Envelope soliton propagation in media with temporally modulated dispersion

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The problem of soliton propagation in media with time dependent second-order dispersion is considered. We present analytical and numerical results for the variation of the soliton parameters and characteristics of the emitted radiation by the soliton, for the cases of a periodic or random modulation of the dispersion. In the case of a periodically modulated dispersion it is shown that resonant emission of linear waves is possible. The law of radiative decay of the soliton is found in both cases. Applications to the propagation of light pulses in optical fibers with variable dispersion are discussed.

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I. INTRODUCTION

The problem of nonlinear wave propagation in media with modulated dispersion appears in many branches of physics: ultrashort pulse propagation in optical fibers with variable dispersion along the fiber [1]- [3], magnetic solitons in modulated films [4], nonlinear plasma waves [5]. The results to be presented here can be applied to problems of propagation in media with either spatially inhomogeneous dispersion or temporally varying dispersion. The first type of dispersion can be found in magnets and long Josephson junctions, while the second appears in optical fibers, plasma waves, etc. Optical fibers represent very interesting media where the evolution of the envelope of the electric field is described by a NLSE (nonlinear Schrödinger equation) where the role of time is played by the spatial coordinate x (corresponding to the first derivative with x). For the fiber case the temporal variation of dispersion corresponds from a physical point of view to the spatial variation of the dispersive properties of fibers. That correspondence is important for the analysis of the influence of randomly varying dispersion, because in virtue of this formal equivalence we are able to apply the methods of the theory of random processes. From a practical point of view, fibers with variable dispersion can appropriately modify the pulse parameters and in particular they can be used for the compression of optical pulses [2,16]. The propagation of an envelope soliton in a medium with temporally varying dispersion was studied by Hasegawa and Kodama [1] for the optical fiber case. They considered the small scale periodic modulation (with characteristic length L_a) that is encountered in picosecond pulses. For this range of pulse duration the dispersion length x_0 is a few hundred meters so that $L_a \ll x_0$. In this case the guiding soliton concept is valid, denoting that solitons are robust against perturbation. More recently Gordon [3] studied the influence of a periodic modulation of the nonlinearity. He found a resonant condition for emission by linearizing the equation around the soliton. Later this approach was associated with the inverse scattering transform (IST) method by Elgin [14], who found the resonant conditions for soliton emission in optical fibers with a periodic amplification and third-order dispersion. Here we consider the cases of periodic and random modulation of the dispersion by using IST perturbation theory. This method essentially simplifies the calculations and represents a natural framework for the study of this nearly integrable system. In particular, it gives the soliton evolution within an adiabatic description of the characteristic dynamical parameters and allows one to estimate the linear wave emission. We compare the results of this analytic approach to the numerical solutions of the partial differential equation. We calculate the radiative damping of solitons for both cases of variable dispersion including large variations of dispersive parameters.

II. FORMULATION OF PROBLEM AND BASIC EQUATIONS

We will study in this work the NLSE soliton dynamics in media with temporally changing dispersion. The problem is described by a modified NLSE,

$$iu_t + a(t)u_{xx} + 2|u|^2u = 0. \tag{1}$$

This modified NLS equation comes up in optical fibers [1], magnets [4], etc. In pulse propagation in optical fibers t plays the role of the spatial variable along the propagation direction and (1) describes the evolution of ultrashort pulses in a fiber with modulation of second-order chromatic dispersion. In the fiber case x is normalized by the pulse duration t_s, the length by the characteristic dispersive length $x_0 = -\frac{t_s^2}{3.1|\kappa''_{\omega\omega}|}$, and the amplitude by $\sqrt{P_0(mW)} = 2.9\lambda^{3/2}\sqrt{|D|S}/t_s$, where $|D| = \frac{2\pi c|\kappa''|}{\lambda^2}$

is the average fiber dispersion, and S is the effective transverse cross section of the fiber. It should be noted that Eq. (1) can be transformed into a NLSE with a nonstationary damping [6] by introducing a new time variable $\tau = \int_0^t a(t')dt'$, and using the transformation $u(x,t) = \psi(x,t)v(t)$, with $v(t) = a^{-1/2}$. Then we have

$$i\psi_t + \psi_{xx} + 2|\psi|^2\psi = i\frac{v_\tau}{2v}\psi. \tag{2}$$

For the investigation of a very slowly varying dispersion Eq. (2) seems more suitable than (1).

