Simulation of phase ordering kinetics in conserved scalar systems with long-range interactions

Takamitsu Ishihara and Hisao Hayakawa*
Department of Physics, Tohoku University, Sendai 980-77, Japan
(Received 31 January 1994; revised manuscript received 12 April 1994)

Two-dimensional phase ordering kinetics is simulated in systems with conserved scalar order parameter with the long-range attractive interactions, falling off with the distance r as $r^{-2-\sigma}$ with $0 < \sigma < 2$. From our simulations, it is found that the dynamical scaling of the scaling function holds and its scaling forms are almost independent of σ for $\sigma \geq 1$. In addition, the growth exponent z of the characteristic length $l(t) \sim t^{1/z}$ is consistent with the theoretical prediction by Bray and Rutenberg [Phys. Rev. E **49**, R27 (1994)].

PACS number(s): 64.60.Cn, 64.60.My

I. INTRODUCTION

The phase ordering kinetics of systems quenched from the disordered phase to the ordered phase has been extensively studied [1]. Typical models of phase ordering kinetics are model A and model B of critical dynamics [2], corresponding to nonconserved and conserved order parameters, respectively. In these processes, the dynamical scaling [3] is a key concept to an understanding of the late stage of domain growth where the equal-time two-point correlation function of the order parameter S has the form

$$\langle S(\mathbf{x},t)S(\mathbf{x}+\mathbf{r},t)\rangle = C(r/l(t)) \quad , \tag{1}$$

where $l(t)$ is the characteristic length at time t after the quench and $C(X)$ is the scaling function. The angular brackets in (1) indicate an average over initial conditions. For systems with the short-range interactions, it is well known that $l(t)$ obeys a power law $l(t) \sim t^{1/2}$ for model A (SRMA) [4] or $l(t) \sim t^{1/3}$ for model B (SRMB) [5]. These power law growths have been confirmed by several numerical simulations [6–9].

When analyzing an experiment, one has to be aware of the possible presence of long-range interactions. Thus, it is of interest to study how the long-range interactions affect the phase ordering processes. Recently, Hayakawa, Rácz and Tsuzuki (HRT) [10] have analyzed the phase ordering kinetics for vector order parameter systems with the long-range attractive interactions, falling off with the distance r as $r^{-d-\sigma}$ with $0 < \sigma < 2$ and the spatial dimension d. After the work by HRT [10], several theoretical investigations have been carried out for systems with long-range interactions. Bray [11] has indicated that the result of HRT which is based on the theory by Kawasaki, Yalabik, and Gunton [12] cannot be used for the scalar order parameter, and predicted $l(t) \sim t^{1/(1+\sigma)}$ for $\sigma < 1$ and $l(t) \sim t^{1/2}$ for $\sigma > 1$ in cases of model A with long interactions (LRMA). Bray and Rutenberg [13] generalized Bray's argument to the model with conserved order parameter. Their prediction for the model B with long-range interactions (LRMB) is given by $l(t) \sim t^{1/(2+\sigma)}$ for $\sigma < 1$ and $l(t) \sim t^{1/3}$ for $1 < \sigma < 2$ [13]. In cases of LRMA, simulations by Hayakawa *et al.* [14] have gotten consistent results with Bray's prediction [11]. Lee and Cardy [15,27] also found $l(t) \sim t^{1/(1+\sigma)}$ in the one-dimensional model. Theoretical arguments [16,17] also support Bray's prediction [11]. In particular, the arguments by Ohta and Hayakawa [16] based on the interfacial picture have shown that the correlation function is scaled for $\sigma < 1$ but not scaled for $\sigma > 1$, though their results are not consistent with those of Hayakawa *et al.* [14] at first sight.

For the conserved case, however, we do not have any argument to support the prediction by Bray and Rutenberg [13]. There are only arguments by HRT [10] and by Hayakawa [18] who have considered a growth kinetics of a system with an O(n) symmetric order parameter with large n cases. Therefore, we need to attempt to check the argument by Bray and Rutenberg [13] in LRMB. In addition, we study the validity of the scaling ansatz of the scattering function or the correlation function (1).

