Prerecorded history of a system as an experimental tool to control chaos

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We present experimental results of synchronizing the current state of a chaotic system with its prerecorded history. This is achieved by a small self-controlling feedback perturbation in the form of the difference between the current state of the system and its past dynamics. The perturbation transforms an unpredictable chaotic behavior into a predictable chaotic or periodic motion via stabilization of unstable, aperiodic, or periodic orbits of the strange attractor. One advantage of the method is its robustness against noise. Furthermore, it does not require any analytical knowledge of the system dynamics and can be simply implemented in experiment by a purely analog technique. The experimental results are supported by a numerical analysis of the conditional Lyapunov exponents and other characteristics of the model equations.

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I. INTRODUCTION

Control of chaotic systems has recently received increased attention. The idea of using an aperiodic perturbation to achieve the desired behavior of a nonlinear system has been formulated by Hübler and Lüscher [1]. The form of this perturbation depends on a goal function defining the desired aim to be achieved. If the equations governing the system dynamics are known, the optimal driving force can be derived from a variational method. In some cases, this force is similar to the transients of the unperturbed system and can be found experimentally from a previous transient response. A nonlinear oscillator, for example, reacts most sensitively, if perturbed by a driving force which mimics the time-reversed dynamics of the unperturbed system.

Another case of driving systems with an aperiodic signal has been considered by Pecora and Carroll [2]. They have investigated the synchronization effect in a chaotic system which can be decomposed into a drive and a response subsystem. As an example of such a compound system one can take any chaotic system, as driving system, and a copy of a part of it as the responding system. To characterize synchronization effects, Pecora and Carroll have introduced the conditional Lyapunov exponents for the response system. They have shown that a necessary condition to synchronize a chaotic system is that all these exponents have to be negative.

For both methods, one has to know the underlying equations, in order to construct the aperiodic perturbation acting upon the system behavior. Moreover, the driving force is typically large and is applied to the system as a perturbation without any feedback, i.e., it does not depend on the current state of the response system. The subject of controlling chaotic systems has become most popular since the paper of Ott, Grebogi, and Yorke (OGY) [3] was published. They have demonstrated that one can convert the motion of a chaotic system to a periodic motion by using only a small feedback perturbation which acts on an accessible system parameter. Their idea is based on stabilizing unstable periodic orbits (UPO), which are dense in a typical strange attractor. The method does not require any knowledge of the system equations and has been successfully applied in various physical experiments, including a magnetic ribbon [4], a spin-wave system [5], a chemical system [6], an electric diode [7], laser systems [8], and cardiac systems [9].

To use the OGY method, one has to analyze the reaction of the system onto a variation of the perturbed control parameter. The method and its modifications [6–13] are discrete in time, since they deal with the Poincaré map of the system and, therefore, they are sensitive to noise [3]. Recently, one of us has suggested an alternative method based on a time-continuous self-controlling feedback with a small perturbation [14–16]. This method is able to stabilize UPO's [14] as well as aperiodic orbits (AO's) [15,16]. The perturbation is applied to the system in such a way that it remains unperturbed for a dynamics on the desired orbit (UPO or AO). There is only a change in the Lyapunov exponents so that this orbit becomes stable. Three different forms of the perturbation, including the method of delayed feedback [14] and external force control [14–17], have been suggested and tested numerically for various chaotic systems. The efficiency of the delayed feedback control has been recently illustrated experimentally for a high-frequency chaotic oscillator, using a simple analog control circuit [18]. In this paper, we present results of the experimental realization of the external force control. This control is based on a synchronization of the current state of the system with its prerecorded history using conventional feedback [19]. To some extent, it is similar to the synchronization done by Pecora and Carroll. However, here, as well as in the OGY method, the stabilization of the system is achieved by a small feedback perturbation which can be

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constructed without any knowledge of the system equations. Unlike the OGY method, no computer analysis of the system dynamics is required, and the method can be implemented in an experiment by a purely analog technique. A similar approach was already applied by Ref. [20], but there it was used to stabilize the steady state of a chaotic system.

The rest of the paper is organized as follows. The method and the experimental system under investigation are described in Secs. II and III, respectively. The experimental results of stabilizing AO's and UPO's are considered in Secs. IV and V, and a conclusion is presented in Sec. VI.

II. THE METHOD

Let us consider a chaotic system which can be described by a set of ordinary differential equations [14–16]:

$$\begin{aligned} \dot{y} &= P(y, \boldsymbol{x}) + F(y,t), \\ \dot{\boldsymbol{x}} &= \boldsymbol{Q}(y, \boldsymbol{x}). \end{aligned} \tag{1}$$

It is supposed that Eq. (1) is unknown, but some scalar variable $y(t)$ can be measured as a system output. Vector $\boldsymbol{x}$ describes the remaining variables of the system that are not available or are not of interest for observation. $F(y,t)$ denotes an external perturbation. Following [14–16], we consider two forms of this perturbation corresponding to the stabilization of AO's and UPO's, respectively.

