# Nonlocal electron transport in the presence of high-intensity laser irradiation

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We investigate electron transport in a plasma heated by spatially modulated laser irradiation. When the heating rate is greater than the electron-electron collision rate, the thermal conductivity is reduced by a factor of 3 to 4 from the Spitzer-Härm [Phys. Rev. **89**, 977 (1953)] value for $K\lambda_e < 0.01$ and is less affected by nonlocal heat-transport effects for $k\lambda_e \gg 1$, where $\lambda_e$ is the electron mean free path and $k$ is the perturbation wave number. Implications for thermal filamentation will be discussed.

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It is well known that inverse-bremsstrahlung (IB) absorption of laser light in plasma can lead to significant departures from a Maxwellian electron distribution function when the heating rate becomes comparable to the electron thermalization rate through self-collisions [1]. The transition is characterized by the parameter $\alpha=$ (heating rate)/(thermalization rate) $=Z^*(v_0/v_t)^2>1$, where $v_0$ is the peak electron oscillatory velocity in the laser field, $v_t=(T/m)^{1/2}$ is the thermal velocity, $T$ is the electron temperature (in ergs), $m$ is the electron mass, and $Z^*=\langle Z^2\rangle/\langle Z\rangle$ (where $\langle\ \rangle$ denotes an average over the ion species). In this limit, and in the absence of heat sinks, the plasma continuously heats up and attains a self-similar distribution function of the form $f_{ib}\propto\exp\{-[v/V(t)]^5\}$. Mora and Yahi [2] have calculated the corresponding electron thermal conductivity $\kappa$ for a collisional plasma and found a reduction of about 3–4 from the Spitzer-Härm [3] (SH) value. If on the other hand the IB heating is weak yet spatially modulated with a wave number $k$, the effective thermal conductivity is found to fall significantly below the SH value for $k\lambda_e>0.01$, where $\lambda_e$ is an electron mean free path [4–6]. This occurs when the heat-carrying electrons, with energies of about $7T$, become effectively delocalized in space and cannot collide fast enough with the background thermal electrons to establish a Maxwellian distribution. Thus we have two distinct mechanisms for conductivity reduction: one is due to the $f_{ib}$ form of the background distribution function in the limit where $\alpha\gg1$ and $k\lambda_e\ll1$, and another is due nonlocal heat-transport effects in the limit where $\alpha\ll1$ and $k\lambda_e\gg1$.

In this paper we investigate the effective value of $\kappa$ for arbitrary $\alpha$ and $k\lambda_e$ (assuming $v_0<v_t$). We are particularly interested in answering the question of whether the reduction in $\kappa$ relative to $\kappa_{\rm SH}$ for $k\lambda_e\gg1$ will be less or more severe as $\alpha$ becomes larger than 1. This can be important for laser-fusion applications, where the thermal filamentation growth rate is intrinsically dependent on the value of $\kappa$ [4].

The approach we use here is similar to the one adopted in Ref. [4]. The electron distribution function is numerically calculated using the electron Fokker-Planck (FP) code SPARK, assuming no ion motion. Although the code is fully nonlinear, the physical processes become more transparent if we refer to the linearized form of the FP equation, assuming a perturbation (denoted by $\delta$) with spatial dependence $\exp(ikx)$,

$$\frac{\partial f_0}{\partial t}=D_{ee}(f_0)+D_{ib}(f_0)\ , \tag{1}$$

$$\frac{\partial\delta f_0}{\partial t}+ikv\delta f_1=D_{ee}(\delta f_0)+D_{ib}(\delta f_0)+\delta D_{ib}(f_0)\ , \tag{2}$$

$$-\nu_{ei}\delta f_1=ikv\delta f_0-\frac{e\delta E}{m}\frac{\partial f_0}{\delta v}\ , \tag{3}$$

where $\delta E$ is the perturbed electric field (calculated assuming quasineutrality), $\nu_{ei}=4\pi ne^4Z^*\ln\Lambda/v^3$ is the electron-ion ($e$-$i$) collision frequency, $n$ is the electron number density, $e$ is the magnitude of the electric charge, and $\ln\Lambda$ is the Coulomb logarithm. These coupled equations are derived from the diffusive form of the FP equation, in the Lorentz limit. The diffusive approximation assumes that $f=f_0+f_1v_x/|v|$ and $\partial/\partial t\ll\nu_{ei}$, which has been shown to be accurate for our type of problem, and the Lorentz limit assumes that $Z\gg1$ (though approximate corrections for low $Z$ may be easily incorporated) [5].

