Discontinuous scaling of hysteresis losses

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We study the dependence of hysteresis loop area A on the frequency Ω and amplitude H of the driving field for several mean-field treatments of the kinetic Ising model. An unusual *discontinuous* double-power-law scaling behavior is found in all cases. In the low-frequency regime, it is found that $A-A_0 \sim H^{2/3}\Omega^{2/3}\mathcal{G}_L(\Omega/H^\gamma)$, where A_0 is the zero-frequency value of the loop area, $\mathcal{G}_L$ is a scaling function, and γ is a model-dependent exponent. In the high-frequency regime, the loop area itself scales with frequency and amplitude as $A \sim H^\alpha \Omega^{-1}$, where α is also a model-dependent exponent. The transition between these extremes is sharp and can be characterized by an amplitude-dependent critical frequency. We also note differences in behavior above and below the critical ordering temperature T_C.

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The phenomenon of hysteresis is notable as an example of a dynamic, nonequilibrium process that is also of technological importance. For example, in the context of bistable optical and magnetic devices, one desires accurate and repetitive switching between two states of a system that are degenerate in the absence of an external drive. Here we focus attention on the energy dissipated per cycle, i.e., the hysteresis loop area, for the case of a periodically driven system. Previous theoretical [1–4] and experimental [5,6] treatments of this problem have demonstrated that this quantity varies as a power law in both the amplitude H and the frequency Ω of the driving signal $\mathcal{H}(t)=H\sin\Omega t$ at low frequency. For a model with continuous symmetry, Rao, Krishnamurthy, and Pandit [1] found that this scaling disappears for an intermediate range of frequencies and then reappears (with different exponents) in the limit $\Omega\to\infty$. Most recently, extensive Monte Carlo simulations of lattice Ising models have been used to suggest [4] that the loop area obeys *dynamical scaling* in the form

$$A \sim H^a\Omega^b\mathcal{G}\left[\frac{\Omega}{H^\gamma}\right] \tag{1}$$

for all values of H and Ω where $\mathcal{G}(x)$ is a scaling function with the properties

$$\mathcal{G}(x)=\begin{cases}\text{const} & \text{for } x \ll 1\\ \to 0 & \text{for } x \gg 1\ ,\end{cases} \tag{2}$$

and a, b, and γ are scaling exponents. It was also claimed in this work that the loop area obtained from the mean-field limit of a kinetic Ising model with Glauber dynamics [7] satisfies (1) as well.

The purpose of this paper is to report the results of numerical computations that reveal the existence of an unusual *discontinuous* dynamical scaling of the hysteresis loop area for three related mean-field models of Ising dynamics. Our results do not agree with those reported in Ref. [4]. Indeed, the fact that no relationship such as (1) can be expected to hold over the entire frequency range is clear already from the following argument. When $\Omega\to\infty$, the system is unable to respond to the input signal and the steady-state magnetization M simply assumes its zero-field equilibrium value. No loop area is generated. Similarly, in the limit $\Omega\to 0$, fluctuations eventually drive the system to the equilibrium state corresponding to the current value of $\mathcal{H}$. The loop area is again zero since the equilibrium value of M is a single-valued function of $\mathcal{H}$ (except at $\mathcal{H}=0$). But if fluctuations are suppressed (as in a mean-field treatment), the system can be trapped in a metastable state where $M<0$ while $\mathcal{H}>0$ (and vice versa) for fields less than the coercive field $\mathcal{H}_C$. The system returns to equilibrium only when the field exceeds $\mathcal{H}_C$, at which point $\mathcal{H}$ and M have the same sign. The result is a nonzero adiabatic loop area A_0:

$$\lim_{\Omega\to 0} A = A_0 > 0\ . \tag{3}$$

Since the right-hand side of (1) goes to zero as $\Omega\to 0$, one must add A_0 to it to correctly reproduce the low-frequency limit. Since this guarantees an incorrect high-frequency limit, there is no single scaling relationship valid for all frequencies.

To discover the correct scaling behavior, we have studied three related mean-field equations of motion for the average magnetization of an Ising model with z nearest neighbors and ferromagnetic exchange J:

$$\frac{dM}{dt}=\frac{1}{\tau}\left[-M+\tanh\left[\frac{zJM+\mathcal{H}}{kT}\right]\right], \tag{4}$$

$$\frac{dM}{dt}=\frac{1}{\tau}\left[-M\cosh\left[\frac{zJM+\mathcal{H}}{kT}\right]+\sinh\left[\frac{zJM+\mathcal{H}}{kT}\right]\right], \tag{5}$$

and

$$\frac{dM}{dt}=\frac{1}{\tau}(BM-CM^3+\mathcal{H})\ . \tag{6}$$

The kinetic equation (4) arises [8] when a mean-field approximation is made to the master equation description of Ising dynamics assuming that the rate to flip a spin is given by Glauber's prescription:

$$W(S_j \rightarrow -S_j) = \frac{1}{\tau} \frac{\exp[(E_I^j - E_F^j)/kT]}{\cosh[(E_I^j - E_F^j)/kT]} . \tag{7}$$