The organization of this paper is as follows. In the next section we will introduce the method of our simulation of LRMB. In Sec. III, we summarize our numerical results, which are almost consistent with the theoretical prediction. We also indicate that the scaling form of the scaling function is almost independent of σ for $\sigma \geq 1$. In the final section, we discuss our results and present concluding remarks. In the Appendix we illustrate the universal behavior of the scattering function for a small wave number based on the argument by Furukawa [19]

II. THE METHOD OF OUR SIMULATION

In this section we will illustrate the method of our simulation of the model B of critical dynamics with long-range interaction (LRMB). In our model, the time evolution of the order parameter obeys

*Electronic address: hisao@cmpt01.phys.tohoku.ac.jp

$$\frac{\partial S(\mathbf{r},t)}{\partial t} = \nabla^2 \mu(\mathbf{r},t) \quad , \tag{2}$$

where μ is the chemical potential given by

$$\mu(\mathbf{r},t) = -\nabla^2 S(\mathbf{r},t) + g(S) + \beta \int d\mathbf{r}' \frac{S(\mathbf{r}',t) - S(\mathbf{r},t)}{|\mathbf{r}-\mathbf{r}'|^{d+\sigma}} \quad , \tag{3}$$

where we assume that $g(S)$ whose zeros exist at $S \simeq +1, -1$ is an odd function of S. We restrict ourselves to the cases of $0 < \sigma < 2$ and $\beta > 0$.

Our numerical method is based on the cell-dynamical systems (CDS) introduced by Oono and Puri [9]. It is possible to discretize (2) as

$$S(\mathbf{n}, t+1) = S(\mathbf{n},t) + \langle\langle \mu(\mathbf{n},t) \rangle\rangle - \mu(\mathbf{n},t) \quad , \tag{4}$$

where $S(\mathbf{n},t)$ is the order parameter at the lattice site $\mathbf{n}$ and time t, and

$$\langle\langle \mu(\mathbf{n}) \rangle\rangle = \frac{1}{6} \sum_{\mathbf{n}' \in \mathrm{NN}} \mu(\mathbf{n}') + \frac{1}{12} \sum_{\mathbf{n}' \in \mathrm{NNN}} \mu(\mathbf{n}')$$

with the summation over the nearest neighbor sites (NN) and the next nearest neighbor sites (NNN) of $\mathbf{n}$. The chemical potential μ in (4) is assumed to be

$$\mu(\mathbf{n},t) = D(-\nabla^2)^{\sigma/2} S(\mathbf{n},t) - A \tanh[S(\mathbf{n},t)] + S(\mathbf{n}) \quad , \tag{5}$$

where $D = 0.5$ and $A = 1.3$ are positive constants, and $(-\nabla^2)^{\sigma/2}$ is a symbolic representation of $V(r) \sim r^{-d-\sigma}$. The operator $(-\nabla^2)^{\sigma/2}$ can be interpreted as an operator in Fourier space [20]

$$\hat{L}(\mathbf{q}) = [1 - \tfrac{1}{3}\cos(q_x) - \tfrac{1}{3}\cos(q_y) - \tfrac{1}{3}\cos(q_x)\cos(q_y)]^{\sigma/2}, \tag{6}$$

where $\mathbf{q} = (q_x, q_y) = (2\pi m_x/N, 2\pi m_y/N)$ with the integers m_x and m_y, which are less than the linear size of the system N.

The method of solving (4) and (5) can be summarized as follows. Let F be the Fourier transform. First, we transform $S(\mathbf{n},t) \xrightarrow{\mathrm{F}} \tilde{S}(\mathbf{q},t)$ into the Fourier space. Second, defining $\tilde{S}'(\mathbf{q},t) \equiv \hat{L}(\mathbf{q})\tilde{S}(\mathbf{q},t)$, we come back to the real space $\tilde{S}'(\mathbf{q},t) \xrightarrow{\mathrm{F}^{-1}} S'(\mathbf{n},t)$ with the introduction of the inverse of the Fourier transform. Thus we get an explicit form of the chemical potential from (5) and obtain $S(\mathbf{n}, t+1)$ from (4). Finally, we come back to the first stage in this paragraph to continue this process.

In our simulation, quenches are carried out at the center of the miscibility gap. The initial value of the order parameter on the cell is distributed at random between -0.125 and 0.125. In order to avoid the dependence of a special initial configuration of the order parameter field, we have performed eight runs for $\sigma = 0.5$, and ten runs for $\sigma = 1.0$ and 1.5. Our results are obtained from simulations up to $t = 15\,000$ with the system size 256×256 in the cell unit for $\sigma = 1.0$ and 1.5, and up to $t = 40\,000$ with the system size 512×512 for $\sigma = 0.5$. Moreover, we have performed ten runs for the system with the size 256×256 up to $t = 15\,000$ and 20 runs for the system with the size 128×128 up to $t = 50\,000$ in the case of $\sigma = 0.5$ to check the finite size scaling. We adopt the periodic boundary condition. Note that we need a larger system size to simulate the dynamics for small σ than a system for large σ [14].

