Solitary electrostatic waves in a thin plasma slab

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The nonlinear properties of the electrostatic perturbations in a thin plasma slab are investigated. A new solitary wave moving with the group velocity is then found.

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Studies of the propagation of guided electromagnetic waves in active dielectrics and in plasmas are of relevance for many modern technological applications [1]. The behavior of surface waves propagating along plasma columns or plasma slabs has consequently been treated in many laboratory experiments [2,3].

The linear theory for surface waves in bounded plasmas is now rather well known [4]. However, the corresponding nonlinear theory is very complex and needs thus much attention. Zhelyazkov, Stoyanov, and Yu [5] studied the nonlinear propagation of a high frequency symmetric surface wave on a thin plasma layer of constant density and sharp boundaries, and found that solitary waves can exist. These calculations were extended to a plasma slab with arbitrary density profile [6], and to the nonlinear propagation of antisymmetric surface waves [7]. Related results originating from a strong striction nonlinearity model can be found in Ref. [8]. Vladimirov [9] derived a generalized nonlinear equation for the interaction of the symmetric and antisymmetric surface waves in a plasma slab, and it was subsequently shown [10] that coupled bright and dark solitary surface waves can propagate on the boundaries of a plasma slab. Taking the singular currents at the boundary layers into account, it turns out that the self-consistent interaction of the symmetric and antisymmetric plasmons can be described by new nonlinear equations [11] which differ significantly from those of previous papers.

In the present Brief Report, we are going to extend the theory further. Thus, considering, for simplicity, the propagation along the x axis of a low-frequency electrostatic wave in a cold plasma slab with a smooth arbitrary density profile $n_0(z)$, where there are no variations in the y direction, and where the ions just play the role of an immobile background, we write the electrostatic potential of the waves as $\phi=\phi_0(x,z,t)\exp(ikx-i\omega t)$, where $\partial \ln\phi_0/\partial x \ll k$ and $\partial \ln\phi_0/\partial t \ll \omega$. If the wavelength $2\pi/k$ is much larger than the width of the slab, i.e., if $(1/n_0)dn_0/dz \gg k$, then it is well known that the dispersion relation is [4]

$$\omega^2 \approx (k/2)\int_{-\infty}^{\infty} dz\, \omega_p^2 \ , \tag{1}$$

where $\omega_p=(n_0q^2/\varepsilon_0 m)^{1/2}$ is the electron plasma frequency, and q/m the electron charge to mass ratio.

We shall now consider the nonlinear propagation of the electrostatic wave, noting that its potential must satisfy the equation [12,13]

$$\nabla\cdot[\varepsilon(\omega)\nabla\phi-(q^2/m^2\omega^4)((\nabla^2|\nabla\phi|^2)\nabla\phi-(\omega_p^2/4\omega^2\varepsilon(2\omega)) \times\{\nabla[(\nabla(\nabla\phi)^2)\cdot\nabla\phi^*]+(\nabla^2(\nabla\phi)^2)\nabla\phi^*/2\})]=0 \ , \tag{2}$$

where $\varepsilon(\omega)=1-(\omega_p^2/\omega^2)+2i(\omega_p^2/\omega^3)\partial\ln\phi_0/\partial t$, and where the star stands for complex conjugate. The $1/\varepsilon(2\omega)$ terms in (2) are obviously due to second harmonic generation [12].

Considering long-wavelength, low-frequency waves, i.e., $k \ll \partial \ln n_0/\partial z$ and $\omega \ll \omega_p$, we rewrite (2) as

$$\partial_z(\hat{\varepsilon}\partial_z\phi)\approx k^2(1-r)\hat{\varepsilon}\phi \ , \tag{3a}$$

where $r=(1/k^2\phi_0)(\partial_x^2\phi_0+2ik\partial_x\phi_0)$, $\partial_x\equiv\partial/\partial x$, $\partial_z\equiv\partial/\partial z$, and

$$\hat{\varepsilon}=\varepsilon(\omega)-(q^2/m^2\omega^4)\{\partial_z^2|\partial_z\phi_0|^2+[(\partial_z\phi_0^*)/\phi_0]\partial_z(\partial_z\phi_0)^2 -(\phi_0^*/2\phi_0)\partial_z^2(\partial_z\phi_0)^2\} \ . \tag{3b}$$

Equation (3) has the approximate solution

$$\partial_z\phi_0\approx(k^2/\varepsilon)\int_{-\infty}^{z}dz'(1-r)\tilde{\varepsilon}(z')\phi_0(z') \ , \tag{4a}$$

where

$$\tilde{\varepsilon}=\varepsilon(\omega)+(q^2/2m^2\omega^4)\partial_z^2(\partial_z\phi_0)^2 \ . \tag{4b}$$

We have in (4a), by means of some partial integrations, obviously rewritten the integral of $\hat{\varepsilon}$ in terms of an integral of $\tilde{\varepsilon}$. Here we can treat ϕ_0 as a real function.