The AO's of the strange attractor can be stabilized by a negative feedback perturbation of the form [15,16]

$$F(y,t) = K\{y_{\mathrm{ap}}(t) - y(t)\}. \tag{2}$$

Here $y_{\mathrm{ap}}(t)$ is a segment of a time trace on an AO corresponding to the previous system behavior and K is an adjustable weight of the perturbation. The experiment has to be carried out in two stages. In the first preparatory stage, a segment of the aperiodic output signal $y_{\mathrm{ap}}(t)$ of the unperturbed ($F = 0$) system has to be recorded in a memory. In the second stage, the system can be forced to repeat exactly the recorded signal $y_{\mathrm{ap}}(t)$ after a transient process, when the perturbation is switched on. The main feature of this perturbation is that it does not change the particular solution of the system corresponding to the recorded signal $y_{\mathrm{ap}}(t)$: $F(y,t) = 0$ at $y = y_{\mathrm{ap}}(t)$. Adjusting the weight K, the stabilization of this particular solution can be achieved with an extremely small perturbation. This stabilization can be interpreted in terms of Pecora and Carroll's method of synchronization [2]. The prerecorded signal $y_{\mathrm{ap}}(t)$ in Eq. (2) can be replaced by the output signal of an additional, identical chaotic system. The latter one can be seen as the driving system, and the original one can be interpreted as a response system [16]. Thus the problem of stabilizing a prerecorded AO is equivalent to the problem of synchronizing two identical chaotic systems. This permits the use of the conditional Lyapunov exponents introduced by Pecora and Carroll, as a criterion of stabilization.

In Ref. [16], the effect of synchronizing the current state of a chaotic system with its prerecorded history is suggested to be used as an alternative approach in the field of forecasting. The small perturbation can transform an unpredictable chaotic behavior into a predictable one via changing the maximal initially positive Lyapunov exponent into a negative one.

The stabilization of the UPO's can be achieved by using the perturbation of the form [14]

$$F(y,t) = K\{y_i(t) - y(t)\}. \tag{3}$$

Here, $y_i(t)$ is the periodic solution $y_i(t) = y_i(t + T_i)$ of the system corresponding to the ith UPO, T_i is its period. The periodic signal $y_i(t)$ can be reconstructed from the chaotic output signal $y(t)$, using the standard method of delay coordinates [21]. In this paper, we emphasize a new aspect in the problem of UPO stabilization. We demonstrate that some of the UPO's can be stabilized without applying the procedure of reconstruction. This is possible due to the fact that the UPO's are robust [22]. They vary slowly with smooth parameter changes, although the attractor can undergo a dramatic change with alternating periodic and chaotic regimes. The sudden changes in the asymptotic behavior of chaotic systems are related to the change of the stability of the periodic orbits and not to the change of their form. Therefore a stable periodic state of the system can be recorded in a memory and then used as an external signal $y_i(t)$ in the perturbation (3) to stabilize the UPO in the neighboring chaotic states, corresponding to a small variation of the control parameter.

III. THE MODEL

We have used an electronic autonomous chaos oscillator suggested by Shinriki *et al.* [23] to test the method at a real experimental system. The circuit is shown in Fig. 1, while the equations of state are

$$\begin{aligned} C_1\dot{V}_1 &= V_1(\tfrac{1}{R} - \tfrac{1}{R_1}) - f(V_1 - V_2) + I_C, \\ C_2\dot{V}_2 &= f(V_1 - V_2) - I_3, \\ L\dot{I}_3 &= -I_3R_3 + V_2, \end{aligned} \tag{4}$$

where V_1, V_2, and I_3 are the voltage across the capacitor

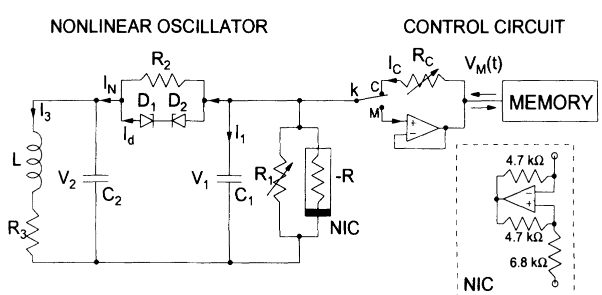

FIG. 1. The scheme of the nonlinear oscillator and the control circuit. At the lower right a scheme of the NIC is plotted. The variable resistor R_1 is a precise potentiometer fixed by hand. The operational amplifier is of the type TL071 biased with ± 15 V. D_1 and D_2 are 3.3 V zener diodes BZX55C 3V3.