We calculate the characteristic length of the system $l(t)$ and the scattering function $I(\mathbf{q},t)$. We have used the circular averaged scattering function $I_c(q,t)$ defined by [7,8]

$$I_c(q,t) = \sum_{\mathbf{q}} I(\mathbf{q},t) \Big/ \sum_{\mathbf{q}} 1 \quad ,$$

where $q = 2\pi n/N, n = 0, 1, 2, \ldots, N$. The summation $\sum_{\mathbf{q}}$ is carried out in the spherical shell defined in $n - \frac{1}{2} \le |\mathbf{q}|N/2\pi \le n + \frac{1}{2}$. Note that the summation has the cutoff $q_c = (2\pi/N)n_c$ with $n_c = N/2$.

The traditional definition of the characteristic length, $k_m(t)^{-1}$, is given by

$$k_m(t) = \frac{\sum_{q=0}^{q=q_c} q I_c(q,t)}{\sum_{q=0}^{q=q_c} I_c(q,t)}. \tag{7}$$

Shinozaki and Oono [21], however, have shown that this definition leads to systematic errors and have introduced a new characteristic length $q_m(t)^{-1}$ defined by

$$q_m(t) = \frac{\sum_{\mathbf{q}\neq\mathbf{0}} q^{-1} I(\mathbf{q},t)}{\sum_{\mathbf{q}\neq\mathbf{0}} q^{-2} I(\mathbf{q},t)}. \tag{8}$$

In our later discussions, we will use $q_m(t)^{-1}$ as the characteristic length $l(t)$, except for the case in which we compare our results with the traditional results.

III. RESULTS

In this section we summarize our numerical results, including the time evolution of $l(t)$ and the scaling form of the scattering function.

First, we present the time dependence of the characteristic length $l(t)$. As mentioned in the Introduction, Bray and Rutenberg [13] have predicted $l(t) \sim t^{1/3}$ for $1 < \sigma < 2$, $l(t) \sim (t \ln t)^{1/3}$ for $\sigma = 1$, and $l(t) \sim t^{1/2+\sigma}$ for $0 < \sigma < 1$. From Fig. 1, we have estimated the growth exponent $1/z$ of the characteristic length defined by $l(t) \sim t^{1/z}$ as $1/z = 0.43 \pm 0.03$ for $\sigma = 0.5$ and $1/z = 0.36 \pm 0.01$ for $\sigma = 1.5$. For $\sigma = 1.0$, on the other hand, on account of the logarithmic correction to the characteristic length $l(t)$ as $l(t) \sim (t \ln t)^{1/z}$, we obtain $1/z = 0.35 \pm 0.01$. Although these growth exponents obtained from our simulation have a little larger values than the theoretically predicted values for each σ, our simulation results almost seem to be consistent with the prediction by Bray and Rutenberg [13]. A little discrepancy between the theory and simulation may come from the following reasons: finite size effects, and the insuffi-

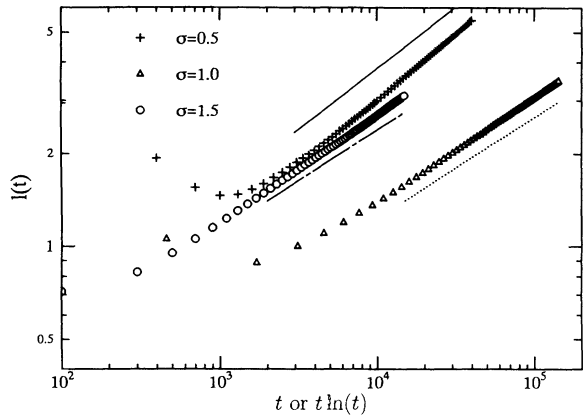

FIG. 1. The double log plots of $l(t)$ versus time where $l(t) = 2\pi/q_m(t)$ in systems with the size 512 × 512 for $\sigma = 0.5$ and with the size 256 × 256 for $\sigma = 1.0, 1.5$, respectively. The solid, dotted, and dashed lines correspond to $l(t) = t^{2/5}$, $[t\ln(t)]^{1/3}$, and $t^{1/3}$, respectively.

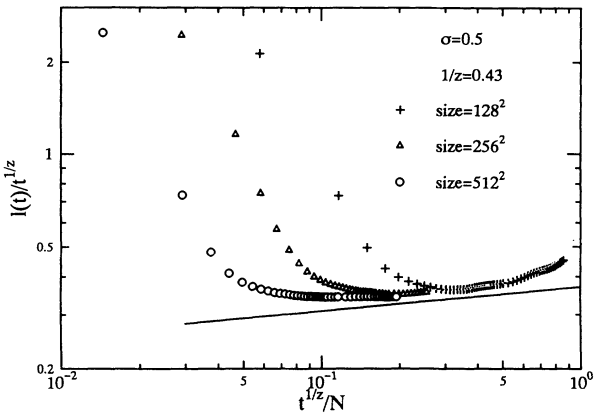

FIG. 2. The double log plots of $l(t)/t^{1/z}$ versus $t^{1/z}/N$ for the system of $\sigma = 0.5$, where we have used a numerically evaluated value for z. The envelope line at the late stage of domain growth has been obtained by connecting the points whose derivative coefficients are nearly zero.