Here we will investigate the evolution of the single soliton solution for the case when $a(t) = 1+\epsilon(t)$, with $\epsilon \ll 1$, and $\epsilon(t)$ is a periodic or a random function, with zero mean.

$$u_s(x,t) = 2i\eta\,\text{sech}[2\eta(x-\zeta(t)] \times \exp\left[-i\frac{\xi}{\eta}z - i\delta(t)\right], \tag{3}$$

where

$$z = 2\eta(x-\zeta), \quad \delta = 4(\xi^2-\eta^2)t + \delta_0, \quad \zeta = -4\xi t + \zeta_0.$$

In (3) $\eta(t)$, $\zeta(t)$, $\xi(t)$, and $\delta(t)$ are the amplitude, position of center, velocity, and phase of the soliton, respectively.

We perform our calculations for the variation of the soliton variational parameters using the perturbation theory for NLSE solitons based on the inverse scattering transform (IST) [7,8]. The results of the calculation are

$$\eta = \eta_0, \quad \xi = \xi_0, \tag{4}$$

$$\frac{d\zeta}{dt} = -4\xi a(t), \quad \frac{d\delta}{dt} = 4(\xi^2-\eta^2)a(t). \tag{5}$$

The calculation for the adiabatic variation of the parameters can be performed explicitly. For the periodic modulation case we consider $\epsilon(t) = \epsilon_0 \sin\omega t$ so that

$$\zeta = -4\xi_0\left(t - \epsilon_0\frac{\cos(\omega t)-1}{\omega}\right), \tag{6}$$

$$\delta = 4(\xi^2-\eta^2)\left(t - \epsilon_0\frac{\cos(\omega t)-1}{\omega}\right). \tag{7}$$

For the random modulation with

$$\langle\epsilon(t)\rangle = 0, \tag{8}$$

$$\langle\epsilon(t)\epsilon(t')\rangle = \frac{\sigma_\epsilon^2}{2t_c}\exp\left(-\frac{|t-t'|}{t_c}\right), \tag{9}$$

we find for the mean square of $\Delta\zeta = \zeta - \zeta_0$ and $\Delta\delta = \delta - \delta_0$, with $\zeta_0 = -4\xi_0 t$ and $\delta_0 = 4(\xi_0^2 - \eta_0^2)t$

$$\langle(\Delta\zeta)^2\rangle = 16\xi_0^2\sigma_\epsilon^2[t + t_c(e^{-t/t_c}-1)], \tag{10}$$

$$\langle(\Delta\delta)^2\rangle = 16\eta_0^4\left(1-\frac{\xi_0^2}{\eta_0^2}\right)^2\sigma_\epsilon^2[t + t_c(e^{-t/t_c}-1)]. \tag{11}$$

For $t \ll t_c$ we obtain the t^2 dependence typical of diffusive motion,

$$\langle(\Delta\zeta)^2\rangle = 8\xi_0^2\sigma_\epsilon^2 t^2/t_c,$$

$$\langle(\Delta\delta)^2\rangle = 8\eta_0^4\left(1-\frac{\xi_0^2}{\eta_0^2}\right)^2\sigma_\epsilon^2 t^2/t_c.$$

When $t \gg t_c$ we have the standard diffusion law with $\langle(\Delta\zeta)^2\rangle, \langle(\Delta\delta)^2\rangle \sim \sigma_\epsilon^2 t$. It should be noted that, when $\xi_0 = 0$, $\langle(\Delta\zeta)^2\rangle = 0$ and only the phase is modulated.

III. CALCULATION OF THE EMISSION FIELD

Along with the variation of the solitonic parameters we must also consider the problem of the emission of linear waves by the soliton under the action of the dispersion modulation. We will calculate the energy of emission using the IST method.

