Synchronization effects in the dynamical behavior of elevators

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We simulate the dynamical behavior of M elevators serving N floors of a building in which a Poisson distribution of persons call elevators. Our simulation reproduces the jamming effect typically seen in large buildings when a large number of persons decide to leave the building simultaneously. The collective behavior of the elevators involves characteristics similar to those observed in systems of coupled oscillators. In addition, there is an apparently rule-free critical population density above which elevators start to arrive *synchronously* at the ground floor.

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The macroscopic effect of collective synchronization is a remarkable phenomenon occurring in several fields and is presently a topic of very active research. Effects of collective synchronization can be observed in situations as diverse as, for example, the dynamics of sliding charge-density waves [1], the phase-locking of relativistic magnetrons [2], and many other situations of interest in physics and engineering [3]. In addition, as early observed by Winfree [4], synchronization is a phenomenon that also occurs in virtually all levels of biological organization. Interesting examples of mutual synchronization involving biological phenomena are epileptic seizures in the brain, electric synchrony among cardiac pacemaker cells, flashing synchrony among swarms of fireflies, crickets chirping in unison, synchronization of menstrual cycles in groups of women, and several others [5]. A common way of investigating the dynamics of such collections of oscillators is by assuming a relatively weak coupling K among them together with a random distribution of eigenfrequencies. Increasing coupling from zero one finds incoherent collective behavior up to a critical threshold after which the system starts to behave in a state displaying relatively high degrees of synchronization among the individual oscillators composing the system. A convenient way of dealing analytically with several coupled oscillators is via a model proposed by Kuramoto [6], who assumed the dynamics to be ruled by the set of equations

$$\dot{\theta}_i = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i), \qquad i = 1, 2, \ldots, N. \quad (1)$$

In this equation N is the number of coupled oscillators while θ_i and ω_i are their phase and natural frequency, respectively. Recent progress in the understanding of the dynamical behavior of systems containing relatively few degrees of freedom along with a strong motivation to understand nontrivial temporal and spatial structures has led to much interest in models for spatially extended systems [7]. In several of these investigations, not only the somewhat more traditional mathematical approach of using sets of either ordinary or partial differential equations has been used, but different aspects have been uncovered by also using coupled map lattices and cellular automata [7–9].

The purpose of this paper is to report synchronization effects observed in a computer simulation of the dynamical behavior of M elevators serving N floors of a building. Specifically, we report here the dynamics of the jamming effect occurring at a certain time when a large number of occupants of a building decide to leave it nearly simultaneously or, in other words, the *Feierabend* effect. Macroscopic synchronization effects between elevators were observed under a number of different conditions, all of them variations around a simple set of "working rules" assumed to define the dynamical behavior of the elevators. These rules were chosen in such a way to provide a reasonably fair representation of what one usually gets from not very sophisticated, not computerized elevators. The definition of a set of rules controlling the behavior of elevators is not a completely trivial task since the movement of elevators depends on a quite large number of factors. To start, while one might easily agree on a working definition for *small* and *large* buildings, it is hard to differentiate unambiguously the "transition" from small to large buildings. This question is relevant because elevator rules good for small buildings are not practical for higher buildings and vice versa. In higher buildings, it is common practice to divide the total number of elevators into smaller groups in order to ensure good service for all floors. This division is not unique: one may use some elevators to serve only odd-numbered floors while the others serve even-numbered floors; or one may decide that a number of elevators will serve only the first, say, ten floors, the next will serve floors 10 to 20, and so on. It is not difficult to imagine a number of other possible combinations. In addition to all this, there are a number of specific details that need to be defined in order to optimize the process, i.e., to minimize the time needed for a person to move between floors and to minimize energy consumption.