ciency of both our simulation time steps and the number of average runs. In particular, the finite size effects may be enhanced in the case of smaller σ. In this case, the concept of finite size scaling may be important if we take into account the system size dependence of the dynamics. Bray [11] has suggested that the characteristic length $l(t)$ may be scaled by the system size N as

$$l(t) = t^{1/z} f(t^{1/z}/N) \quad , \tag{9}$$

where f is a scaling function and its asymptotic behavior is expected to be

$$f(x) \sim \text{const} \quad \text{for} \quad x \lesssim 1, \tag{10}$$

where $x = t^{1/z}/N$. Note that for small x the time evolution of the system is not sufficient to realize the scaling regime, so in this case seeking the asymptotic form of f is almost meaningless. To examine the ansatz (9) by Bray, we plot $l(t)/t^{1/z}$ versus $t^{1/z}/N$ for the system of $\sigma = 0.5$ with the sizes 128 × 128, 256 × 256, and 512 × 512 in Fig. 2 with the estimated value of z from our simulation. From Fig. 2 it is certain that a simple ansatz (9) does not hold. By looking for the envelope line at the late stage of growth, however, we may expect the existence of some sort of the finite size scaling instead of (9), because if we use other values of z, even the theoretical value, there is no such envelope line. Note that according to (10) the envelope line in Fig. 2 should tend to be a constant. Though in Fig. 2 the envelope line has the small slope, our simulation results may not get rid of the possibilities of a constant $f(x)$ for $x \lesssim 1$ from the insufficiency of our simulation.

The scaling ansatz (1) can be rewritten as

$$I_c(q,t) = l(t)^2 I_{sc}(q/q_m(t)), \tag{11}$$

where $I_{sc}(x)$ is the scaling function. In other words, if the scaling is violated, then a single characteristic length $l(t)$ does not make sense. To check whether the scaling holds we plot $l(t)^{-2} I_c(q,t)$ at various time steps in the late stage of the domain growth in Figs. 3–5 as functions of $x = q/q_m(t)$ for $\sigma = 0.5, 1.0$ and $\sigma = 1.5$, respectively. Note that we adopt the following normalization:

$$\int_0^\infty I_{sc}(x) x dx = 1. \tag{12}$$

Figures 3–5 suggest the existence of a universal scaling function. We must confess ourselves that our result does not have enough accuracy to discuss the weak violation of scaling for $\sigma > 1$ suggested by Ohta and Hayakawa [16] (see also Sec. IV).

Figure 6 scales plots of data for $\sigma = 0.5$, 1.0, and 1.5, with a comparison of the data for model B with short-range interaction (SRMB) [22]. We adopt the result of Shinozaki and Oono [22] as the data of model B. In Fig. 5 we use (7) as the characteristic length because they adopted $k_m(t)^{-1}$ defined in (7) in their paper [22]. To emphasize the differences among them we use

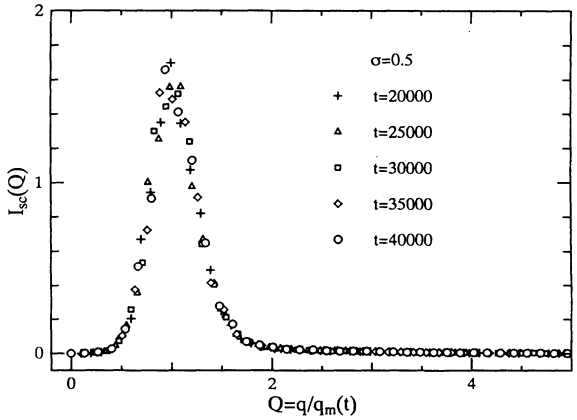

FIG. 3. The scaling plots of the scaling function at t=20 000, 25 000, 30 000, 35 000, and 40 000 for $\sigma = 0.5$ with the system size 512 × 512.

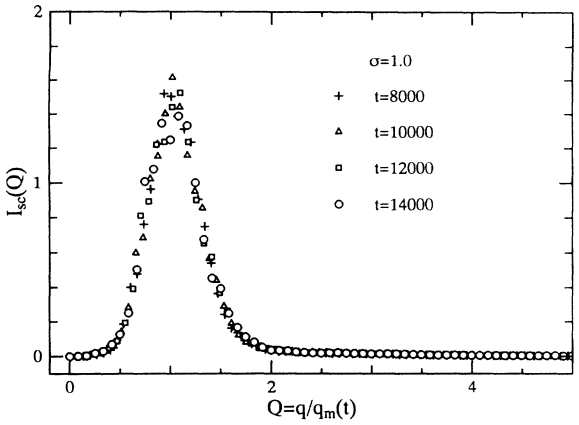

FIG. 4. The scaling plots of the scaling function at t=8000, 10 000, 12 000, and 14 000 for $\sigma = 1.0$ with the system size 256×256.