The energy of emission is defined by [10]

$$E_{\text{rad}} = \frac{2}{\pi}\int_{-\infty}^{+\infty}\ln(|\,a(\lambda)\,|^{-2})d\lambda. \tag{12}$$

The $a(\lambda,t)$ coefficient is related with the $b(\lambda,t)$ coefficient by the condition

$$|\,a\,|^2 + |\,b\,|^2 = 1. \tag{13}$$

For $\epsilon_0 \ll 1$, Eq. (12) has the form

$$E_{\text{rad}} = \frac{2}{\pi}\int_{-\infty}^{\infty}(|\,b(\lambda)\,|^2)d\lambda, \quad t \gg 1. \tag{14}$$

The scattering data coefficient $b(\lambda,t)$, where λ is the spectral parameter, is calculated by perturbation theory according to the formula [5,7,8]

$$\frac{\partial b(\lambda,t)}{\partial t} = -4i\lambda^2 b + \frac{i\epsilon e^{i\delta - 2i\lambda\zeta}}{2\eta(\Delta^2+\eta^2)}A(\lambda,\xi,t), \tag{15}$$

where $\Delta = \lambda - \xi$ and

$$A = \int_{-\infty}^{+\infty} dz\left([\Delta - i\eta\tanh(z)]^2 f(t)R(u)e^{-i\theta} - \frac{\eta^2}{\cosh^2(z)}f^*(t)R^*(u)e^{i\theta}\right)e^{-i\frac{\Delta}{\eta}z}, \tag{16}$$

where $\theta = (\xi/\eta)z + \delta$ and $f(t) = \epsilon(t)$ and $R(u) = -u_{xx}$ correspond to the perturbation terms to the canonical NLSE.

Next we present the results for the two different modulations considered.

(i) For the periodic modulation with $\epsilon(t) = \epsilon\sin(\omega t)$ we obtain from (15) after performing the integration in (16)

$$\frac{\partial\bar{b}(\lambda,t)}{\partial t} = -\frac{2\pi\epsilon e^{4i(\lambda^2+\eta^2)t}\sin(\omega t)(\Delta^2+\eta^2)}{\cosh(\frac{\pi\Delta}{2\eta})}, \tag{17}$$

where we used $b = \bar{b}e^{-4i\lambda^2 t}$.

For the spectral density of the emitted power averaged over one period we find

$$P(\lambda) = \left\langle \frac{d}{dt}(\mid \overline{b(\lambda)} \mid^2) \right\rangle = \left\langle 2\,\mathrm{Re}\left[\bar{b}\frac{\partial \bar{b}^*}{\partial t}\right] \right\rangle. \quad (18)$$

After integration we obtain for $A(\lambda, \eta, \xi)$ and the spectral power $P(\lambda)$

$$A = \frac{4\pi\eta i(\lambda^2+\eta^2)^2}{\cosh(\frac{\pi\Delta_0}{2\eta})},$$

$$P(\lambda) = \frac{2\pi^3\epsilon^2(\Delta_0^2+\eta^2)^2}{\cosh^2(\frac{\pi\Delta_0}{2\eta})}[\delta_D(4(\Delta_0^2+\eta^2)-\omega) +\delta_D(4(\Delta_0^2+\eta^2)+\omega)], \quad (19)$$

where $\Delta_0 = \lambda - \xi_0$ and δ_D is the Dirac delta function.

We observe that the emission is Doppler shifted by the constant initial velocity ξ_0 and is exponentially narrow. Substituting into (15) the adiabatic expressions for the phase $\delta(t)$ from (6) and (7) and using [11]

$$e^{iz\sin(\theta)} = \sum_{-\infty}^{+\infty}(-1)^n \mathcal{J}_n(z)e^{-in\theta},$$

where $\mathcal{J}_n$ is the Bessel function of order n, we obtain the condition for a resonant emission

$$4(\lambda^2+\xi^2-\eta^2)+n\omega = 0, \quad (20)$$

where the amplitude of a higher harmonic is $\approx \mathcal{J}_n^2(\frac{4\eta^2\epsilon}{\omega})$. This formula agrees with the one obtained by Gordon [3] and Elgin [14]. Figure 1 shows the power spectrum of the numerical solution of (1) for the resonant case $\omega = 4\eta^2$. Higher harmonics of emission can clearly be seen.