C_1, the voltage across the capacitor C_2, and the current through the inductor L, respectively. The negative impedance converter (NIC) has been constructed on the basis of the operation amplifier TL071. It can be approximated by a linear resistor with the negative resistivity $-R$ in the working range of the voltage ΔV. The parallel resistor R_1 compensates the negative slope of the NIC and serves as a basic control parameter of the oscillator. The only nonlinearity in the system has its origin in two zener diodes. Their current versus voltage characteristic can be approximated as follows:

$$I_d \equiv f_d(V) = \begin{cases} 0, & \mid V \mid < V_d \\ \mathrm{sgn}(V)\{A\Delta V^3 + B\Delta V^4 + C\Delta V^5\}, & \mid V \mid \geq V_d \end{cases} \tag{5}$$

where $V_d = 2.5$ V, $A = 2.2500$, $B = -1.9460$, $C = 0.8188$, $\Delta V = \mid V \mid - V_d$, and $\mathrm{sgn}(V) = \pm 1$ for $V > 0$ and $V < 0$, respectively. The voltage V is given in volts, and the current I_d is given in milliamps. The parallel resistor R_2 serves to vary the shunt of the capacitor C_1 and the resonant circuit LC_2:

$$I_N \equiv f(V) = f_d(V) + \frac{V}{R_2}. \tag{6}$$

Note that a similar oscillator where the diodes are removed and the nonlinearity is carried in the NIC is known as Chua's circuit or double-scroll oscillator [24].

In our experiment, we have fixed the parameters as follows: $C_1 = 10$ nF, $C_2 = 100$ nF, $L = 270$ mH, $R = 6.91$ kΩ, $R_2 = 12$ kΩ, $R_3 = 100$ Ω. R_1 was used as a control parameter. Figure 2(a) shows the experimental bifurcation diagram of the unperturbed ($I_C = 0$) system. With increasing R_1, the system behavior develops via the following scenario: Stable fixed point (I), period

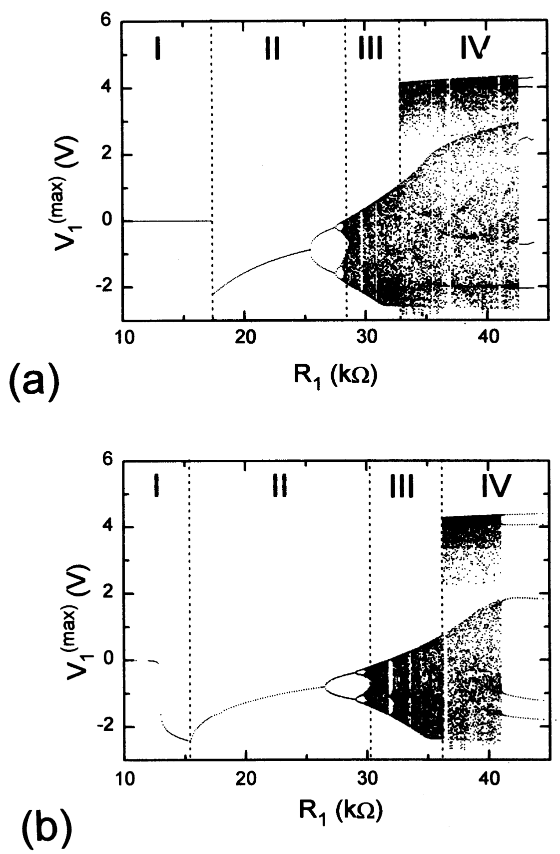

FIG. 2. (a) Calculated and (b) experimental bifurcation diagrams. The distribution of the maxima of the voltage V_1 is plotted as a function of the control parameter R_1. Roman numerals mark distinct regimes: (I) stable fixed point, (II) period doubling scenario, (III) mono-scroll chaos, (IV) double-scroll chaos.

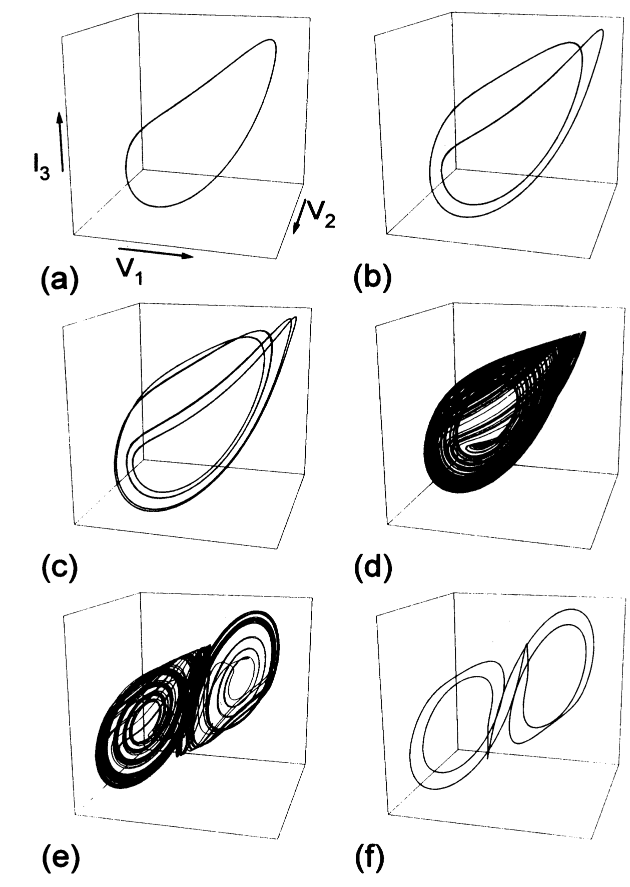

FIG. 3. Typical experimental phase portraits of the system corresponding to different regimes: (a) period-1 cycle taken from region II in Fig. 2 ($R_1 = 24.8\,\mathrm{k}\Omega$), (b) period-2 cycle from region II ($R_1 = 27.8\,\mathrm{k}\Omega$), (c) period-4 cycle from region II ($R_1 = 29.0\,\mathrm{k}\Omega$), (d) mono-scroll chaos from region III ($R_1 = 33.8\,\mathrm{k}\Omega$), (e) double-scroll chaos from region IV ($R_1 = 39.8\,\mathrm{k}\Omega$), (f) double-scroll cycle of period 5 from region IV ($R_1 = 41.8\,\mathrm{k}\Omega$).

