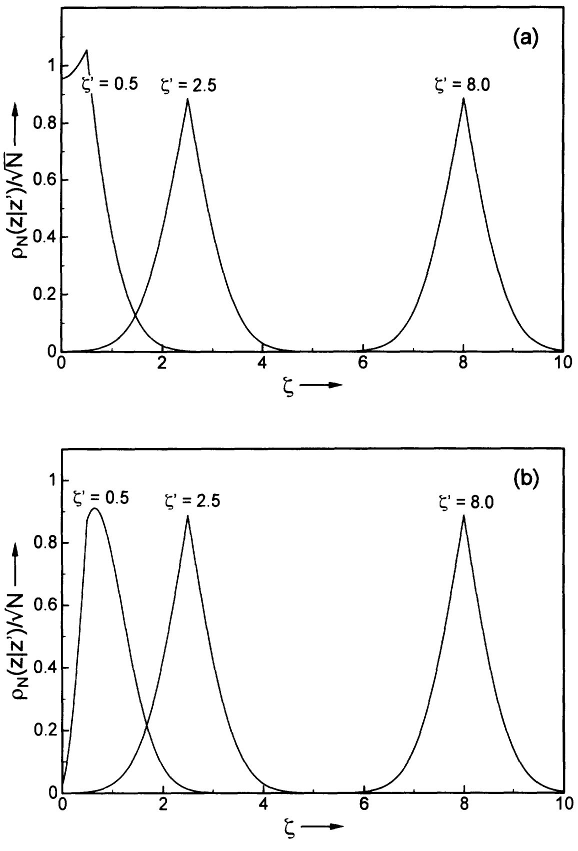

FIG. 2. (a) shows the normalized monomer density profile, Eqs. (53) and (62), versus ζ for different values of the root $\zeta' = (0.5, 2.5, 8.0)$ at the adsorption threshold $\Gamma = 0$. (b) Same as in (a) in the nonadsorbed region $\Gamma = 10$. Compare these drawings with Fig. 1.

$$(1-3\sqrt{\pi}\zeta/2)\,, \quad (1+\zeta^2 e^{-\zeta^2})\,, \quad (76)$$

and

$$2\overline{R}_\perp(z)_N/\sqrt{N} = \sqrt{\pi}\frac{(1+\zeta^2)\operatorname{erfc}(\zeta) - 2\zeta e^{-\zeta^2}/\sqrt{\pi}}{1+e^{-\zeta^2}} \quad (77)$$

with the limiting behavior

$$(\sqrt{\pi}/2-2\zeta)\,, \quad -\zeta e^{-\zeta^2}\,. \quad (78)$$

We turn now to the most interesting adsorbing case $E > E_c$. Then we get from (68)

$$\overline{R_\perp^2}(z)_N/\overline{R_\perp^2}(\infty)_N = \frac{1+6\sqrt{\pi}|\Gamma|\zeta^2 e^{|\Gamma|(|\Gamma|-2\zeta)}\operatorname{erfc}(\zeta-|\Gamma|)}{1+2\sqrt{\pi}|\Gamma| e^{|\Gamma|(|\Gamma|-2\zeta)}\operatorname{erfc}(\zeta-|\Gamma|)}\,, \quad (79a)$$

$$2\overline{R}_\perp(z)_N/\sqrt{N} = \frac{\sqrt{\pi}\zeta^2\operatorname{erfc}(\zeta) - 2\zeta e^{-\zeta^2} + 2\sqrt{\pi}(1+\zeta^2-2|\Gamma|\zeta)e^{|\Gamma|(|\Gamma|-2\zeta)}\operatorname{erfc}(\zeta-|\Gamma|)}{1+2\sqrt{\pi}|\Gamma| e^{|\Gamma|(|\Gamma|-2\zeta)}\operatorname{erfc}(\zeta-|\Gamma|)}\,. \quad (79b)$$

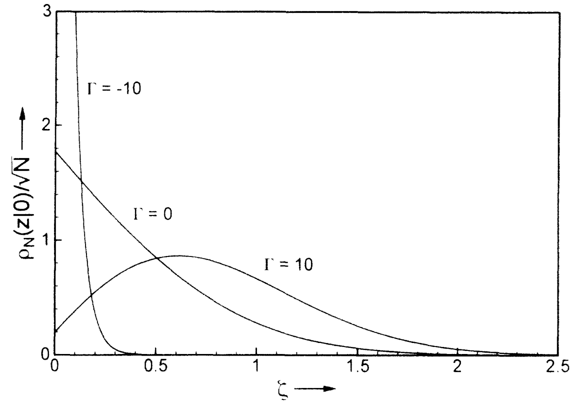

FIG. 3. Density of monomers rooted in the surface $\rho_N(z|0)/\sqrt{N}$ is plotted as a function of scaled variable ζ for nonadsorptive $\Gamma=10$ and critical $\Gamma=0$, as well as for the adsorptive $\Gamma=-10$ case.