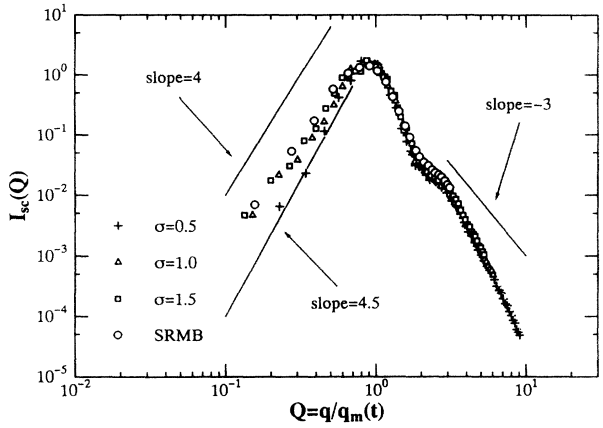

FIG. 6. The double log plots of the normalized scaling function versus scaled wave number Q for systems of LRMB for $\sigma = 0.5, 1.0, 1.5$ and for the system of SRMB. We adopt the result of Shinozaki and Oono [22] as the data of SRMB. To compare their old data [22], we have used k_m^{-1} defined by (7) as the characteristic length. Slopes 4 and -3 represent the conventional Q^4 law and Porod's law, respectively, while slope 4.5 corresponds to $Q^{4+\sigma}$ for $0 < \sigma < 1$ predicted from (18).

double log plots for the normalized scaling function and $Q = q/k_m(t)$. It is striking that all of data except for $Q \simeq 0$ in $\sigma = 0.5$ can be represented by a single universal function.

The most essential part of the scaling function is determined by two asymptotic limits $Q \to 0$ and ∞. In ordinary short-range cases, it is well known that $I_{sc}(Q) \sim Q^4$ for small Q [23,19] and $I_{sc}(Q) \sim Q^{-3}$ (Porod's law) [24] for large Q. For small Q we may expect $I_{sc}(Q) \sim Q^4$ even for the long-range model from the generalized Furukawa argument (see Appendix) [19]. The interpretation of an anomalous behavior near $Q \sim 0$ for $\sigma = 0.5$ will be presented in the next section.

For large Q, we expect that Porod's law

$$I_c(q,t) \sim \Sigma(t) q^{-3} \tag{13}$$

with the interface density $\Sigma(t)$ is still valid in our model, because the profile of the order parameter determined by the static equation (2) is steep in the vicinity of the interface [16]. Even in the short-ranged model, however, it is difficult to observe Porod's law from simulations and experiments, because the interfaces have finite widths. Thus, the following modification:

$$I_c(q,t) \propto \Sigma(t) q^{-3} \exp(-\lambda_I{}^2 q^2) \tag{14}$$

is often used by several authors [25,26], where λ_I is proportional to the interface width. The ratio $\Sigma(t)/q_m(t)$ should be constant in time for the validity of the modified Porod law. We valuate $\Sigma(t)$ from the cross point of $\ln[q^3 I_c(q,t)]$ at $q = 0$ as in Fig. 7, where we note that $I_c(q,t)$ is not normalized as in Eq. (12). We also plot $\Sigma(t)/q_m(t)$ for $\sigma = 0.5, 1.0$ and 1.5 as a function of time (Fig. 8). From this figure, the ratio $\Sigma(t)/q_m(t)$ is almost constant in time for $\sigma = 0.5$. On the other hand, the ratio depends on time for $\sigma = 1.0$ and 1.5. This insufficiency may stem from the following reason. Our simulations for

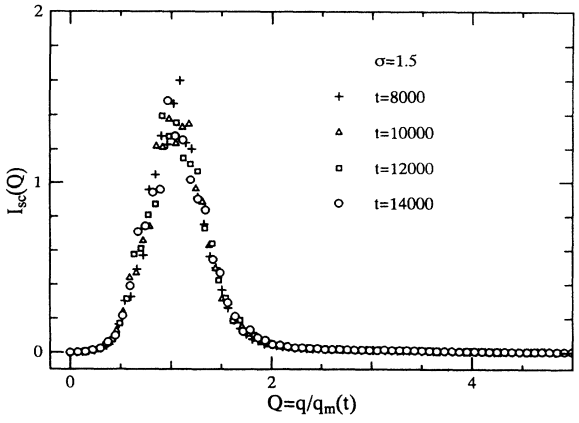

FIG. 5. The scaling plots of the scaling function at t=8000, 10 000, 12 000, and 14 000 for $\sigma = 1.5$ with the system size 256×256.