The $e$-$e$ collision operator $D_{ee}$ and the IB heating operator $D_{ib}$ are described in detail in Ref. [1]. For our purposes it suffices to know that $D_{ib}/D_{ee}\sim\alpha$. The other important scaling parameter can be obtained by comparing the heat-transport term with the $e$-$e$ collision term, i.e., $ikv\delta f_1/D_{ee}\delta f_0\sim(k\lambda_e)^2$, where $\lambda_e=T^2/4\pi ne^4(Z^*)^{1/2}\ln\Lambda$. The significance of these two parameters can be explained as follows: By considering first Eq. (1), which describes the evolution of the homogeneous background plasma, we can see that the parameter $\alpha$ represents the ratio of IB heating to electron thermalization through $e$-$e$ collisions. When $\alpha\ll1$, the $D_{ee}$ operator drives the distribution function toward a Maxwellian, $f_M\propto\exp(-v^2/2v_t^2)$. However, when $\alpha\gg1$, the distribution function evolves (after a brief transient period) to a self-similar state of the form $f_{ib}$. The significance of parameter $k\lambda_e$ lies in the fact that when $k\lambda_e\ll1$ (and assuming $\alpha\ll1$), $e$-$e$ collisions are able to maintain $\delta f_0$ close to a perturbed Maxwellian

 

$$\delta f_M = \frac{\delta T}{2T}\left(\frac{v^2}{v_t^2} - 3\right) f_M . \qquad (4)$$

This represents the so-called collisional limit assumed by fluid theory, which leads to the SH thermal conductivity. In the opposite limit ($k\lambda_e \gg 1$) the high-velocity electrons in the tail of the distribution function diffuse in $x$ faster than they are able to diffuse in $v$, thus leading to a departure from a Maxwellian. The resultant breakdown of the fluid approximation is manifested by a severe reduction in the effective $\kappa$.

Before calculating the general dependence of $\kappa$ on $\alpha$ and $k\lambda_e$, let us first review the results based on the following two limits: First, we consider the case of $k\lambda_e \ll 1$ but arbitrary $\alpha$. Mora and Yahi proposed a test isotropic distribution of the form

$$f_{0\mu} = \frac{n}{4\pi V^3}\frac{\mu}{\Gamma(3/\mu)}\exp[-(v/V)^\mu] \qquad (5)$$

to analyze the transport under these conditions. Here $V = v_t[3\Gamma(3/\mu)/\Gamma(5/\mu)]^{1/2}$ and $\mu$ is a coefficient such that $f_{0\mu=2} = f_M$ and $f_{0\mu=5} = f_{ib}$. The heat flow can be calculated by linearizing Eq. (5) and substituting the result in Eq. (3). The effective conductivities, defined by

$$\delta q = \frac{2\pi}{3} m \int_0^\infty \delta f_1 v^5 dv = -ik\kappa\delta T - ikT\kappa_n \delta n/n , \qquad (6)$$

are then given by

$$\frac{\kappa(\mu)}{\kappa_{SH}} = \frac{a(7b-5c)}{64}\sqrt{\pi/2} ,$$

and (7)

$$\frac{\kappa_n(\mu)}{\kappa_{SH}} = -\frac{a(b-c)}{32}\sqrt{\pi/2} ,$$

where $a = [\Gamma(3/\mu)]^{5/2}[3/\Gamma(5/\mu)]^{7/2}$, $b = \Gamma(10/\mu)/12$, and $c = [\Gamma(8/\mu)]^2/9\Gamma(6/\mu)$. We note that for $\mu > 2$ there is a heat flow arising from a density gradient, though for $\delta T/T = -\delta n/n$ and $\mu = 5$ its contribution is only about 25% of the total heat flux. In this paper we consider the case of $\delta n = 0$ only.

The two important limits of Eq. (7) are $\kappa(\mu=2) = \kappa_{SH}$ and $\kappa(\mu=5) \approx 0.25\kappa_{SH}$. As shown by Mora and Yahi the reduction in conductivity may be explained by plotting (see Fig. 2 ) $Q(v) = v^5\delta f_1$ as a function of $v/v_t$ for $\mu=2$ (dashed curve) and $\mu=5$ (solid curve). Since the maximum value of $Q$ shifts to a lower velocity for $\mu=5$, the heat-carrying electrons become more collisional and thus less effective at transporting heat.