(ii) For the random modulation we assume that (8) and (9) are satisfied. It is necessary to calculate the mean spectral density of emitted power, i.e.,

$$P(\lambda) = 2\mathrm{Re}\left\langle \left[\bar{b}\frac{\partial \bar{b}^*}{\partial t}\right] \right\rangle, \quad (21)$$

where $\langle\cdots\rangle$ denotes ensemble averaging. Using (8), (9),

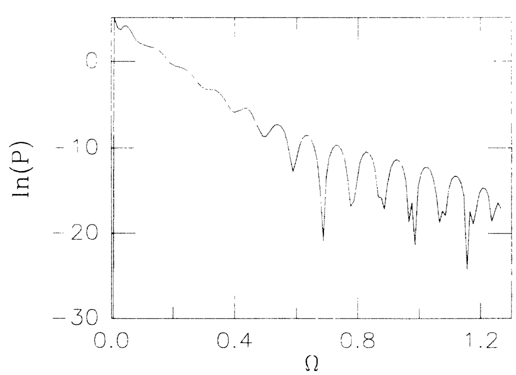

FIG. 1. Power spectrum in log-linear scale for the resonant case.

(13), (14), and (21) we have

$$P(\lambda) = \frac{4\pi^2\sigma_\epsilon^2(\Delta_0^2+\eta^2)^2}{[1+16t_c^2(\Delta_0^2+\eta^2)^2]\cosh^2(\frac{\pi\Delta_0}{2\eta})}. \quad (22)$$

Let us now calculate the correction to the soliton profile $u = u_{ad}(x,t) + \delta u(x,t)$ where $\delta u(x,t) = 2\eta i e^{-i\theta}(\overline{\omega} + \tilde{\omega})$, where $\overline{\omega}$ describes the part of the correction that moves together with the soliton and changes its shape and $\tilde{\omega}$ is the oscillating part of the radiation [8]. The first expression can be calculated analytically:

$$\overline{\omega} = \epsilon\,\mathrm{sech}(z)[1-2z\tanh(z)],$$

where $z = 2\eta x$. Substituting $\overline{\omega}$ into the expression for u we obtain

$$u_{\rm loc} = 2i\eta e^{-i\theta}[1+\epsilon(t)]\mathrm{sech}\{[1+\epsilon(t)]^2 z\}. \quad (23)$$

Thus the soliton duration in media with variable dispersion is changed. A similar phenomenon for the sine-Gordon kink was noted in [9]. The oscillating part is calculated in the Appendix by using the stationary phase method and is equal to

$$\tilde{\omega} = -\frac{\epsilon\sqrt{\pi}}{2\cosh(\frac{\pi x_c}{2})(x_c^2+1)\sqrt{4\eta^2 t}} \times[x_c + i\tanh(z)]^2 e^{-i\delta_1 - i\frac{\pi}{4}} + \frac{e^{i\delta_1 + i\frac{\pi}{4}i}}{[\cosh(z)]^2}, \quad (24)$$

where $\delta_1 = 4\eta^2 t - x^2/4\eta t$ and $x_c = x/4\eta t$.

In the following we compare the evolutions given by (23) and (24) with the numerical solution of Eq. (1) which was obtained by the method of lines using a finite difference discretization in space with Dirichlet boundary conditions. The nonlinear term was given the standard form. The resulting system of ordinary differential equations was solved by a predictor corrector method of fourth order. The accuracy of the computation has been checked by monitoring the values of the mass and momentum which remain conserved in the perturbed case (and energy in the unperturbed case). In all cases the relative error for these quantities was less than 10^{-3}. It has been pointed out by Herbst and Ablowitz [12] that this nonintegrable discretization of the NLSE could lead to numerical chaos while the integrable variant does not. To check for this we have systematically changed the form of the nonlinear term and compared the results. In order to avoid reflections of the radiation component from the boundaries we have added to the equation a damping term acting only close to the boundaries. Figures 2(a) and 2(b) show the profiles given by (23) and (24) and the numerical solutions of (1) for $\omega = 1$ and $\omega = 0.5$ for three different values of t. We see that (23) and (24) approximate well the changes of the soliton shape in the nonresonant case. For $\omega = 4\eta^2$, the resonant value predicted by the theory, (23) and (24) cannot describe the solution of (1). This is very apparent in Fig. 3 which shows the maximum of the modulus of the solution for the numerical simulation and the perturbation result.