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In what follows we will assume the ground floor to be the only possible way to exit the building and that all calls for elevators are from passengers wanting to exit by going to this floor. This choice reflects closely the phenomena that we want to consider. Each elevator can carry no more than 20 passengers simultaneously. The specific working rules of the elevators are assumed to be as follows. Whenever free elevators are available, every call originating from a floor from which there were no previous unresolved calls will set the closest free elevator in motion. This will occur regardless of the relative position of the floor originating the call and of any preexisting movement of elevators. After fulfilling the original call that set it moving, every elevator will always try to attend all other calls appearing in the floors lying below it, namely, it will stop at all levels lying below. While in practice one knows that an elevator already full cannot accept new passengers, *real* elevators are frequently not able to recognize and use the information that they are full to avoid stopping unnecessarily at lower floors. We simulate both situations: "naive" elevators, elevators which even after being full will continue to stop for every unfulfilled call from below, and "smart" elevators, elevators that once full will ignore all subsequent calls, moving straight to the lowest floor. Empty elevators arriving at empty floors, i.e., floors that have already been served, or on the ground floor will stop and either wait for a new call or move to floors where there are passengers calling, which could not be served earlier.

The updating of the dynamics is done at regularly spaced time intervals, controlled by the discrete ticking of a clock. At every clock step we assume $p_\mu(n_k)$ new passengers to call elevators from the kth floor, distributed according to a Poisson law

$$p_\mu(n_k) = \mu^{n_k}\frac{e^{-\mu}}{n_k!}, \qquad (2)$$

where μ is the Poisson parameter. At every clock step the location and movement of every elevator will also be updated. The speed of the elevators is one floor per time step, either up or down. Whenever an elevator stops in some floor, it remains at the floor during a few clock units. This time is intended to represent an average time consumed in opening the door, moving passengers and closing the door. In the present simulations we assumed this time to be five clock units. In absence of any new call, elevators will either preserve their previous state of movement or stop, if they reach the first floor. Otherwise, elevators will react to new calls as described above. To investigate the dynamical behavior as a function of μ we perform simulations over relatively long time intervals (typically $t = 5000$ clock steps) accumulating several quantities at every clock step, e.g., the total number of elevators that arrive at the ground floor, the number of times that L elevators arrive *simultaneously* at the ground level, the cumulative number of passengers transported, the number of passengers waiting in each queue, etc.

Based on these assumptions we wrote a FORTRAN program to simulate the dynamics of a few different buildings

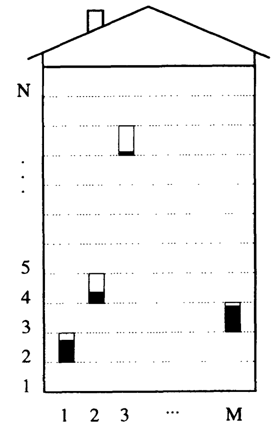

FIG. 1. Schematic representation of a typical building. The black shading of each elevator is proportional to the number of passengers that it carries.

with several possible number of elevators. Simulations were done for different values of the Poisson parameter μ. Apart from storing the aforementioned relevant quantities describing the process, the program could also be run interactively, generating a postscript on-screen animation of the dynamics, which could be easily video recorded. An schematic snapshot of a video frame is given in Fig. 1. The shading of each individual elevator is proportional to the number of passengers that it carries. Further (not shown in the schematic figure), the actual number of passengers waiting on every floor was displayed on the left of the rectangle representing the building by a line segment with length proportional to the population density. Line segments on the right side indicated the *cumulative* number of passengers transported since the beginning of every run. The majority of runs was done for $M = 10$ elevators and $N = 50$ floors. For numerical values of this order, the position, the population density inside every elevator (their shading) and the population density outside every floor (line segments) may be comfortably updated on screen at every time step, producing an actual movie of the dynamics. In this way we are always able to visualize simultaneously the up-and-down movement of all elevators and to monitor the behavior of the population

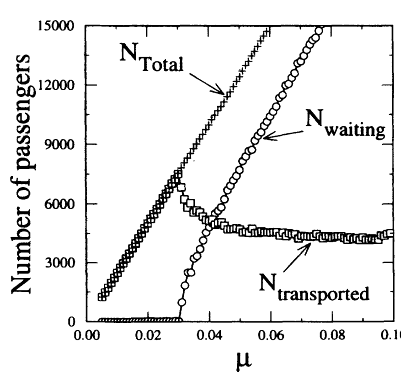

FIG. 2. Accumulated population density after 5000 time steps. Above $\mu \simeq 0.03$ the transport is not efficient anymore.