doubling scenario (II), mono-scroll (Rössler type) chaos (III), double-scroll chaos (IV). The typical experimental phase portraits corresponding to these distinct states are presented in Fig. 3. We have also calculated the bifurcation diagram using Eqs. (4)–(6) and the above values of parameters [Fig. 2 (b)]. The calculated and the experimental bifurcation diagrams are in good qualitative agreement. The quantitative difference can be explained by an extreme sensitivity of the system to small differences of the parameters, particularly to a small variation of the function (5).

The control of the oscillator has been achieved by a simple control circuit shown in Fig. 1. The key k is used to switch the circuit from the data-collecting regime M to the control regime C. In the control regime, this circuit provides the required form of the control current:

$$I_C = \frac{V_M(t) - V_1}{R_C} \equiv K\{V_M(t) - V_1\}. \qquad (7)$$

Here, $V_M(t)$ is the prerecorded output signal. The control resistor R_C determines the weight of the perturbation ($K \equiv 1/R_C$).

IV. STABILIZATION OF APERIODIC ORBITS

An experimental control of AO's is very easy. At first k (Fig. 1) is switched into position M, to record a chaotic signal $V_M^{(\mathrm{ap})}(t)$ of arbitrary length in the memory. Afterwards, k is switched to the position C, to synchronize the system with this prerecorded AO. The results of such a synchronization for two distinct chaotic regimes (mono-scroll and double-scroll) are presented in Fig. 4. Before activating the control, the system generates a signal different to that recorded in the memory. The magnitude of the difference $\Delta V(t) = V_1(t) - V_M^{(\mathrm{ap})}(t)$ is of the order of the output signal $V_1(t)$. This means there is no correlation between these signals. After activating the control, this difference decreases rapidly to a small value. The system starts to repeat exactly its previous behavior corresponding to the recorded signal $V_M^{(\mathrm{ap})}(t)$. After a transition process, the control current $I_C = \Delta V/R_C$ is extremely small, $I_C \approx 2\ \mu\mathrm{A}$ (root mean square value), compared with the total current I_1 through capacitor C_1, $I_1 \approx 3$ mA. Therefore the synchronization of the system with its prerecorded history is provided by a very small control signal with a relative value less than 0.1%.

For some experimental situations the large transient value of perturbation can be undesired. There are two possibilities to solve this problem. First, as in the OGY method, the perturbation can be switched on at the moment when the system comes close to the state corresponding to the beginning of the recorded trajectory. Second, one can introduce in the feedback a limitation of the perturbation. As shown in [14–16] a stabilization in this case is also possible but this leads to a longer transient.

In the experiment, the synchronization was possible for $R_C \leq 10$ kΩ ($K > 0.1$ kΩ^{-1}). For values of R_C larger than 100 kΩ ($K < 0.01$ kΩ^{-1}), no synchronization can be observed. The threshold of synchronization can be determined numerically from the variational equations:

$$\begin{aligned} C_1\delta\dot{V}_1 &= \left(\frac{1}{R} - \frac{1}{R_1}\right)\delta V_1 - f'(V_1^{(\mathrm{ap})} - V_2^{(\mathrm{ap})}) \\ &\quad \times(\delta V_1 - \delta V_2) - K\delta V_1, \\ C_2\delta\dot{V}_2 &= f'(V_1^{(\mathrm{ap})} - V_2^{(\mathrm{ap})})(\delta V_1 - \delta V_2) - \delta I_3, \\ L\delta\dot{I}_3 &= -R_3\delta I_3 + \delta V_2. \end{aligned} \qquad (8)$$

Here, $\delta V_1 = V_1 - V_1^{(\mathrm{ap})}(t)$, $\delta V_2 = V_2 - V_2^{(\mathrm{ap})}(t)$, $\delta I_3 = I_3 - I_3^{(\mathrm{ap})}(t)$ are the small deviations from the AO $\{V_1^{(\mathrm{ap})}(t), V_2^{(\mathrm{ap})}(t), I_3^{(\mathrm{ap})}(t)\}$ and f' denotes the derivative of the function (6). These equations define the conditional Lyapunov exponents [2,16]. At $K = 0$, these exponents coincide with the usual Lyapunov exponents