To obtain a relationship between $\mu$ and $\alpha$, Matte *et al.* [7] numerically solved Eq. (1) and fitted their results with the formula $\mu(\alpha) = 2 + 3/(1 + 1.66/\alpha^{0.724})$. Based on this result we now propose the following $\alpha$-dependent conductivities:

$$\frac{\kappa(\alpha)}{\kappa_{SH}} = 1 - \frac{0.751}{[1 + (0.25/\alpha)^{0.75}]} ,$$

and (8)

$$\frac{\kappa_n(\alpha)}{\kappa_{SH}} = -\frac{0.0658}{[1 + (0.153/\alpha)^{0.75}]} ,$$

where $\kappa/\kappa_{SH}$ is plotted in Fig. 1. In terms of useful parameters we have $\alpha = 0.042 I_{14}\lambda^2 Z^*/\varepsilon[T(\text{keV})]$, where $I_{14}$ is the laser intensity in units of $10^{14}$ W/cm$^2$, $\lambda$ is the laser wavelength in $\mu$m, $\varepsilon = (1 - n/n_c)^{1/2}$, and $n_c$ is electron critical number density. As expected, other electron transport processes, such as viscosity and resistivity, are also affected by the change in $f_0$. These have been discussed in great detail by Dum [8].

Now we consider the case of $\alpha \ll 1$ and arbitrary $k\lambda_e$. This was done in Ref. [4] and corresponds to situation where Eq. (1) yields the results $f_0 = f_M$, and Eqs. (2) and (3) are solved self-consistently. Naturally the background distribution is not a true equilibrium (since there are no energy sinks), but its temporal variation rate is assumed slower than the rate it takes for $\delta f_0$ to reach equilibrium. In other words the IB heating rate has to be less than the thermal conduction rate across $k^{-1}$. The resultant values of $\kappa/\kappa_{SH}$ are plotted in Fig. 3 as functions of $k\lambda_e$.

The results for the general case of arbitrary $k\lambda_e$ and $\alpha$ are shown in Fig. 3. Since for a finite $\alpha$ the distribution function is not in a steady state, the curves essentially show the instantaneous value of $\kappa/\kappa_{SH}$ as a function of $k\lambda_e$ and $\alpha$. In practice, to keep the value of $\alpha$ constant during a SPARK simulation the laser intensity had to be increased in time at the same rate as the fractional increase in the background temperature. From Fig. 3 we see that for $k\lambda_e < 0.01$, $\kappa/\kappa_{SH}$ displays the behavior shown in Fig. 1. The main results, however, are that for a given $k\lambda_e \gg 1, \kappa/\kappa_{SH}$ actually increases with $\alpha$, and that for given $\alpha$ the reduction with $k\lambda_e$ starts at larger values of $k\lambda_e$. Part of the explanation for this can be traced back to Fig. 2. Since the velocity of the heat-carrying electrons of an IB-heated plasma is reduced from the classical value, these more collisional electrons have an effectively smaller mean free path and are hence less susceptible to nonlocal transport effects. A similar physical argument has also been put forward by Yahi and Mora for the case of a plasma driven with a strong heat flow. A complementary explanation is based on the fact

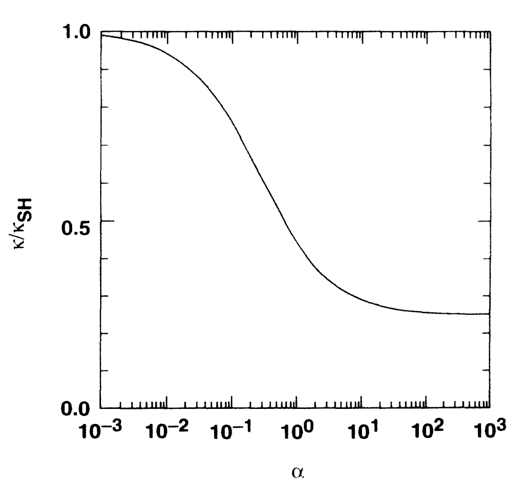


FIG. 1. Plot $\kappa/\kappa_{SH}$ as a function of $\alpha$, for $k\lambda_e \ll 1$, where $k$ is the perturbation wave number and $\lambda_e$ is the electron mean free path.

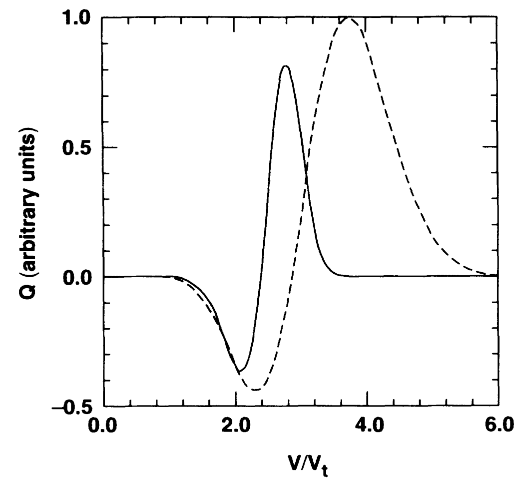


FIG. 2. Plots of $Q$ (in arbitrary units) as functions of $v/v_t$. The dashed curve refers to the classical fluid result, and the solid curve refers to the case where $\alpha \gg 1$. Note that the area under the curve is proportional to the heat flow.