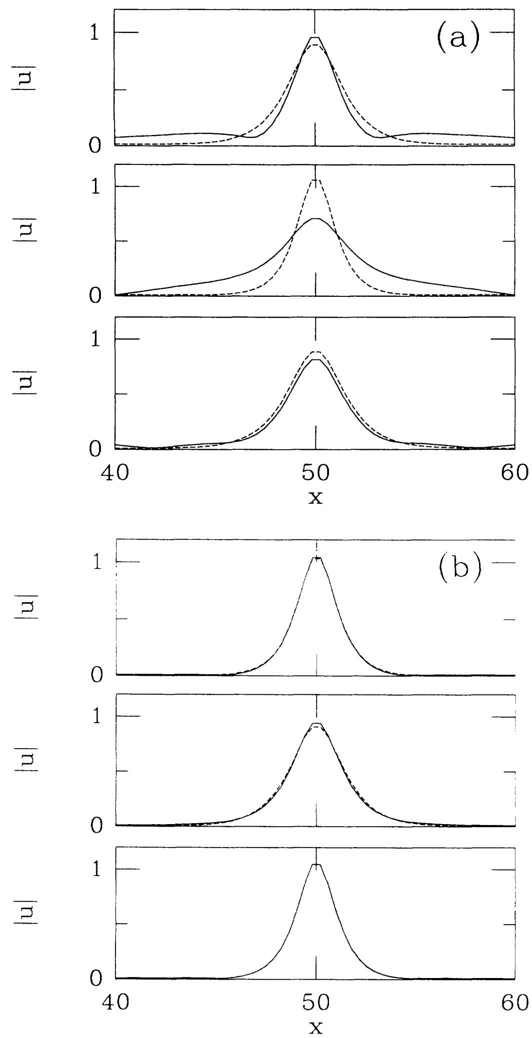

FIG. 2. (a) Profile of $|u|^2$ as a function of x for different times t=10 (top), t=42 (middle), and t= 58 (bottom) for a periodic modulation of dispersion, for $2\eta = 1, \epsilon = 0.1$, and $\omega = 1$ (resonant case) The solid line corresponds to the numerical solution of (1) and the dashed line to (23) and (24). (b) Same as (a) for $\omega = 0.5$.

IV. CALCULATION OF THE RADIATIVE DECAY OF SOLITON

The emission of waves by the soliton under the action of modulation of the dispersion in principle can lead to two effects: radiative damping and soliton deceleration. For an estimation of these effects we use two conserved integrals for Eq. (1): the number of quanta

$$N = \int_{-\infty}^{+\infty} | u |^2 \, dx = \text{const}, \tag{25}$$

and the field momentum Q

$$Q = \frac{i}{2} \int_{-\infty}^{+\infty} (u_x u^* - u_x^* u) dx = \text{const}. \tag{26}$$

For these integrals the following IST formulas are valid:

$$N = 4\eta + \int_{-\infty}^{+\infty} (| b(\lambda) |^2) d\lambda, \tag{27}$$

$$Q = 8\xi\eta + \int_{-\infty}^{+\infty} 2\lambda (| b(\lambda) |^2) d\lambda. \tag{28}$$

Differentiating with respect to time, we have the relations

$$\frac{dN}{dt} = 4\frac{d\eta}{dt} + \int_{-\infty}^{+\infty} P(\lambda) d\lambda = 0, \tag{29}$$

$$\frac{dQ}{dt} = 8\frac{d(\xi\eta)}{dt} + \int_{-\infty}^{+\infty} 2\lambda P(\lambda) d\lambda = 0. \tag{30}$$

Using relation (29) the radiative damping of the solitons can be calculated.

Let us first consider the periodic modulation. We have

$$\mathcal{P} = \int_{-\infty}^{+\infty} P(\lambda) d\lambda = \frac{\pi^3 \epsilon^2 \omega^2}{32 (\cosh[\frac{\pi}{2} \sqrt{\frac{\omega}{4\eta^2} - 1}])^2 \sqrt{\frac{\omega}{4\eta^2} - 1}}. \tag{31}$$