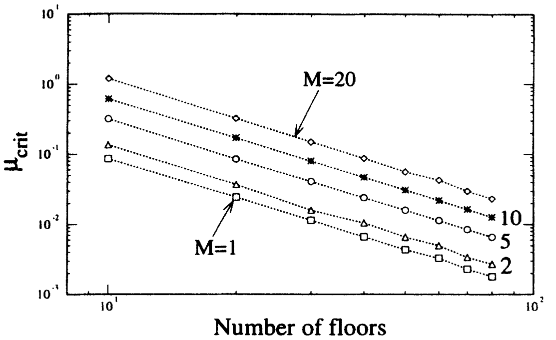

FIG. 3. Critical values of μ as functions of the number of floors and number of elevators. As seen in Fig. 2, beyond μ_{crit} the transport is not efficient anymore. The characteristic exponent underlying all curves is $\mu \simeq N^{-1.88}$.

inside every elevator and on the several waiting queues. By monitoring the number of passengers in the waiting queues and inside elevators, it was possible to recognize many interesting effects such as, for example, when the number of elevators was well adapted to the size of a building, to the population density living in the building, etc. Simulation of a particular building consisted essentially of "playing the elevator game" for different values of μ and observing the dynamics on screen while cumulating important numbers for later analysis.

Figure 2 shows for μ between 0 and 0.1 the number of passengers transported and waiting elevators after 5000 clock steps. One clearly sees the existence of an operational "jamming threshold." For μ slightly above the *critical* value $\mu_c \simeq 0.03$ the ten elevators are not able to cope with the traffic anymore: the number of waiting passengers increases rapidly while the number of passengers transported tends rapidly to the maximum limit afforded by the capacity of the total number of elevators. Figure 3 shows a plot the critical values of μ obtained by simulation the dynamics of buildings containing $10 \leq N \leq 80$ floors and served by either $M = 1, 2, 5, 10$, or 20 elevators. As one sees from the graph, the behavior on a log-log plot may be very well fit by straight lines. By changing the size of the building, the capacity of the elevators and/or the number of elevators serving the building one obtains curves displaying qualitatively identical behaviors. For example, for $N = 50$, we find $\mu_{crit} \simeq 0.003\,12 \cdot M$ for $1 \leq M \leq 25$. A typical recording of the time evolution of *all* elevators is shown in Fig. 4, which displays simultaneously the movement of all ten elevators as a function of time. Each line in this graph represents the actual trajectory followed by an individual elevator. The ten lines are not individually discernible in Fig. 4: for $\mu = 0.01$ because the ten elevators move in an uncorrelated way with their individual trajectories significantly overlapping each other, and for higher μ values because

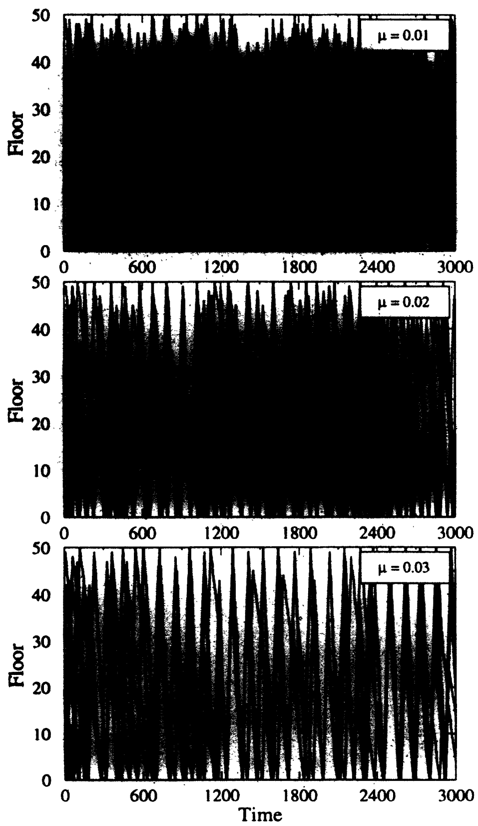

FIG. 4. Up-and-down movement of ten elevators as a function of time for three different Poisson distributions of calls. In this and subsequent figures, the time is measured in arbitrary "clock" units, used to update the dynamics.