The limiting forms are now

$$\overline{R_\perp^2}(z)_N=\begin{cases}\overline{R_\perp^2}(\infty)_N & \text{for } z>z_0 \, ,\\ z^2 & \text{for } \Delta z_0 \ll z<z_0 \, ,\end{cases} \tag{80a}$$

$$\overline{R}_\perp(z)_N=\begin{cases}0 & \text{for } z>z_0 \, ,\\ -z & \text{for } \Delta z_0 \ll z<z_0 \, ,\end{cases} \tag{80b}$$

apart from the rounding region of width Δz_0 around z_0. Since $z_0^2 \gg \overline{R_\perp^2}(\infty)_N=N/3$, the perpendicular length rapidly increase by a factor $\gg 1$ within the rounding region. As already discussed above, the form of (80) suggests that, as the position z of the root is decreased through $z_0 \sim N$, the branched polymer is suddenly stretched and is captured by the adsorbing wall. It remains there as z is further decreased.

As a last point in our consideration of the adsorption phenomena of the single branched polymer at a wall, we calculate properties which should easily be accessible to enumeration or Monte Carlo simulation methods. We consider the number $\mathcal{A}_{N,n}$ of possible configurations of branched polymers made from N monomers with $n \geq 1$ monomers at the surface. The various weighted moments are defined by

$$m_p=\lambda^{-N}\sum_{n=1}^{\infty} n^p \mathcal{A}_{N,n} e^{nE} \, , \tag{81}$$

factoring out the nonuniversal growth factor λ^N. Going over to the continuum description, with the help of (11) we have

$$m_p=Z_N^{(1)}(z_1=0,\ldots,z_p=0) \, . \tag{82}$$

The moments can be derived by successive differentiation with respect to the variable $E-E_c \sim -\Gamma/\sqrt{N}$. The general relation is

$$m_p=(-\sqrt{N})^{p-2}\frac{d^p}{d\Gamma^p}F_N \, , \tag{83}$$

where we have defined a generating function

$$F_N(\Gamma)=Nm_0=e^{\Gamma^2}\mathrm{erfc}(\Gamma) \, . \tag{84}$$

This function can be inferred from (53a) with $z=0$ and (83) with $p=1$. Of course, we have not displayed any nonuniversal factor. Thus one can equally well use redefined quantities $F_N \rightarrow \alpha F_N$, and $m_p \rightarrow \beta^p m_p$, where the constants α and β are arbitrary. Mean values may be defined by

$$\langle n^p \rangle = m_p/m_0 \, . \tag{85}$$

The ratio of the second and the first moment is equal to the monomer density $\rho_N(0|0)$ (65). The asymptotic expressions can be read off from (66). A quantity that is universal up to the scale of the scaling variable $\Gamma \sim (E_c-E)\sqrt{N}$ is defined as

$$\begin{aligned}\mathcal{R}_{31}=\frac{\langle n^3\rangle\langle n\rangle}{(\langle n^2\rangle)^2}&=\frac{[1+\Gamma^2-\sqrt{\pi}\Gamma(\Gamma^2+\frac{3}{2})e^{\Gamma^2}\mathrm{erfc}(\Gamma)][1-\sqrt{\pi}e^{\Gamma^2}\Gamma\,\mathrm{erfc}(\Gamma)]}{[\Gamma-\sqrt{\pi}(\Gamma^2+1/2)e^{\Gamma^2}\mathrm{erfc}(\Gamma)]^2}\\ &=\left\{\begin{matrix}\frac{3}{2}\, , & E<E_c\\ 4/\pi\, , & E=E_c\\ 1\, , & E>E_c\end{matrix}\right\} \text{ for } N\gg 1 \, .\end{aligned} \tag{86}$$

As a last topic, let us now consider properties of dilute solutions of branched polymers. We will follow Eisenriegler's discussion of the case of ideal linear polymers [22]. The monomer density $\rho(z)$ of the solution is given by the monomer density $\rho_N(z|z')$ of the single polymer rooted in the layer z', times the number of polymers with that root and integrated over z'. The normalization has to be chosen so that the density is equal to the bulk density ρ_b if $z \rightarrow \infty$. As a consequence of the dissipation-fluctuation theorem and (62), we get

$$\rho(z)=\sqrt{\pi N}Z_N^{(1)}(z)\rho_b \, , \tag{87}$$

where $\sqrt{\pi N}Z_N^{(1)}(z)$ is given by (53a). The asymptotic forms of the density in the three cases may be inferred from (57) (see Fig. 4). We discuss a few limiting cases. At the adsorption threshold $E=E_c$ the density decreases from twice the bulk density at the surface to the bulk density within a surface layer of the thickness $\xi=\sqrt{N}$. For the case $E<E_c$, we have the limiting behavior near the wall