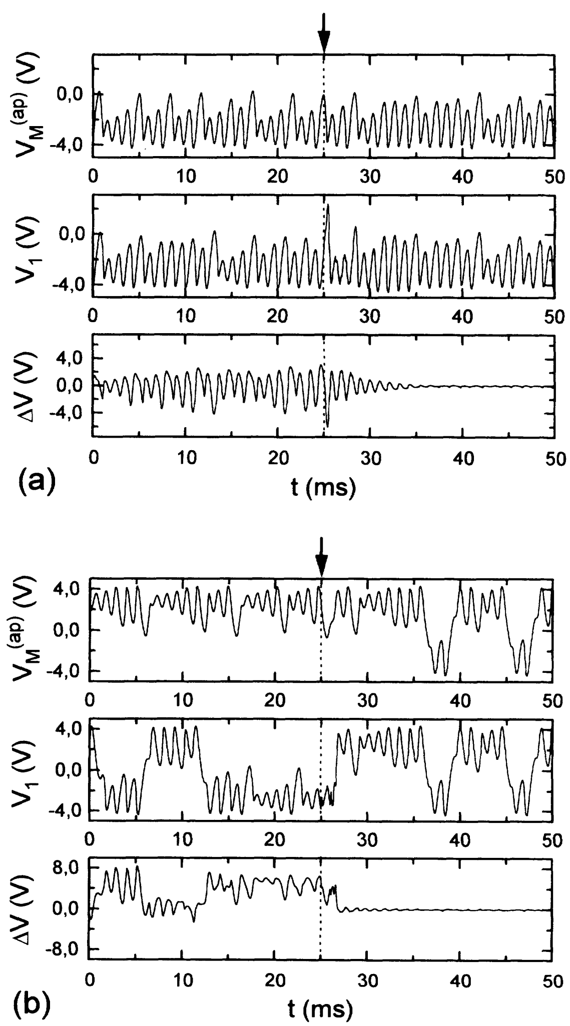

FIG. 4. Time traces of the recorded aperiodic output signal $V_M^{(\mathrm{ap})}(t)$, the dynamics of the output signal V_1, and the difference $\Delta V = V_1 - V_M^{(\mathrm{ap})}$ for mono-scroll chaos regime (a) ($R_1 = 33.8\,\mathrm{k}\Omega$, $R_c = 10\,\mathrm{k}\Omega$) and double-scroll chaos regime (b) ($R_1 = 39.8\,\mathrm{k}\Omega$, $R_c = 10\,\mathrm{k}\Omega$). The arrows and the dashed lines mark the moment of switching onto the control.

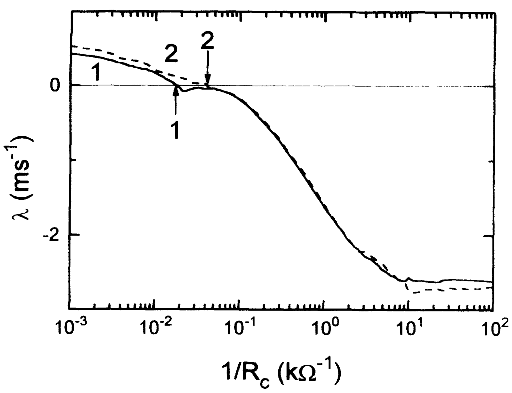

FIG. 5. The maximal conditional Lyapunov exponent λ calculated from Eq. (9) as a function of the reciprocal of the control resistor R_C. Curves 1 and 2 correspond to mono-scroll chaos ($R_1 = 32\,\mathrm{k}\Omega$) and double-scroll chaos ($R_1 = 40\,\mathrm{k}\Omega$), respectively. The arrows show the thresholds of the synchronization.

of the unperturbed system ($\lambda = 0.50\,\mathrm{ms}^{-1}$ for the mono-scroll regime at $R_1 = 32\,\mathrm{k}\Omega$ and $\lambda = 0.63\,\mathrm{ms}^{-1}$ for the double-scroll regime at $R_1 = 40\,\mathrm{k}\Omega$). As well as usual Lyapunov exponents, they do not depend on the initial conditions of AO, because they are averaged along the whole strange attractor. Figure 5 shows the dependence of the maximal conditional Lyapunov exponent on K for two distinct chaotic regimes (mono-scroll and double-scroll). This characteristic defines the operation range of the method. The synchronization is possible only in the intervals of K where the maximal conditional Lyapunov exponent is negative [$\lambda(K) < 0$] and, therefore, a threshold of synchronization is defined by $\lambda(K) = 0$. The calculated threshold values are in agreement with those obtained in experiment. However, the calculated values of the Lyapunov exponent remain small in some interval beyond the threshold (Fig. 5). The rapid decrease of $\lambda(K)$ begins at the characteristic value of $K \approx 0.1\ \mathrm{k}\Omega^{-1}$. Because of that, the experimental threshold of synchronization may be shifted towards greater values of K.