Here, the subscripts I and F refer to the initial state with spin S_j and the final state with spin $-S_j$, respectively, and E_I^j is the energy associated with the jth site, i.e.,

$$E_I^j = -S_j \left[\mathcal{H} + \sum_{nn} J S_{nn} \right] . \tag{8}$$

The notation nn refers to nearest neighbors of the site j, and the time scale is set by the constant τ, which we take to be unity.

The kinetic equation (5) arises similarly [9] if the single spin flip rate is assumed to take the form

$$W(S_j \rightarrow -S_j) = \frac{1}{\tau} \exp\left[\frac{E_I^j}{kT} \right] . \tag{9}$$

This choice satisfies detailed balance as well and may be interpreted as an Arrhenius-type rate if the zero of energy is presumed to coincide with the energy of the activated transition state. Note the contrast with the Glauber rate, which depends on the energy change associated with the process. One easily checks that (4) and (5) have the same metastable and equilibrium states.

With the choices $B \propto T_C - T$ and $C > 0$, the final kinetic equation (6) arises most naturally when the double-well free energy function $F\{M\} = -\frac{1}{2}BM^2 + \frac{1}{4}CM^4 - \mathcal{H}M$ appropriate to systems with Ising symmetry [10] is inserted into a purely relaxational version of the time-dependent Ginzburg-Landau (TDGL) equation of motion

$$\frac{dM}{dt} = -\frac{1}{\tau} \frac{\delta F}{\delta M} . \tag{10}$$

Indeed, if one integrates the right-hand side of (4) or (5) with respect to M, a qualitatively similar double-well function results.

Equations (4), (5), and (6) were integrated numerically subject to a sinusoidal applied field $\mathcal{H}(t)$. After initial transients died away, the area of the hysteresis loop over one complete cycle was computed from

$$A = \oint M \, d\mathcal{H} . \tag{11}$$

This is numerically equal to the energy dissipated as heat by the system per field cycle.

We consider the case of $T < T_C$ first. For all three models, we observed *two* different scaling relationships of the form (1): one valid from the lowest frequencies up to an amplitude-dependent critical value of the frequency and the other valid from this critical value up to the highest frequencies. The low-frequency and high-frequency data collapses for the TDGL equation (10) with $B = C = 1$ are illustrated in Figs. 1 and 2, respectively, for amplitudes $\mathcal{H} > \mathcal{H}_C$ ranging from 1.0 to 7.0 and for frequencies ranging from 0.05 to 20.0. These results were obtained as follows. First, the exponents a, b, and γ [cf. (1)] in Fig. 2 were chosen to yield the best collapse of the data with the largest values of the scaling variable Ω/H^γ. The vertical dashed line indicates where data collapse breaks for lower values of this variable and thus

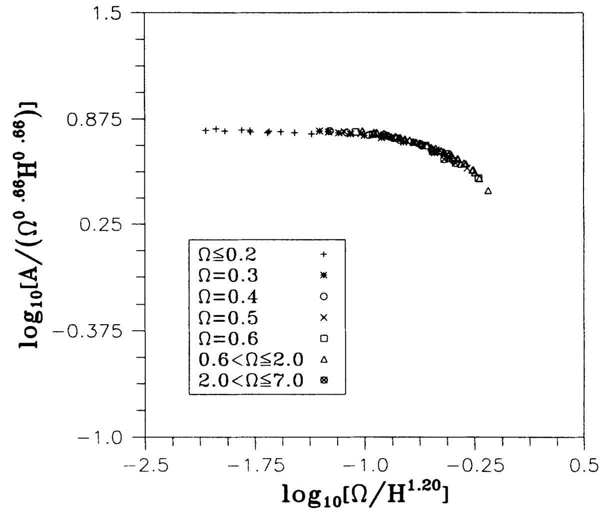

FIG. 1. Log-log plot of the low-frequency data collapse for the TDGL model. The data which fit (14) are not plotted.

defines an amplitude-dependent critical frequency $\Omega_C(H)$. Next, we eliminate all the data with frequencies *greater* than Ω_C. Finally, the exponents a, b, and γ in Fig. 1 were chosen to produce the best collapse of the remaining (low)-frequency data. Similar results were found for the other two kinetic models. *All* the low frequency data are well fit by the scaling form

$$A = A_0 + H^{2/3}\Omega^{2/3} g_L \left[\frac{\Omega}{H^\gamma} \right] , \tag{12}$$

with a value of γ that depends on the details of the kinetics: $\gamma \simeq 1.2$, 0.5, and 0.5 for the TDGL, Glauber, and Arrhenius models, respectively. The scaling function