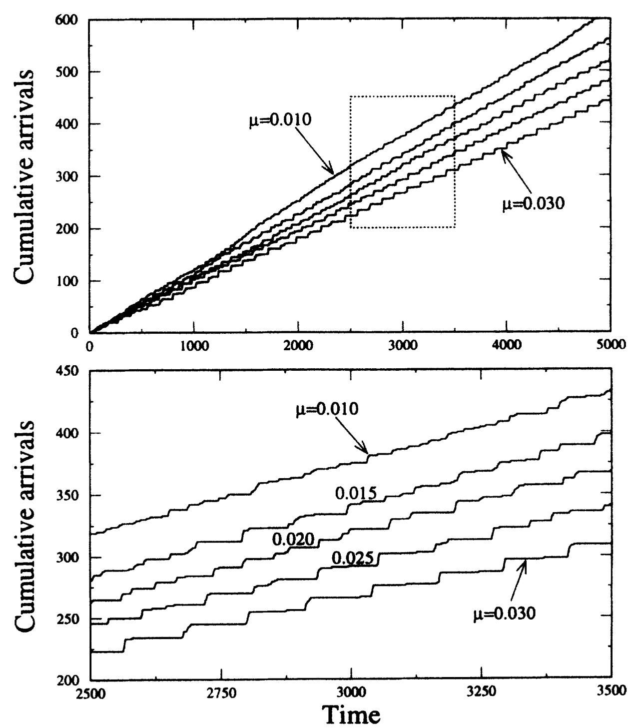

FIG. 5. Time evolution of the cumulative number of elevators arriving on the ground floor. The lower picture is a magnification of the portion inside the rectangle in the upper one.

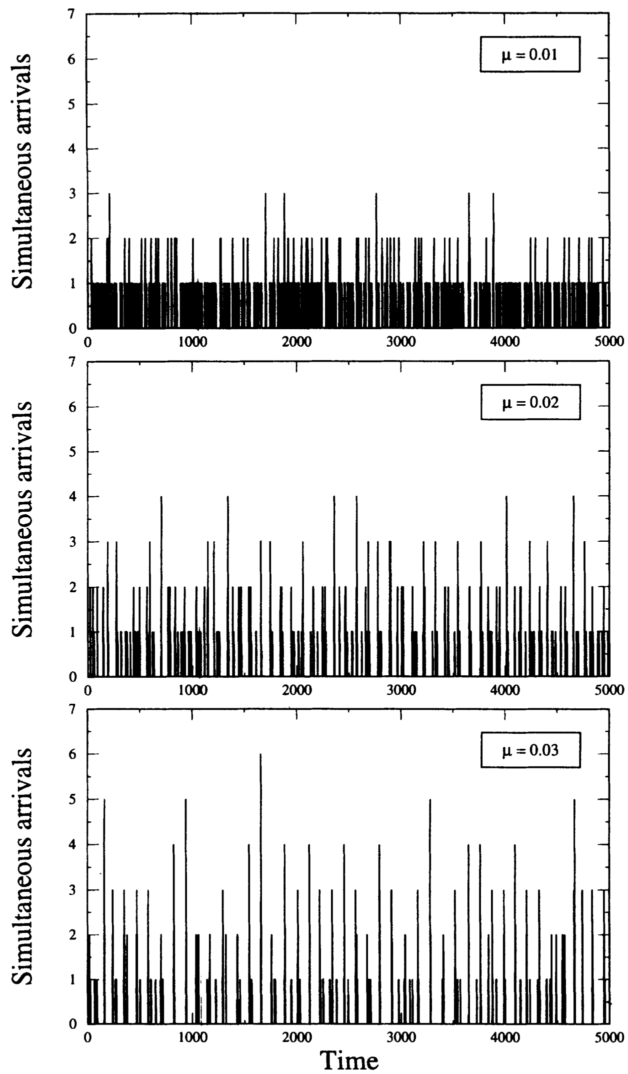

FIG. 6. Histograms showing the number of simultaneous arrivals as a function of time and population density.