The behavior of $\mathcal{P}$ as a function of ω is shown in Fig. 4 for

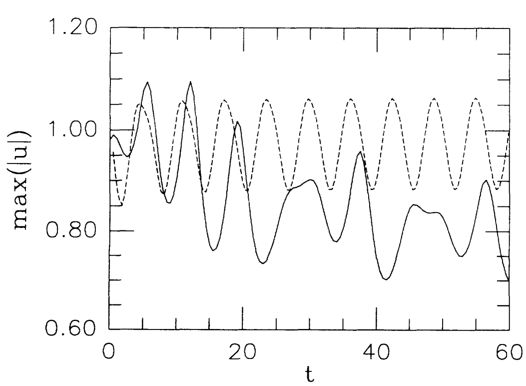

FIG. 3. Maximum of $|u|$ vs time for the resonant case of Fig. 2(a)

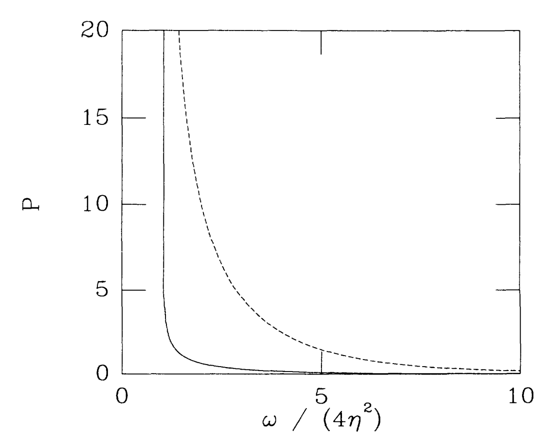

FIG. 4. Total radiation emitted from (31) as a function of ω for $\eta = 1$ and 0.5.

$\epsilon = 1$ for different values of η. It is easy to see that there is emission only for $\omega \geq 4\eta^2$. Similar results have been obtained for Langmuir solitons interacting with sound waves by Malomed [13]. The above restriction shows that the important modulation periods $\lambda_a = \frac{2\pi}{\omega}$ are the ones that are smaller than the period of internal oscillation of the soliton $\frac{2\pi}{4\eta^2} = \lambda_s^{(i)}$. If we consider the fundamental soliton in the fiber with $2\eta = 1$, we see that $\lambda_a = 8z_0$ where $z_0 = \frac{\pi}{4}$ is the soliton period. This agrees with previous results [3,14]. On the other hand, for $\lambda_a \ll x_0$ the emission is exponentially small and the soliton remains robust with small losses, so that it can be described by averaging out the high frequency oscillations of Eq. [1]. For the picosecond duration solitons, $x_0 = \frac{\pi}{4}$ is about a few hundred meters and the influence of variation of k'' for the emission can be neglected. However for femtosecond (fs) pulses x_0 is a few meters and this effect must be taken into account. In particular the intensive emission can lead to the decay of the soliton. This effect can be important in optical fiber loop devices with a variable dispersion part. The radiative damping of the soliton easily follows from (26) and (28) and is given by the equation

$$\frac{d\eta}{dt} = -\frac{\pi^3\epsilon^2\omega^2}{2^7(\cosh[\frac{\pi}{2}\sqrt{\frac{\omega}{4\eta^2}-1}])^2\sqrt{\frac{\omega}{4\eta^2}-1}}. \quad (32)$$

Here the variable t corresponds to a space variable along the fiber. This equation is difficult to integrate analytically, and we perform a numerical integration. The results for $\epsilon = 0.3$ and $\omega = 1.01$, 1.2, 2, and 5 are presented in Fig. 5. It is clear that there is resonant emission for $\omega = 1$ and that the emitted power decreases with ω. It seems that the long distance behavior is described by a logarithmic law. This phenomenon can be observed in an optical loop with a local change of the dispersion. Consider, for example, the fundamental soliton ($2\eta = 1$) propagation in a fiber loop with slowly varying dispersion. Let the loop length be equal to L, the second-order dispersion $\beta_2 = -20$ ps^2 / km, the second-order dispersion changes be $\Delta\beta = 5$ ps^2 / km,

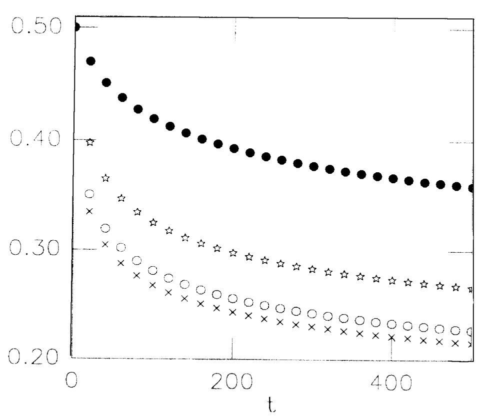

FIG. 5. Radiative damping in time of the soliton amplitude for the periodic case for $\epsilon = 0.3$ and $\omega = 1.01$ (×), 1.2 (∘), 2 (⋆), and 5 (•).