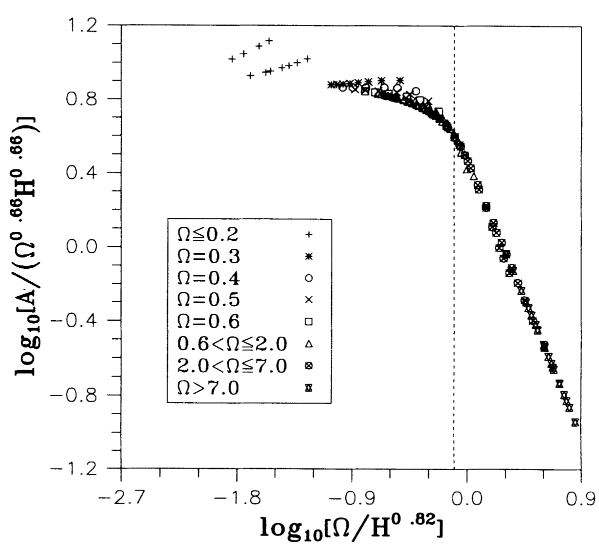

FIG. 2. Log-log plot of the high-frequency data collapse for the TDGL model. The vertical line marks the transition from the low-frequency scaling form (12) to the high-frequency scaling form (14).

$\mathcal{G}_L(x)$ is slightly different in the three cases as well. These results disagree with those reported by Acharyya and Chakrabarti [4] for the Glauber model but agree with the analytic results of Jung *et al.* [5] for the TDGL model at very low frequency, where $\mathcal{G}_L(x)$ can be taken to be a constant.

All the high-frequency data are well fit by the formula

$$A \sim H^{2/3}\Omega^{2/3}\mathcal{G}_H\left[\frac{\Omega}{H^{\gamma}}\right], \tag{13}$$

where $\gamma \simeq 0.82$, 0.3, and 1.1 for the TDGL, Glauber, and Arrhenius models. The scaling functions are again different in the three cases. In fact, they differ in just such a way so that all of the high-frequency data are well described by the expression

$$A \sim H^{(2+5\gamma)/3}\Omega^{-1} . \tag{14}$$

As noted by Acharyya and Chakrabarti [4], the simple Ω dependence in (14) follows immediately from (11) if the free energy is quadratic in M. The response is linear in this case and the Fourier spectrum of the time-dependent magnetization $M(t)$ contains only the fundamental frequency of the driving field. This situation will occur if the system remains in the immediate vicinity of one of the zero-field maxima or minima of $F\{M\}$. The system will then generate highly elliptical hysteresis loops centered around one of the extrema, and this is in fact what we observe. Careful study of Fig. 2 reveals a curious feature of the high-frequency data. For sufficiently large values of Ω (*independent* of H), some of the loop data appear to collapse onto a "knee" that joins smoothly onto the curves (13) or (14). Unfortunately, we have been unable to identify a distinguishing physical characteristic of these "schizophrenic" points which, of course, technically belong to the low-frequency data collapse.

So long as $T < T_C$, all the scaling exponents reported above are independent of temperature. In fact, the absolute value of the loop area itself is temperature independent in the high frequency regime, despite the fact that some of the loops are symmetric while others are asymmetric. This is not the case at low frequency where, e.g., $A_0 = A_0(T)$.

When $T > T_C$, $F\{M\}$ has a single minimum for all values of $\mathcal{H}$ and the adiabatic loop area A_0 is identically zero. We find that the high-frequency scaling behavior is exactly the same as for $T < T_C$. Moreover, the loop area remains temperature independent and equal to the value found below T_C. However, in contrast to the results reported in [4] for the Glauber model, we were unable to obtain data collapse of any of the low-frequency data when $T > T_C$.

In conclusion, we have found that the hysteresis loop area exhibits *discontinuous* double-power-law scaling behavior in several mean-field approximations to the periodically driven kinetic Ising model. Two of three scaling exponents are found to take values independent of the kinetic details. Above T_C, scaling is obtained at high frequency only. But below T_C, an amplitude-dependent critical frequency separates two distinct scaling regimes. This discontinuity arises as a consequence of the presence of a nonzero adiabatic loop area A_0. Although formally A_0 must vanish if fluctuations are taken into account, there will be experimental situations where fluctuation effects will not be observable on the time scale of the dynamic measurements. This appears to be the case for the bistable laser system reported in Ref. [5], and we predict discontinuous scaling in that instance.

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