Integrating both sides of (4a), and then following closely the algebra of Ref. [4], we obtain

$$\phi_0(x,z,t)\approx\phi_0(x,0,t)\left[1-k^2z\int_{-\infty}^{\infty}dz'(1-r)(1-\tilde{\varepsilon})/2\right] \ , \tag{5}$$

which is valid inside the plasma slab because $\omega \ll \omega_p$ and $kz<1$.

Outside the plasma slab, we note that the potential satisfies the Laplace equation, and that it thus can be written in the Taylor expanded form [4]

$$\phi_0(x,z,t)\approx\phi_0(x,0,t)[1-kz(1-(i/k)\partial \ln\phi_0/\partial x)] \; . \quad (6)$$

The solutions (5) and (6) have to agree in the intermediate region [4]. We then note that the imaginary parts of (5) and (6) are identical if ϕ_0 is a function of $x-v_g t$, where $v_g \equiv \omega/2k$. This value for the group velocity is obviously consistent with (1). The real parts of (5) and (6) turn out to be equal if

$$\partial^2\phi_0/\partial x^2+(\beta^2/96)\phi_0^3+k^2\Delta\phi_0=0 \; , \quad (7)$$

where ϕ_0 now stands for $\phi_0(x-v_g t, z=0)$, $\beta=4qk^3/m\omega^2$ and $\Delta=1-(k/2\omega^2)\int_{-\infty}^{\infty} dz\, \omega_p^2$.

A solution of (7) is

$$\phi_0=\phi_0(0)/\cosh[(x-v_g t)/L] \; , \quad (8)$$

where $1/L=k(-\Delta)^{1/2}$ and $\phi_0(0)=(8k/\beta)(-3\Delta)^{1/2}$. Obviously Δ must be a slightly negative quantity.

We think that our solution (8), which is supposed to describe the nonlinear propagation of an electrostatic wave along a thin plasma slab, will not differ qualitatively from the corresponding solution for wave propagation along a thin plasma column. The present theory must thus be of interest when the results of laboratory experiments are discussed [2].

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[1] See, e.g., A. Hasegawa, *Optical Solitons in Fibers* (Springer, Berlin, 1990).
[2] D. Grozev, A. Shivarova, and S. Tanev, J. Plasma Phys. **45**, 297 (1991).
[3] J. Margot and M. Moisan, J. Plasma Phys. **49**, 357 (1993).
[4] O. M. Gradov and L. Stenflo, Phys. Rep. **94**, 111 (1983).
[5] I. Zhelyazkov, O. Stoyanov, and M. Y. Yu, Plasma Phys. Contr. Fusion **26**, 813 (1984).
[6] L. Stenflo and M. Y. Yu, in *Surface Waves in Plasmas and Solids,* edited by S. Vuković (World Scientific, Singapore, 1986), p. 645.
[7] I. Zhelyazkov, T. Vodenicharova, and M. Y. Yu, J. Plasma Phys. **36**, 143 (1986).
[8] A. Shivarova and N. Dimitrov, Plasma Phys. Contr. Fusion **27**, 219 (1985).
[9] S. V. Vladimirov, Fiz. Plazmy **12**, 961 (1986) [Sov. J. Plasma Phys. **12**, 552 (1986)].
[10] S. V. Vladimirov, L. Stenflo, and M. Y. Yu, Phys. Lett. A **153**, 144 (1991).
[11] S. V. Vladimirov and M. Y. Yu, Phys. Lett. A **184**, 459 (1994).
[12] L. Stenflo and O. M. Gradov, IEEE Trans. Plasma Sci. **PS-14**, 554 (1986).
[13] I. Zhelyazkov, in *The Physics of Ionized Gases,* edited by J. Purić and D. Belić (World Scientific, Singapore, 1987), p. 459.