$\gamma = \frac{n_2\Omega}{cA_{\text{eff}}} \approx 2$ W^{-1} km^{-1} for $\lambda = 1.55$ μm, the fiber have a core diameter $a = 9.3$ μm, and the soliton pulse duration be $T_0 = 1$ps. Then for $L = \pi \times 50$ m ≈ 155.2 m we obtain from Fig. 5 that after 500 round trips the soliton amplitude decreases due to radiative damping by 77%. This apparently strong damping would be interesting to observe experimentally.

The decay time can easily be estimated to be $t_d \sim \frac{1}{\epsilon^2\omega^2}$. We obtain here a very interesting result: the decay of solitons when the number of quanta is conserved. Because the total integral $N = N_d + N_c$ is conserved, the growth of the continuum component N_c leads to the decrease of the discrete component N_d and η correspondingly.

From (30) one can infer the influence of the perturbation on the velocity (radiative deceleration):

$$2\int \lambda P(\lambda)d\lambda + (8\xi\eta)_t = 0. \quad (33)$$

The calculation showed that

$$8\xi_t\eta = 0 \text{ and } \xi = \text{const.} \quad (34)$$

The soliton is not decelerated under emission. This is related to the effect that the number of quanta emitted forward and backward are equal.

We now study a medium with random dispersion.

(i) Let us begin our consideration with the white noise case. From Eq. (22) we obtain

$$\int_{-\infty}^{+\infty} P(\lambda)d\lambda = 4\pi^2\sigma_\epsilon^2\mu\eta^5, \quad (35)$$

$$\frac{d\eta}{dt} = -\pi^2\sigma_\epsilon^2\mu\eta^5, \quad (36)$$

where

$$\mu = \int_{-\infty}^{\infty} \frac{(z^2+1)^2}{[\cosh(\frac{\pi z}{2})]^2}dz = \frac{128}{15\pi}. \quad (37)$$

Integration gives

$$\eta = \frac{\eta_0}{(1+4\pi^2\mu\eta_0^4 t)^{\frac{1}{4}}}, \quad (38)$$

so that the decay length is

$$t_d = \frac{1}{4\pi\sigma_\epsilon^2\mu\eta_o^4}.$$

(ii) For a noise with a finite correlation length ($t_c \neq 0$) we obtain

$$\frac{d\eta}{dt} = \int_{-\infty}^{+\infty} \frac{\pi^2\sigma^2(\lambda^2+\eta^2)^2}{[1+16t_c^2(\lambda^2+\eta^2)^2][\cosh(\frac{\pi\lambda}{2\eta})]^2}d\lambda. \quad (39)$$

This equation is difficult to solve explicitly, so we integrate it numerically. Results for $\epsilon = 0.3$ and $t_c = 0$, 0.2, 3, and 5 are presented in Fig. 6. The first value of

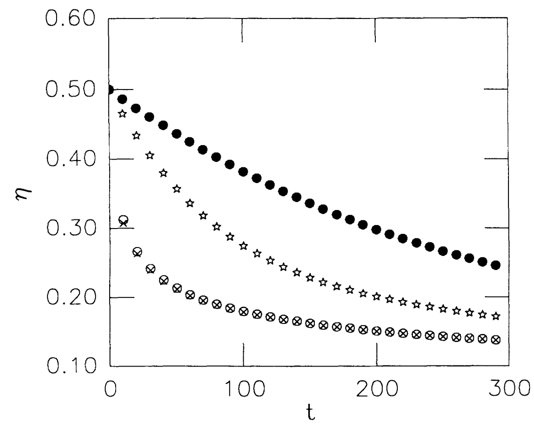

FIG. 6. Radiative damping in time of the soliton amplitude for a random ϵ with correlation time $t_c = 0$ ($\times$), 0.2 ($\circ$), 3 ($\star$), 5 ($\bullet$).

t_c corresponding to the white noise case agrees very well with (38). The increase of the correlation length leads to an increase of the soliton radiative decay length.