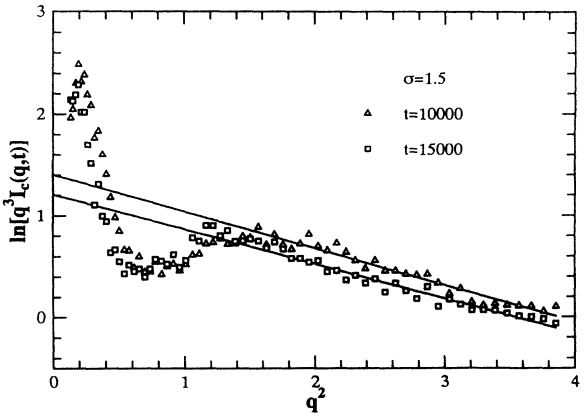

FIG. 7. The plots of $\ln[q^3 I_c(q,t)]$ versus q^2 for $\sigma = 1.5$ at t=10 000 and 15 000.

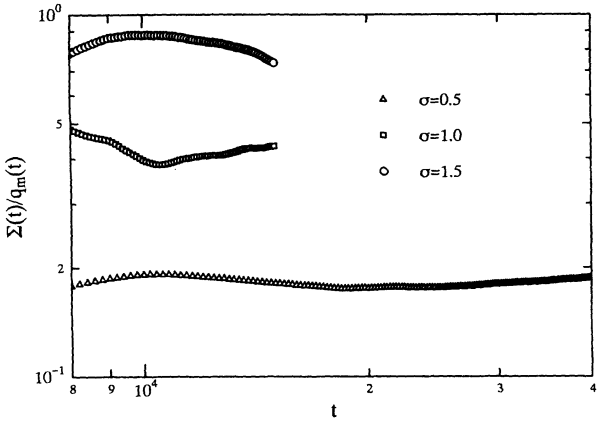

FIG. 8. The plots of $\sigma(t)/q_m(t)$ versus time where $\sigma(t)$ is the interfacial area density at time t for $\sigma = 0.5, 1.0$, and 1.5.

$\sigma = 1.0$ and 1.5 have been performed for smaller systems with shorter simulation time than those of $\sigma = 0.5$.

IV. DISCUSSION

We now consider the anomalous behavior of small wave numbers in $I_{sc}(Q)$ for $\sigma \leq 1$. For the nonconserved model with long-range interactions (LRMA), Ohta and Hayakawa [16] have shown that the asymptotic form of the correlation function $C^{NC}(r,t)$ is given by

$$C^{NC}(r,t) \sim r^{-d-\sigma} \qquad (\text{as} \quad r \to \infty) \tag{15}$$

for $0 < \sigma < 1$. This long tail must be an origin of singularities for $I_{sc}(Q)$. In fact, $\langle|\mu(\mathbf{q},t')|^2\rangle$ may be approximated by $\langle|\mu^{NC}(\mathbf{q},t')|^2\rangle$ with the chemical potential $\mu^{NC}(\mathbf{r},t)$ for LRMA. Therefore, the asymptotic behavior of $\langle|\mu(\mathbf{q},t)^2|\rangle$ for $q \to 0$ in systems of LRMB with $0 < \sigma < 1$ may be evaluated,

$$\begin{aligned}\langle|\mu(\mathbf{q},t)|^2\rangle &\simeq \langle|\mu^{NC}(\mathbf{q},t)|^2\rangle \\ &\simeq \langle|\mu(0,t)|^2\rangle - C_2(ql(t))^{\sigma} + \cdots,\end{aligned} \tag{16}$$

where we have used (A8). C_2 is a constant in both q and t. The leading singularity term $(ql(t))^{\sigma}$ comes from the inverse of the Fourier transform of (15). If we substitute (16) into (A5) or

$$I^{NC}(\mathbf{q},t) \simeq \int^t dt' \langle|\mu^{NC}(\mathbf{q},t')|^2\rangle ct' \tag{17}$$

with a constant c [see (A7)], we obtain the asymptotic form of the scaling function $I_{sc}(ql(t))$ as

$$I_{sc}(Q) \simeq C_1 Q^4 - C_2 Q^{4+\sigma} + \cdots \tag{18}$$

for small $Q = ql(t)$ where $C_1 = \langle|\mu(0,t)|^2\rangle$. This (18) suggests that the singular term $Q^{4+\sigma}$ can be observed in the scaling form of $I_{sc}(Q)$ for $0 < \sigma < 1$. On the other hand, by using an argument similar to both the above to obtain (18) and the result of Ohta and Hayakawa [16], the scaling function for $1 < \sigma < 2$ can be evaluated as

$$I_{sc}(Q) \simeq C_1 Q^4 - C_2' Q^{4+\sigma} t^{\frac{1-\sigma}{2}} + \cdots \tag{19}$$

for small Q, where C_2' is a constant. Equation (19) suggests that the scaling is violated for $\sigma > 1$ as in Ref. [16]. Thus the leading singular term disappears in the scaling limit. This kind of rough discussion leads to qualitatively consistent results with our simulation results (see Fig. 6).

