$1/f$ noise in the β decay of ^{90}Sr-^{90}Y

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The β^- decay statistics of the radioactive ^{90}Sr-^{90}Y source below 546 keV has been studied to search for the existence of $1/f$ noise. The presence of a flicker floor in a graph of relative Allan variance versus inverse mean count $\langle M_T \rangle^{-1}$ shows the existence of $1/f$ noise.

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INTRODUCTION

The spontaneous emission of charged particles in radioactivity can be considered to be a current flowing. This current should show fluctuations due to random emission. Any emission process is a scattering event in one way or other. In such a process, infraquanta are generated by bremsstrahlung. Handel's quantum theory of $1/f$ noise [1] applies to such a situation and accordingly one should expect a $1/f$ type fluctuation in the particle current. This fluctuation would of course be over and above the well known Poissonian behavior of radioactivity.

As the radioactive current is very weak, one takes recourse to a series of successive measurements to determine the noise spectral density of the flux fluctuations. Allan variance [2] comes in handy for such an analysis. The Allan variance theorem [3] connects the noise spectral density with the Allan variance. The Allan variance $A(T)$ is given by

$$A(T)=[1/2(N-1)]\sum_{i=1}^{(N-1)}|M_T^i-M_T^{i+1}|^2\ , \qquad (1)$$

where N is the number of measurements taken for analysis, M_T^i is the number of counts recorded for a period T, and i refers to the ith such measurement. Therefore $A(T)$ can be calculated by a series of successive count measurements M_T^i.

Relative Allan variance $R(T)$ is defined as [4]

$$R(T)=A(T)/\langle M_T \rangle^2\ , \qquad (2)$$

where $\langle M_T \rangle$, the mean count, is given by

$$\langle M_T \rangle=(1/N)\sum_{i=1}^{N} M_T^i\ . \qquad (3)$$

The mean count can also be written as equal to $m_0 T$ where m_0 is the mean count rate. For a fluctuation having both shot noise and $1/f$ noise, it has been shown that [3]

$$R(T)=[1/\langle M_T \rangle]+F\ , \qquad (4)$$

where F is the flicker floor, defined as

$$F=4Z^2\zeta\alpha A\ln(2)/k\ ,$$

where Z is the charge of the particles, ζ is the coherence factor, α is the fine structure constant, $A=2/3\pi\{1-[1+E_\beta/m_e c^2]^{-2}\}$, E_β is the kinetic energy of the β particles, $m_e c^2$ is the rest energy of β particles, and k is the dielectric constant of the material of the source. We see that a log-log plot of $R(T)$ versus $\langle M_T \rangle^{-1}$ should tend to a constant value F as $\langle M_T \rangle \to \infty$.

EXPERIMENTAL METHOD

In the present study, the decay characteristics of the radioactive isotopes ^{90}Sr-^{90}Y were studied with regard to its noise performance. ^{90}Sr decays to ^{90}Y by 100% β emission with an end-point energy of 546 keV. It has a half-life of 28.6 yr. The daughter nucleus ^{90}Y is also radioactive, emitting β particles of end-point energy 2279 keV with a half-life of 64.1 h. In the present study we have considered only the spectrum up to 540 keV which is a mixture of β particles due to the decay of both ^{90}Sr and ^{90}Y.

Figure 1 shows the experimental setup for β counting.

TABLE I. Presently obtained values of F. The source is ^{90}Sr-^{90}Y. The end-point energy of ^{90}Sr is 546 keV.

Kinetic energy of the β^- particles E_β (in keV)	Experimental flicker floor 10^5F	$A=(2/3\pi)[1-(1+E_\beta/m_e c^2)^{-2}]$	$10^4F/A$
450	2.6	0.1522	1.7
495	2.4	0.1575	1.5
540	2.8	0.1620	1.7

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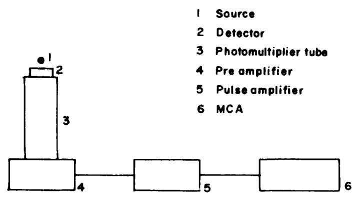

FIG. 1. Experimental setup for the β counting of ^{90}Sr-^{90}Y.

An NE-102 plastic scintillator was mounted on a photomultiplier. Good optical coupling between the two was achieved by using silicone oil. Pulses from the preamplifier of the photomultiplier are fed to a linear amplifier type PA-521 [Electronic Corporation of India Limited (ECIL), Hyderabad, India] and are then analyzed by a multichannel analyzer type MCA-38 (ECIL). To obtain the mixed spectrum, the upper discriminator level of the MCA was set such that pulses beyond 550 keV were not allowed. Several sets of spectral data were taken with counting time intervals ranging from 5 to 1000 min.