V. CONCLUSION

The above results show that for $\lambda \gg x_0$ the emission is exponentially small and the adiabatic description is valid. When the modulation length is compatible with x, i.e., $\lambda = x_0$ we have resonant emission of the soliton. This can be important for the propagation of femtosecond solitons in fibers and in optical loop devices with a variable dispersion. From the expression of the emitted power $\mathcal{P}$ it follows that for $\lambda \ll x_0$ the emission is small and the guiding center soliton description is valid. This confirms the analysis of [1]. The analysis also showed that there is a radiative damping of the soliton. This phenomenon can be observed experimentally in an optical loop device with locally varying dispersion after a few hundred round trips of the pulse.

Note added. When this work was completed we learned about the study of Malomed *et al.* [15], who formulated the adiabatic dynamics of a pulse with chirped frequency by a variational approach. Our expressions (23) and (24) are compatible with the results of this approach. We are grateful to these authors for providing a copy of their work prior to publication.

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APPENDIX

The field up to the first-order correction is defined by $u = u_{ad} + \delta u$, where

$$\delta u = 2i\eta e^{i\theta}(\overline{\omega} + \tilde{\omega}), \tag{A1}$$

$$\overline{\omega} = \frac{\epsilon}{8\pi i\eta^2}\int_{-\infty}^{+\infty} \frac{A(\lambda,\eta)[\lambda + i\eta\tanh(z)]^2}{(\lambda^2+\eta^2)^3} e^{i\frac{\lambda}{\eta}x} d\lambda - \frac{\epsilon}{8\pi i[\cosh(z)]^2}\int_{-\infty}^{+\infty} \frac{A^*(\lambda,\eta)}{(\lambda^2+\eta^2)^3} e^{-i\frac{\lambda}{\eta}x} d\lambda, \tag{A2}$$

and $z = 2\eta x$.

For $\epsilon R = \epsilon(t) u_{xx}$ we obtain [see (16) and (18)]

$$A(\lambda,\eta) = \frac{4\pi i\eta(\lambda^2+\eta^2)^2}{\cosh(\frac{\pi\lambda}{2\eta})}. \tag{A3}$$

From (A2) and (A3) we have, using the integral

$$\int_{-\infty}^{+\infty} \frac{\cos(xz)}{(x^2+1)\cosh(\frac{\pi}{2}x)} dx = ze^{-z} + \cosh(z)\ln(1+e^{-2z}), \tag{A4}$$

that the localized part of the correction is

$$\overline{\omega} = \epsilon \operatorname{sech}(z)[1 - 2z\tanh(z)]. \tag{A5}$$

For the oscillating part $\tilde{\omega}$ we find

$$\tilde{\omega} = -\frac{\epsilon e^{-i\delta}}{8\pi i\eta^2}\int_{-\infty}^{+\infty} \frac{A(\lambda,\eta)[\lambda + i\eta\tanh(z)]^2}{(\lambda^2+\eta^2)^3} \times e^{-4i\lambda^2 t + i\frac{\lambda}{\eta}x} d\lambda + \frac{\epsilon}{8\pi i[\cosh(z)]^2} e^{i\delta}\int_{-\infty}^{+\infty} \frac{A^*(\lambda,\eta)}{(\lambda^2+\eta^2)^3} e^{4i\lambda^2 t - i\frac{\lambda}{\eta}x} d\lambda. \tag{A6}$$

Applying the stationary phase method, we obtain for $|z| \ll 8\eta^2$

$$\tilde{\omega} = -\frac{\epsilon\sqrt{\pi}}{2\cosh(\frac{\pi x_c}{2})(x_c^2+1)\sqrt{4\eta^2 t}} \times [x_c + i\tanh(z)]^2 e^{-i\delta_1 - i\frac{\pi}{4}} + \frac{e^{i\delta_1 + i\frac{\pi}{4}i}}{[\cosh(z)]^2}, \tag{A7}$$

where $\delta_1 = 4\eta^2 t - x^2/4\eta t$ and $x_c = \frac{x}{4\eta t}$.

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