they then synchronize. The point of the figure is to show that for μ values of the order of $\simeq 0.1$ the elevators move always stochastically, i.e., independently of each other, reflecting essentially the randomness of the arrival of calling passengers in each floor. As μ increases one sees the appearance of white intervals along the lowest level.

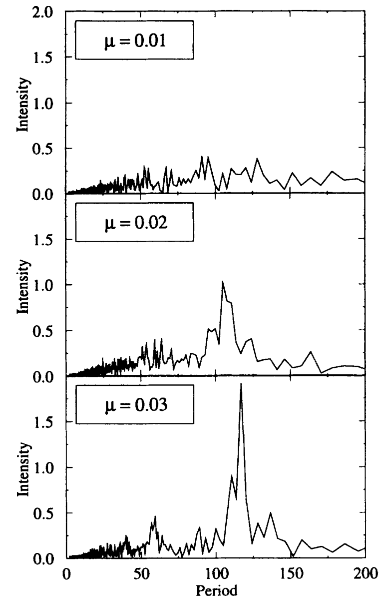

FIG. 7. Fourier transform for three of the time series given in Fig. 5, as indicated by the value of μ.

Such intervals represent *quiescent* periods, when all elevators are busy elsewhere. Thus, as μ increases there are sequences of roughly periodic time intervals where the majority of elevators *bunch* at the lowest quota (ground level), indicating a relatively high synchronization of the arrivals on the ground floor as the number of calls increases.

An alternative way to recognize the synchronization is by monitoring the time evolution of the cumulative number of elevators arriving at the ground floor. This time evolution is recorded in Fig. 5. For small population densities one observes essentially a linear increase in arrivals vs time. For higher densities the number of arrivals increases discontinuously, roughly periodically, via "quantum" jumps. These increases occur in the relatively short time intervals when several elevators arrive simultaneously. The smallest (and barely visible) vertical jump corresponding to the arrival of a *single* elevator may be more easily recognized from the curve corresponding to the lowest density, i.e., $\mu = 0.01$. The synchronization and its corresponding characteristic frequency may

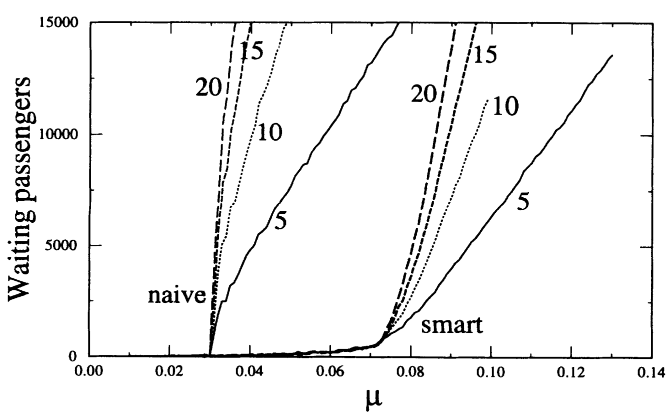

FIG. 8. Total number of waiting passengers for both *naive* and *smart* elevators. Numbers refer to the elapsed time after which measurements were done, in units of 1000 clock steps.

be also recognized from Fig. 6. As a function of time, this figure shows the number of coincidental arrivals at the ground floor for three different values of μ. For low μ, the most salient feature is that elevators tend to arrive independently, as indicated by the large number of single events recorded. As μ increases, there is a redistribution, with many more elevators arriving simultaneously. For $\mu = 0.03$, near the jamming threshold, one may already recognize visually the presence of an underlying periodicity in arrivals. With respect to Fig. 6, it is important to notice that the figure gives the total number of elevators arriving at every single clock step. This is a very stringent definition of synchronization because it only considers as "simultaneous" those arrivals happening at exactly one *single* time step. A more tolerant characterization would very likely involve considering as "simultaneous" all arrivals occurring within a time window larger than a single clock unit. The increase in the number of quasi-simultaneous arrivals seen in Fig. 6 is responsible for the regularly spaced jumps appearing in Fig. 5.