For the results presented in Fig. 4, we have used an analog/digital converter (100 kHz sampling rate, 16 bit resolution, and record length of 200 000 samples), a computer, and a digital/analog converter (same specification) as a memory element to record the AO's to be stabilized. In our experiment, we also have replaced the computer by a tape recorder, and have obtained similar results. Moreover, besides the driving with the prerecorded data, we have driven the system with a replica of the system and with the numerical data calculated from Eq. (3). In both cases, we have achieved the synchronization, but with a larger control signal ($\leq 1\%$, relative value) compared to the case above.

V. STABILIZATION OF UNSTABLE PERIODIC ORBITS

The UPO's of a chaotic system can be stabilized by using the perturbation (3). The external periodic signal $y_i(t)$ can either be reconstructed from a chaotic output signal or it can be obtained directly from the experiment with a shifted value of the control parameter at which the corresponding periodic orbit $y_i(t)$ is stable.

We start the synchronization with the reconstructed UPO's. An asymptotic return map of our oscillator is nearly one dimensional (Fig. 6). This permits a simple procedure to reconstruct the UPO's. The period-i UPO's can be obtained from the intersection points of the ith order return map $V_1^{(n+i)}$ versus $V_1^{(n)}$ with the identity line $V_1^{(n+i)} = V_1^{(n)}$ ($V_1^{(n)}$ denotes the nth maximum of a time trace of the voltage V_1). The results of stabilizing the period-1 and period-8 UPO's using the described procedure of reconstruction are presented in Fig. 7. Even higher periodic orbits were stabilized by an extremely small control signal, the relative value of which in the posttransient regime did not exceed 0.1%.

In order to investigate the influence of noise on the control, we have connected a white noise generator (HP 3562 with a frequency span of 100 kHz) in series with the memory element. The dependence of the control current on the noise amplitude at two different values of the control

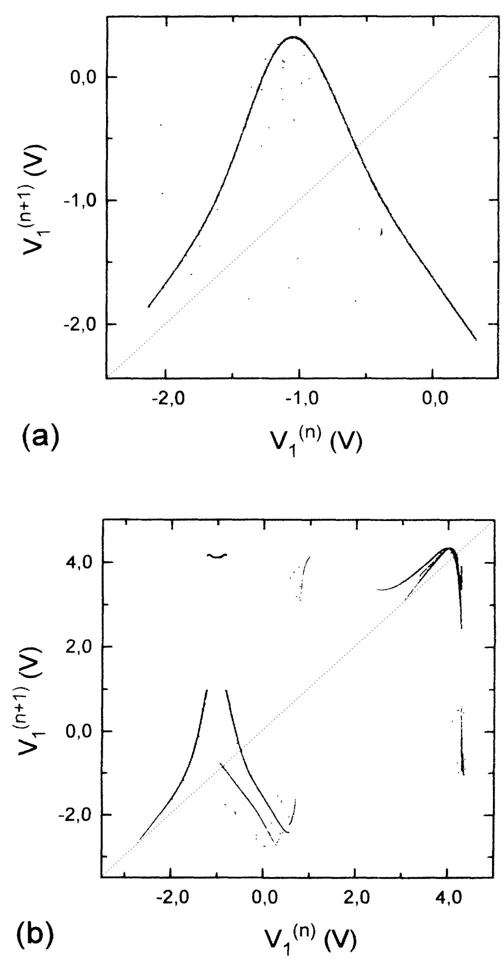

FIG. 6. Experimental return maps for the mono-scroll chaos regime (a) ($R_1 = 33.8\,\mathrm{k}\Omega$) and the double-scroll chaos regime (b) ($R_1 = 39.8\,\mathrm{k}\Omega$).

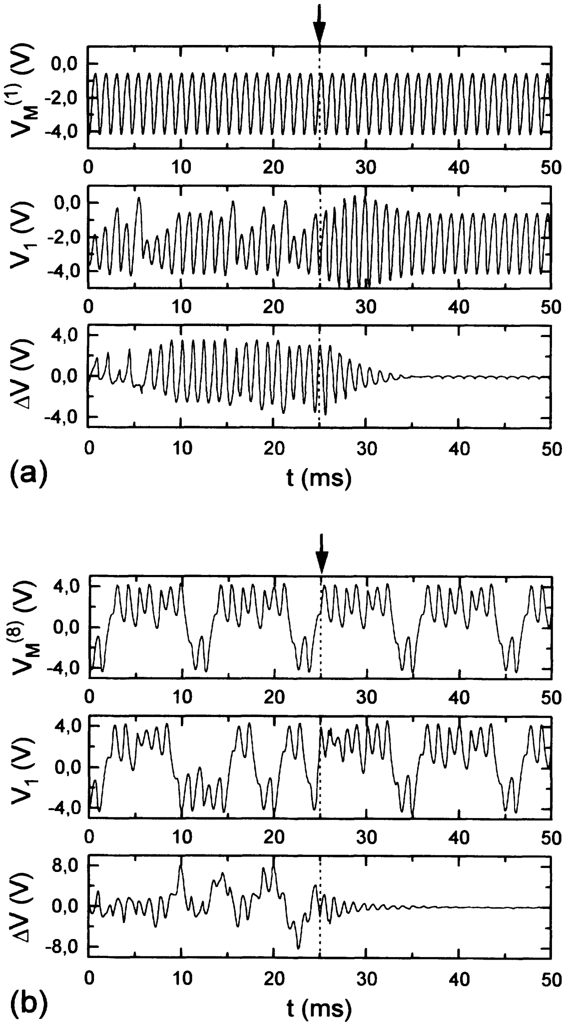

FIG. 7. Experimental results of stabilizing a period-1 orbit (a) (R_1 = 33.8 kΩ, R_c = 10 kΩ) and a period-8 orbit (b) (R_1 = 39.8 kΩ, R_c = 10 kΩ). The upper curves $V_M^{(i)}$ represent the reconstructed UPO's, the middle ones the output signal V_1 of the system, and the lower ones the difference ΔV between these two.