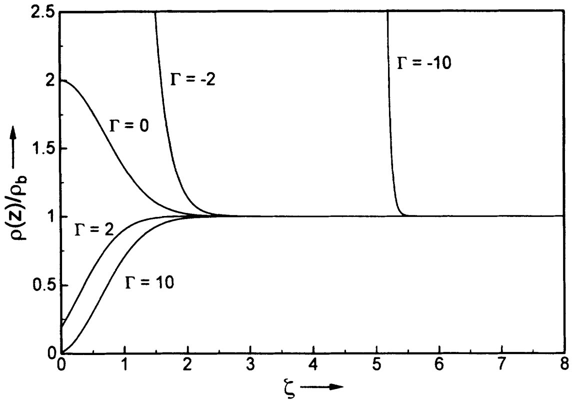

FIG. 4. Monomer density profile in a semidilute solution of branched polymers as a function of the scaled variable ζ [see Eq. (87)] for several values of the parameter Γ as indicated. The density is normalized by the bulk density.

$$\rho(z)/\rho_b=(\zeta+\Gamma^{-1})^2 \, . \tag{88}$$

Thus the density decreases quadratically in z up to a surface layer of thickness Δz_0. This is different from the behavior $\sim(\zeta+\Gamma^{-1})$ for the monomer density of a single chain fixed with one end at the wall in the same limit; see (64). Finally consider the case $E>E_c$. Then the limiting behavior is

$$\rho(z)/\rho_b-1=4\sqrt{\pi}|\Gamma|e^{(z_0-z)/\Delta z_0} \, . \tag{89}$$

On approaching the wall from large z where $\rho=\rho_b$, the density increases dramatically at the distance z_0 from the wall. This phenomenon is of course generated by the fact that the single polymer already senses the surface for $z\approx z_0$. For the monomer density at the surface we arrive at

$$\begin{aligned}\rho(0)/\rho_b&=2-2\sqrt{\pi}\Gamma e^{\Gamma^2}\mathrm{erfc}(\Gamma)\\ &=\begin{cases}\Gamma^{-2}\, , & E<E_c \, ,\\ 2\, , & E=E_c \, ,\\ 4\sqrt{\pi}|\Gamma|e^{\Gamma^2}\, , & E>E_c \, .\end{cases}\end{aligned} \tag{90}$$

Finally, the total monomer excess is defined by

$$\rho_{\mathrm{exc}}=\int_0^\infty dz[\rho(z)-\rho_b] \, . \tag{91}$$

Its explicit form is

$$\begin{aligned}\rho_{\mathrm{exc}}/\rho_b&=\sqrt{\pi N}\,(e^{\Gamma^2}\mathrm{erfc}(\Gamma)-\tfrac{1}{2})\\ &=\sqrt{\pi N}\,/2\begin{cases}-1\, , & E<E_c \, ,\\ 1\, , & E=E_c \, ,\\ 4e^{\Gamma^2}\, , & E>E_c \, .\end{cases}\end{aligned} \tag{92}$$

VI. CONCLUSION

In this paper the statistical properties of large randomly branched polymers in a three-dimensional bulk system near a two-dimensional adsorbing wall were investigated. We started with modeling the branched polymers by an epidemic process and showed by some transformations using supersymmetry that our model can be represented as a semi-infinite Yang-Lee singularity problem in one dimension. Therefore we were able to obtain a lot of exact results for the single polymer and polymer solutions in three dimensions. But we have to admit that with the typical supersymmetric dimensional reduction we lost all information on the structure in layers parallel to the surface.

It would be interesting to compare our results, e.g., with Monte Carlo simulations. We are aware of only two such simulations. In [6] the authors "measure" a type of specific heat $(\langle n^2\rangle-\langle n\rangle^2)/N$ and find a value $\phi=0.714$ for the crossover exponent. In [23] the authors simulate vesicles and "measure" the mean value $\langle n\rangle$ of the number of plaquettes on the surface. This mean value should be proportional to N^{ϕ} at the adsorption transition (N is the number of all plaquettes). The authors find $\phi=0.70\pm0.06$. Both values for ϕ are inconsistent with our result $\phi=\frac{1}{2}$. But in a recent analysis of the comparable case of adsorption of linear polymers Hegger and Grassberger [24] have obtained $\phi=0.496\pm0.005$ for the crossover exponent which is in contrast to earlier results. Their improvements were possible by simulating rather long chains (up to $N=2000$). The authors conclude that all their data show substantial correction to scaling. Thus in the more complicated case of branched polymers and their modeling we cannot trust the values produced in [6,23]. There the comparable values of N are much lower and the authors do not subject their data to any correction to scaling analysis.

In a forthcoming paper we plan to present the mean field theory of our model and the renormalized perturbation theory leading to the ϵ expansion around eight spatial dimensions. Using results from conformal invariance of the Yang-Lee problem we get some information on the four-dimensional branched polymer problem near the surface. From all these results we find reliable extrapolations for the statistical properties of large branched polymers in a two-dimensional bulk system near a one-dimensional adsorbing wall. The exact results for dimensions $d=3$, $d=4$, $d\geq8$, and the $O(\epsilon)$ result in $d=8-\epsilon$ lead to our conjecture on the superuniversality of the crossover exponent

$$\phi=\tfrac{1}{2} \tag{93}$$

in all spatial dimensions.

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