To recognize quantitatively the frequencies underlying the regularities seen in the spacings of Fig. 6 we computed Fourier transforms for the time series shown in Fig. 5. Three such transforms are given in Fig. 7, for values of μ as indicated. From these figures one recognizes the appearance of a clear peak as μ increases, giving the characteristic frequency with which elevators synchronize among themselves.

So far, all results presented were obtained assuming "naive" elevators, i.e., elevators that while going down will stop for every call originating from floors below them, even after being full. Although this is a quite common behavior in real elevators, it is of interest to investigate what happens if elevators are smart enough to notice when they are full and use this information to avoid unnecessary stops. This question is important because full elevators will not further contribute to the transport and their excessive stops will only lead to a decrease of the average speed and to a bunching at the highest elevator velocity (versus floor coordinate) gradient. From these heuristic considerations one may expect synchronization to disappear for smart nonstopping elevators. Figure 8 shows the evolution of the total number of passengers waiting for elevators as a function of μ for both types of elevators. The four sets of curves display the total of passengers waiting for elevators after the indicated number of clock steps, divided by 1000. It is clear that beyond $\mu_{\rm crit}$ the transport is not efficient anymore and the number of passengers waiting will diverge if one waits long enough. These curves are given to indicate how this divergence sets in four both types of elevators. Notice the differences in the behavior around $\mu_{\rm crit}$. The differences may be also recognized from Fig. 9, which shows on a single plot two sets of curves: the total number of passengers transported after 5000 clock units and the total number of arrivals at the bottom of the building.

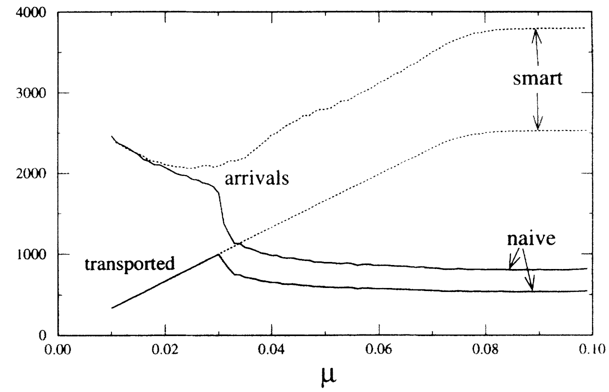

FIG. 9. Total number of transported passengers and total number of arrivals at the ground floor for both rules, naive and smart, measured after 5000 clock steps.

As expected, one sees that smart elevators have larger $\mu_{\rm crit}$, being therefore able to provide a much more efficient transport. We also made plots similar to Fig. 5 to check that synchronization is still happening. The figure obtained is similar to Fig. 5 with smart elevators showing synchronization over a very small μ interval.

One interesting aspect of the simulation is the way in which the asymptotic number of transported passengers is reached as μ increases. While the asymptotic number of transported passengers is clearly limited by the number of elevators and their capacity, it is somewhat surprising to find for naive elevators that as μ increases, this limit is attained after an "overshoot" of the number of transported passengers occurring exactly at the jamming threshold. As clearly seen from Fig. 2, rather than converging monotonically to the saturation level dictated by the number and capacity of the elevators, the most efficient transport occurs around the jamming threshold where one finds a marked discontinuity in the derivative of the number of transported passengers as a function of μ. Beyond the jamming threshold, the simulations indicate the existence of two basic behaviors while passengers start to accumulate in all floors, waiting for elevators that tend more and more to appear full. Immediately after the jamming threshold there is a "relaxation interval" of μ values characterized by negative derivatives in the plots of passengers waiting vs μ. After this regime the number of transported passengers becomes independent of μ. All in all, the best compromise from the point of view of the passengers wanting to leave the building using naive elevators is to plan the building to operate at the jamming threshold. In contrast, for smart elevators there is no advantage in operating at the jamming threshold.