The basic assumption used here is that $\mu(q = 0, t)$ approaches a nonzero time independent constant. The assumption essentially states that μ is not conserved and justifies the usage of the theory by Ohta and Hayakawa [16]. Although this assumption seems to be quite reasonable for the short-range interactions, it may be dangerous for long-range cases. Therefore, the argument presented here should notify the reader that the anomalous behavior like (18) is not derived from a reliable assumption.

We now conclude our paper. The purpose of this paper is to clarify the scaling behavior of the conserved model with long-range interaction. From our CDS simulation, we obtain the following. (i) The time evolution of the characteristic length is consistent with the theoretical prediction by Bray and Rutenberg [13]. (ii) Finite size scaling may exist for the system of smaller σ. (iii) The scattering function satisfies the dynamical scaling for all of σ. (iv) Our simulation suggests that the scaling form of the scattering function $I_{sc}(Q)$ seems to be independent of σ except for small Q behavior in $\sigma = 0.5$. The singular behavior of small Q in $\sigma = 0.5$ may come from the leading singularity of small Q for $\sigma < 1$. We need simulations for larger system size and enough number of average of runs to confirm the above conclusion.

ACKNOWLEDGMENTS

We would like to thank T. Ohta and A.D. Rutenberg for stimulating discussions and useful comments. One of the authors (H.H.) thanks A.J. Bray for sending him the relevant papers prior to their publication. This work was partially supported by a Grand-in-Aid of the Ministry of Education, Science and Culture of Japan.

APPENDIX

In this appendix, we apply the argument of Furukawa [19] for SRMB to our case (LRMB). The equation of motion to describe the time evolution of the order parameter is given by

$$\frac{\partial S_{\mathbf{q}}}{\partial t} = -q^2 \mu_{\mathbf{q}}(t) \quad , \tag{A1}$$

where $\mu_{\mathbf{q}}(t)$ is the Fourier transform of the chemical potential. Therefore, the equation of motion for the scattering function $I(\mathbf{q},t)$ is

$$\frac{\partial I(\mathbf{q},t)}{\partial t} = 2q^2 \langle \mu_{\mathbf{q}}(t) S_{-\mathbf{q}}(t)\rangle, \tag{A2}$$

where the angular brackets express the average over initial configurations. Furukawa [19] assumed the multi-time-scaling of μ as

$$\langle \mu_{\mathbf{q}}(t)\mu_{-\mathbf{q}}(t')\rangle = [l(t)]^{d-\alpha}\tilde{G}_{\mu\mu}(ql(t))U_{\mu\mu}(ql(t), l(t')/l(t)), \tag{A3}$$

where α represents the effects of the long-range interaction as

$$\alpha = 2 \quad \text{for } \sigma > 1, \qquad \alpha = 2\sigma \quad \text{for } \sigma < 1. \tag{A4}$$

Using $I(0,t) \to 0$ in the scaling limit (A2) with (A3) leads to

$$\frac{\partial I(\mathbf{q},t)}{\partial t} = 2q^4\langle|\mu_{\mathbf{q}}(t)|^2\rangle \int_0^t U_{\mu\mu}(ql(t), l(t')/l(t))dt', \tag{A5}$$

where we have used (A3). When $q \to 0$ we obtain from (A5)

$$\frac{\partial I(\mathbf{q},t)}{\partial t} = 2q^4\langle|\mu_0(t)|^2\rangle ct \quad , \tag{A6}$$

where

$$c \equiv \frac{1}{t}\int_0^t U_{\mu\mu}\left(0, \frac{l(t')}{l(t)}\right)dt' \tag{A7}$$

is a constant because $U_{\mu\mu}(0, \frac{l(t')}{l(t)})$ is only a function of t'/t. Setting $\Lambda^2 \equiv \tilde{G}_{\mu\mu}(0)$, we obtain

$$\langle|\mu_0(t)|^2\rangle \simeq \Lambda^2[l(t)]^{d-\alpha} \quad . \tag{A8}$$