The instruments were switched ON at least one hour before the actual start of the experiment. The MCA was calibrated by taking the Compton electron spectrum pro-

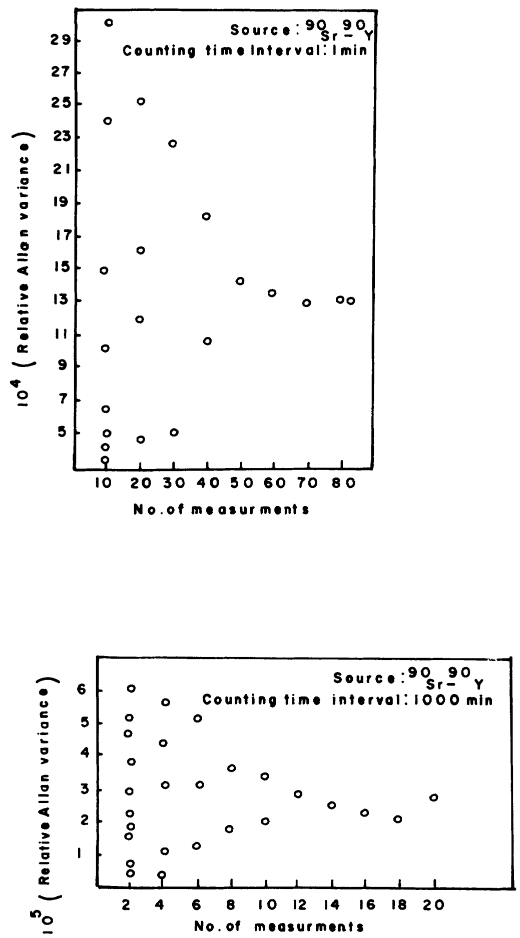

FIG. 2. Relative Allan variance vs number of measurements for two different counting time intervals, for the β counting of ^{90}Sr-^{90}Y.

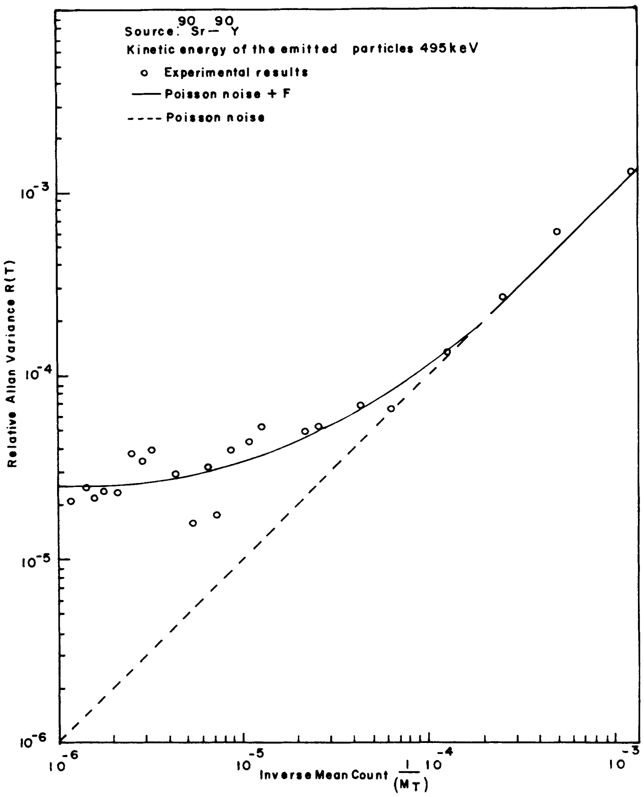

FIG. 3. Relative Allan variance $R(T)$ vs inverse mean count $\langle M_T \rangle^{-1}$.

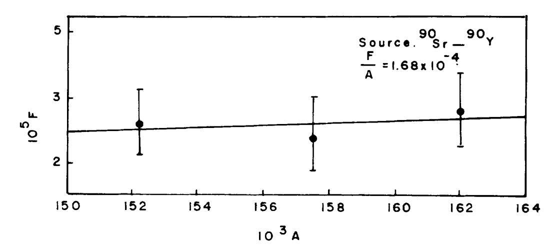

FIG. 4. F vs A for ^{90}Sr-^{90}Y.