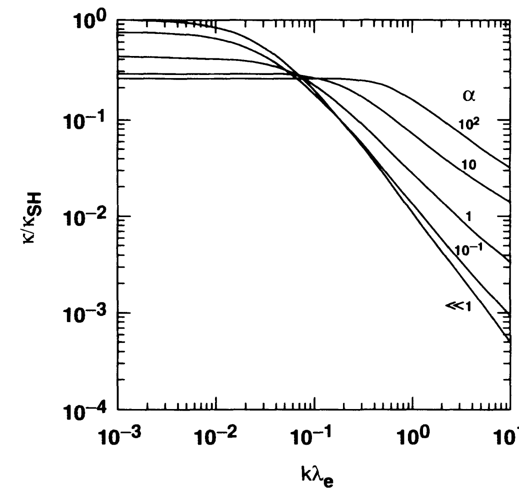


FIG. 3. Plots of $\kappa/\kappa_{\rm SH}$ as functions of $k\lambda_e$, for $\alpha \ll 1$, and $\alpha = 0.1$, 1, 10, and 100.

that for $\alpha \gg 1$ the relative importance of nonlocal transport effects can be estimated from $ikv\delta f_1/D_{ib}\delta f_0 \sim (k\lambda_e)^2/\alpha$. Thus as $\alpha$ increases so does the value of $k\lambda_e$ at which nonlocal transport effects start to dominate.

To assess the importance of these results for the growth rate of thermal filamentation [4–6] we recall the fact that for a given amplitude of laser intensity modulation $\delta I$ the thermal response $\delta T$ is inversely proportional to $\kappa$. This is demonstrated by the steady-state energy balance equation

$$k^2 \kappa T \frac{\delta T}{T} = S_{ib} R \left[ \frac{\delta I}{I} + 2\frac{\delta n}{n} - \frac{3}{2}\frac{\delta T}{T} \right],$$

where $S_{ib}$ is the IB heating and $R$ is the Langdon correction factor $[R(\alpha \ll 1) = 1, R(\alpha \gg 1) = 0.5]$. Assuming pressure balance, the density response may be written as

$$\left| \frac{\delta n}{n} \right| = \left| \frac{\delta I}{I} \right| \Big/ \left[ \gamma + \frac{7}{2} \right],$$

where $\gamma = k^2 \kappa T / S_{ib} R \approx 384(\kappa/\kappa_{\rm SH})(1/R)(k\lambda_e)^2/\alpha$, for $Z \gg 1$. In the limit as $\alpha \gg 1$ and $k\lambda_e \ll 0.01, \gamma$ is negligible and $|\delta n/n|$ becomes insensitive to both $\kappa/\kappa_{\rm SH}$ and $R$. For $k\lambda_e \gg 1$, however, $\gamma$ is large and $|\delta n/n|$ becomes proportional to $R(\kappa_{\rm SH}/\kappa)$. Hence, for a given $k\lambda_e$, the finite-$\alpha$ transport corrections will act to reduce the value of $|\delta n/n|$.

Therefore, we reach the conclusion that in the collisional limit the $\alpha$ corrections to the transport are likely to have a negligible effect on thermal filamentation, whereas in the weakly collisional limit the $\alpha$ corrections are likely to reduce the level of thermal filamentation. However, to accurately assess the overall effectiveness of laser filamentation one should also investigate the contribution from the ponderomotive force.

The results presented in this report refer specifically to sinusoidal perturbations in laser irradiation. To apply them to spatially isolated hot spots in the laser profile requires the use of Fourier decomposition followed by the transport corrections as shown in Fig. 3 (assuming that the transport is still reasonably linear).

In summary, we have discussed the transport properties of a plasma heated by spatially nonuniform high-intensity laser irradiation. When the heating rate is greater than the electron-electron thermalization rate, the thermal conductivity is reduced by a factor of 3–4 from the Spitzer-Härm value of $k\lambda_e < 0.01$ and is less affected by nonlocal heat-transport effects for $k\lambda_e \gg 1$.

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