We assume that Λ does not vanish because we regard $\mu_{\mathbf{q}}$ as a nonconservative quantity. This assumption may be crucial, as mentioned in Sec. IV. Thus from (A6) and (A8) we obtain

$$I(\mathbf{q},t) \sim 2c\Lambda^2 t^2[l(t)]^{d-\alpha}q^4 \quad . \tag{A9}$$

The asymptotic form of the scale function $I_{sc}(ql(t)) = l(t)^{-d}I(\mathbf{q},t)$ for small $ql(t)$ is now given by

$$I_{sc}(ql(t)) \sim t^2[l(t)]^{-4-\alpha}(ql(t))^4 \quad . \tag{A10}$$

We can see that $t^2[l(t)]^{-4-\alpha}$ is a constant by substituting the value of α. Therefore, we obtain

$$I_{sc}(Q) \sim Q^4 \tag{A11}$$

for small Q where $Q = ql(t)$.

[1] For reviews, see, e.g., J.D. Gunton, M. San Miguel, and P.S. Sahni, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and J.L. Lebowitz (Academic, New York, 1983), p. 267; H. Furukawa, Adv. Phys. **34**, 703 (1985); K. Binder, Rep. Prog. Phys. **50**, 783 (1987); A.J. Bray, in *Phase Transitions and Relaxation in Systems with Competing Energy Scales*, Vol. 415 of *NATO Advanced Study Institute, Series C: Mathematical and Physical Sciences*, edited by T. Riste and D. Sherrington (Kluwer Academic, Norwell, MA, 1993).

[2] P.C. Hohenberg and B.I. Halperin, Rev. Mod. Phys. **49**, 435 (1977).

[3] K. Binder and D. Stauffer, Phys. Rev. Lett. **33**, 1006 (1978); H. Furukawa, Prog. Theor. Phys. **59**, 1072 (1978).

[4] S.M. Allen and J.W. Cahn, Acta Metall. **27**, 1085 (1979).

[5] I.M. Lifshitz and V.V. Slyozov, J. Phys. Chem. Solids **19**, 35 (1961); K. Kawasaki and T. Ohta, Physica A **118**, 175 (1983).

[6] K. Kaski, M.C. Yalabik, and J.D. Gunton, Phys. Rev. B **28**, 5263 (1983); E.T. Gawlinski, M. Grant, and K. Kaski, *ibid.* **31**, 281 (1985).

[7] C. Roland and M. Grant, Phys. Rev. B **39**, 11 971 (1989).

[8] J. Amar, F. Sullivan, and R. Mountain, Phys. Rev. B **37**, 196 (1988).

[9] Y. Oono and S. Puri, Phys. Rev. A **38**, 434 (1988).

[10] H. Hayakawa, Z. Rácz, and T. Tsuzuki, Phys. Rev. E **47**, 1499 (1993).

[11] A.J. Bray, Phys. Rev. E **47**, 3191 (1993).

[12] K. Kawasaki, M.C. Yalabik, and J.D. Gunton, Phys. Rev. A **17**, 455 (1978).

[13] A.J. Bray and A.D. Rutenberg, Phys. Rev. E **49**, R27 (1994).

[14] H. Hayakawa, T. Ishihara, K. Kawanishi, and T.S. Kobayakawa, Phys. Rev. E **48**, 4257 (1993).

[15] B.P. Lee and J.L. Cardy, Phys. Rev. E **48**, 2452 (1993). Their numerical result is not consistent with the theoretical prediction for $\sigma = 0.5$. This may come from the finite size effects [27].

[16] T. Ohta and H. Hayakawa, Physica A **204**, 482(1994).

[17] J.A.N. Filipe and A.J. Bray (unpublished).

[18] H. Hayakawa, Fractals **1**, 947 (1993).

[19] H. Furukawa, J. Phys. Soc. Jpn. **58**, 216 (1989).

[20] H. Hayakawa and T. Koga, J. Phys. Soc. Jpn. **59**, 3542 (1990).

[21] Y. Shinozaki and Y. Oono, Phys. Rev. E **48**, 2622 (1993).

[22] Y. Shinozaki and Y. Oono, Phys. Rev. A **45**, R2161 (1992).

[23] C. Yeung, Phys. Rev. Lett. **61**, 1135 (1988).

[24] G. Porod, in *Small Angle X-ray Scattering*, edited by O. Glater and L. Krattky (Academic, New York, 1983).

[25] T. Hashimoto, M. Takenaka, and H. Jinnai, J. Appl. Cryst. **24**, 457 (1991).

[26] T. Koga, K. Kawasaki, M. Takenaka, and T. Hashimoto, Physica A **198**, 473 (1993).

[27] A.D. Rutenberg and A. J. Bray, Phys. Rev. E (to be published).