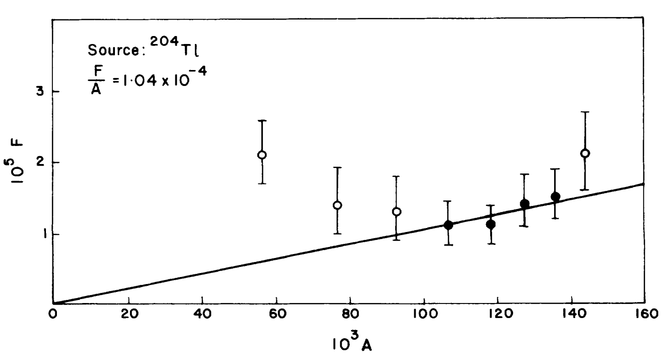

FIG. 5. F vs A for ^{204}Tl. Only filled circles are considered with the origin for linear regression.

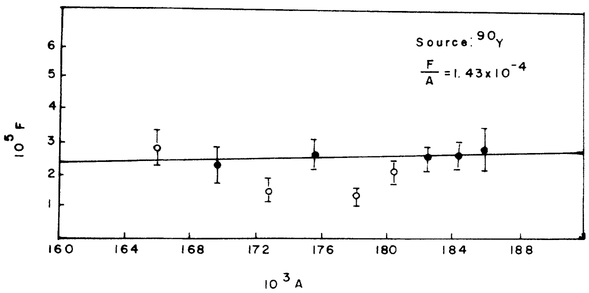

FIG. 6. F vs A for ^{90}Y. Only filled circles are considered with the origin for linear regression.

duced in the scintillator by the 662 keV γ rays of ^{137}Cs. This spectrum was taken prior to and after every set of readings. If there were any changes in the location of the Compton edge then that particular set of readings was rejected during the analysis. Due to the presence of "variance noise" the values of $A(T)$ and hence $R(T)$ fluctuate if the number of count measurements taken for their calculation are less. Therefore, to obtain a consistent value of $R(T)$, a large number of measurements have to be taken for analysis, especially when the counting time intervals are short, as seen in Fig. 2(a). When T is large a small number of count measurements would be sufficient to give a reliable value of $R(T)$, as in Fig. 2(b).

The analysis was carried out at three channels corresponding to energies 450, 495, and 540 keV.

EXPERIMENTAL RESULTS AND DISCUSSION

Figure 3 shows experimentally obtained values of $R(T)$ versus $1/\langle M_T \rangle$ for $E_\beta = 495$ keV. It is clear from the figure that there is a deviation from Poisson statistics. As $1/\langle M_T \rangle$ decreases, it can be seen that $R(T)$ approaches the flicker floor. The solid curve is obtained by computer fitting of the experimental values to the equation

$$R(T) = 1/\langle M_T \rangle + F \ .$$

In Fig. 3 the value of F is 2.4×10^{-5}. Similar figures were obtained for the other β energies as well. The values of F for other β energies are given in Table I.

Figure 4 is a plot of F versus A. We have also shown in Figs. 5 and 6 similar plots of our earlier experimental results [6–8]. It can be seen that there is a trend for the points to follow a bowl shaped curve. This is not in accordance with the existing theory. Further experiments have to confirm this observation. However, we have drawn a straight line passing through the origin and considering a maximum number of points, as this is the current theoretical prediction, and have determined F/A values in each case.

TABLE II. Experimentally obtained values for the reduction factor K'.

Sources	End-point energy (in keV)	$10^4 F/A$	K'	Reference
^{90}Sr-^{90}Y	546	1.68	120	Present
^{204}Tl	770	1.04	194	6
^{90}Y	2279	1.43	141	7

It is now believed that the inclusion of dielectric constant k in the expression for F is incorrect [5]. Therefore we have at present tried to calculate the factor K', called the reduction factor, which is defined as

$$K' = [4Z^2 \alpha \ln(2)] / [(F/A)_{\rm expt}] \ . \qquad (5)$$

The values of K' are given in Table II for the sources we have investigated.

CONCLUSIONS

We have detected $1/f$ fluctuations in yet another β^- decay statistics. Our results on the other two β sources, namely, ^{204}Tl and ^{90}Y, also yielded similar results [6,7]. Also the values of F/A for the three sources which we have investigated until now are fairly constant, as shown in Table II. Thus the values of the flicker floor seem to depend only on the energy of the emitted β^- particle, whatever the source from which it is emitted. A combined source such as ^{90}Sr-^{90}Y giving roughly the same value of F seems to indicate that it is not the source that is responsible for $1/f$ noise but the process of emission.

The experimental finding that a flicker floor exists in the emission of β^- particles shows that a limit exists in the precision with which one takes a count of β^- particles. Considering only two adjacent count measurements, at $T \to \infty$

$$R(\infty) = \tfrac{1}{2} |M_T^i - M_T^{i+1}|^2 / \langle M_T \rangle^2 \ .$$

Taking $R(\infty) \approx 10^{-5}$, the limit in the error will be 0.45%.

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