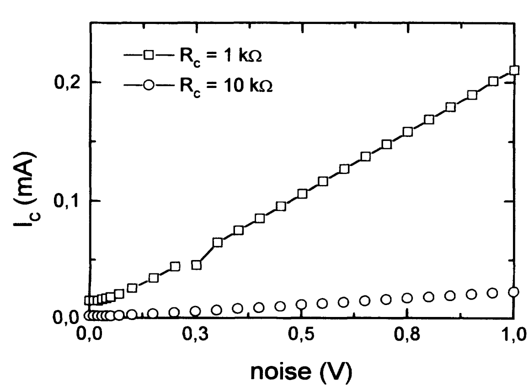

FIG. 8. Observed dependence of the control current I_C on the amplitude of noise at two different values of the control resistor R_C (R_1 = 33.8 kΩ).

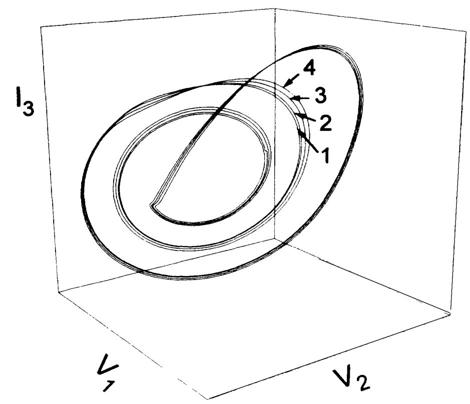

FIG. 9. Evolution of the period-3 orbit calculated from the unperturbed ($I_C = 0$) Eq. (5). The cycle (1) corresponds to the stable state of the orbit (R_1 = 29.6 kΩ) and the remaining cycles represent reconstructed unstable periodic orbits: (2) R_1 = 30 kΩ, (3) R_1 = 31 kΩ (4) R_1 = 32 kΩ.

resistor R_C is presented in Fig. 8. The control tracks the system onto the desired orbit, even at noise comparable to the amplitude of the signal V_1. An increase in noise leads only to an increase of the control current I_C and to a blurred periodic orbit: $\Delta V = V_1 - V_M^{(i)} = I_C R_C$.

The UPO's which are stable at certain values of the control parameter R_1 can be stabilized without examining the procedure of reconstruction. This approach is based on the assumption that the periodic orbits vary gradually with the change of the control parameter. An example supporting this assumption is presented in Fig. 9. The period-3 orbit is shown for four different values of the control parameter, including the regions where it is a stable and an unstable one. Indeed, this orbit varies slightly in the chaotic regime close to the period-3

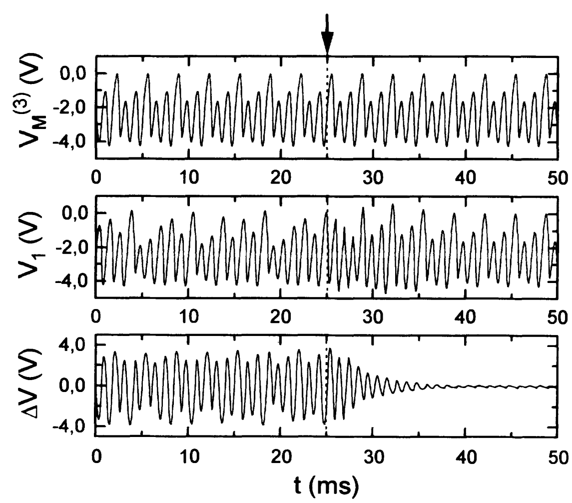

FIG. 10. Experimental results of stabilizing period-3 (R_1 = 31.4 kΩ, R_c = 10 kΩ) without using the procedure of reconstruction. The upper curve $V_M^{(3)}$ represents the recorded output signal corresponding to the mono-scroll period-3 window for a slightly shifted value of the control parameter.

window. This permits the use of the stable periodic orbits recorded inside the periodic window instead of the reconstructed UPO's. Figure 10 displays the results of stabilizing the period-3 UPO in the chaotic regimes close to the period-3 window making use of the recorded time trace of the system corresponding to this window. The above periodic window is obvious in the bifurcation diagram which is presented in Fig. 2(b). In this case, the relative value of the control current was less than 0.2%.