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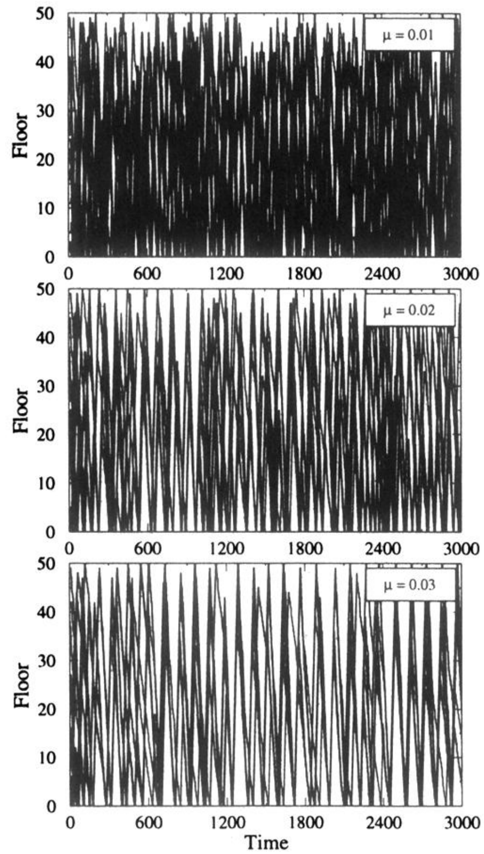

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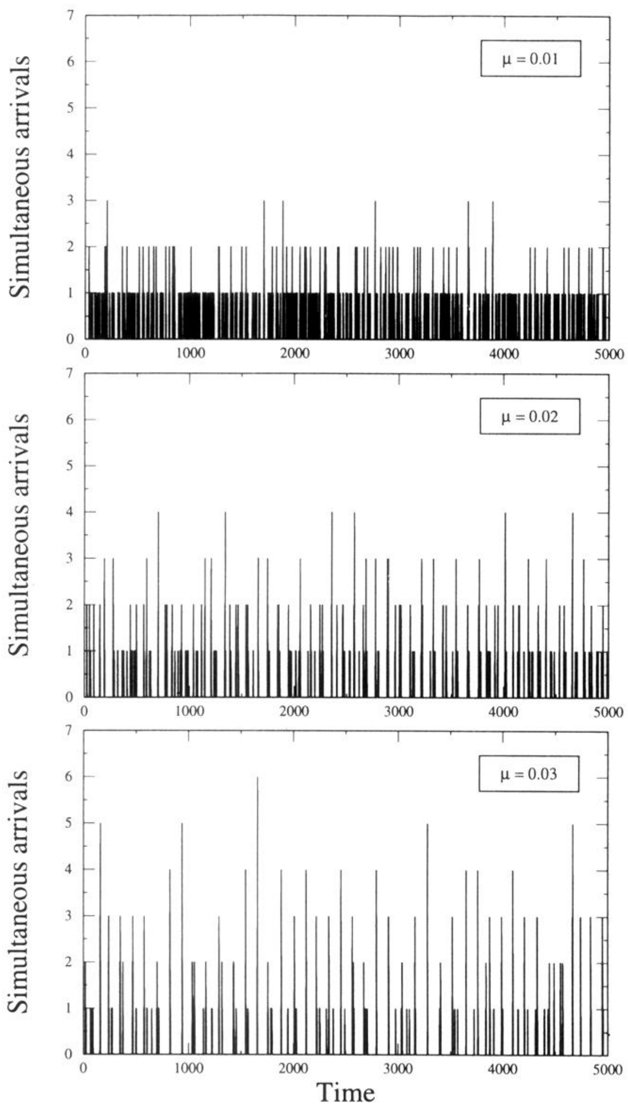

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