VI. CONCLUSION

We have implemented a recently proposed method of chaos control [14–16] in an experimental system. The advantage of the method is its simple realization by the synchronization of the current state of the system with its pre-recorded history. It permits the stabilization of unstable periodic orbits as well as aperiodic orbits of a chaotic system by an extremely small perturbation. Therefore it is not necessary to know any equation which describes the system or to analyze the system. In addition, the method only needs a small perturbation to achieve the synchronization. Furthermore, we were able to demonstrate the robustness of the method against noise, which is necessary for the control of many systems. A particularly interesting application of this method can be expected in the field of forecasting. Using only a small perturbation, an unpredictable chaotic dynamics can be transformed into a predictable one via synchronizing the system with its prerecorded history.

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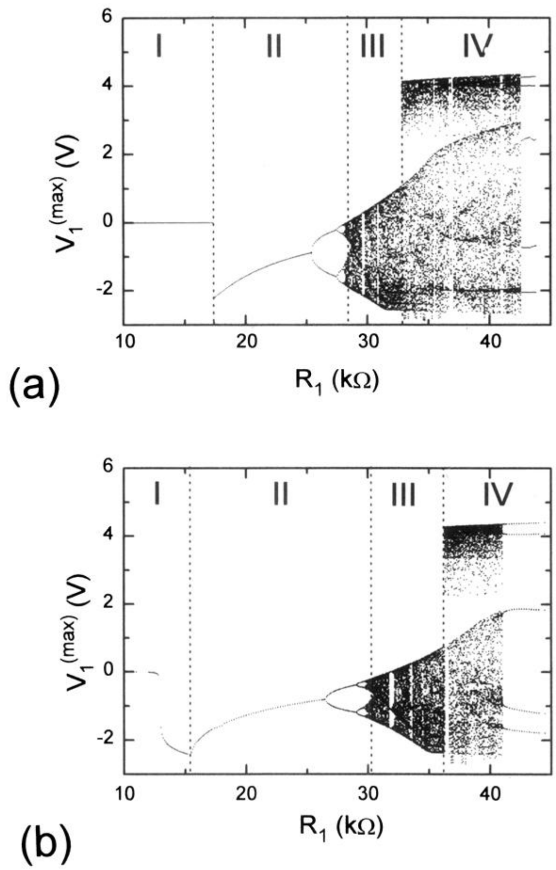

FIG. 2. (a) Calculated and (b) experimental bifurcation diagrams. The distribution of the maxima of the voltage V_1 is plotted as a function of the control parameter R_1. Roman numerals mark distinct regimes: (I) stable fixed point, (II) period doubling scenario, (III) mono-scroll chaos, (IV) double-scroll chaos.

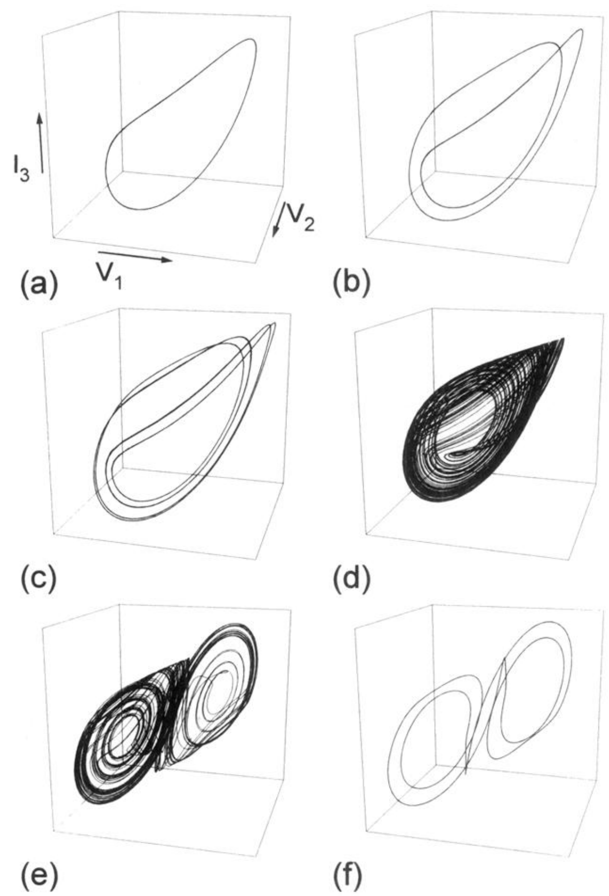

FIG. 3. Typical experimental phase portraits of the system corresponding to different regimes: (a) period-1 cycle taken from region II in Fig. 2 ($R_1 = 24.8\,\mathrm{k\Omega}$), (b) period-2 cycle from region II ($R_1 = 27.8\,\mathrm{k\Omega}$), (c) period-4 cycle from region II ($R_1 = 29.0\,\mathrm{k\Omega}$), (d) mono-scroll chaos from region III ($R_1 = 33.8\,\mathrm{k\Omega}$), (e) double-scroll chaos from region IV ($R_1 = 39.8\,\mathrm{k\Omega}$), (f) double-scroll cycle of period 5 from region IV ($R_1 = 41.8\,\mathrm{k\Omega}$).