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Sound propagation in magnetic fluids

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In magnetic fluids, sound waves are coupled to the magnetic variables since the magnetization oscillates with the fluid density. Here, explicit formulas for the velocity and the damping of sound are determined from the hydrodynamic equations. Thermal expansion and heat diffusion are also included. The magnetic field is not restricted to be homogeneous; arbitrary magnetic forces are allowed in the analysis. The ambiguities inherent in the notion of pressure are shown to have no effect on the sound propagation. The sound velocity and the heat diffusion constant are derived to be anisotropic. This is a truly macroscopic result. The underlying mechanism is not any model assumption on the magnetic fluid particles, but demagnetization, i.e., the coupling of the wave geometry to the Maxwell equations.

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I. INTRODUCTION

One should expect that sound propagation in magnetic fluids is very different from sound in normal, nonmagnetic fluids. In normal liquids, the velocity of sound is usually stated as $u^2=(\partial p/\partial\rho)$, where p is the pressure. In magnetized materials, p is not uniquely defined, so this definition cannot be applied here [1,2]. In addition, there is the variation in $\mathbf{M}$: With particle-diffusion times being slow, the density wave of sound takes the magnetic particles with it and induces an oscillation in the magnetization. Due to the Maxwell equations, this magnetization wave is accompanied by waves in the magnetic field $\mathbf{H}$ and the magnetic induction $\mathbf{B}$, i.e., sound is coupled to electrodynamics.

On the other hand, from experiments it is known that sound in magnetic fluids is essentially the same as in the carrier liquid. Neither a magnetic field as such, nor field gradients causing a ponderomotive force density, have any noticeable influence.

In this work, sound propagation in magnetic fluids is studied theoretically, based on the hydrodynamic equations of motion. It turns out that u is a function of the magnetic field, but the effect is small. Basically, u is given by the compressibility, just as in normal liquids.

The velocity shift derived here is anisotropic; it depends on the angle ψ between the magnetic field and the wave vector of the sound mode $\mathbf{k}$. This is due to demagnetization effects, i.e., to the coupling to the Maxwell equations. This mechanism had never been considered before. It is truly hydrodynamic in nature; no model assumptions are made on the shape or behavior of the magnetic monodomain particles suspended in the liquid carrier. The fluid would be treated as being homogeneous and isotropic—were it not for the magnetic field, which breaks the isotropy and, if inhomogeneous, also the homogeneity. Considering the long wavelength of sound, the continuum assumption should be a valid approximation even for materials as complex as magnetic fluids. On the other hand, ignoring the anisotropy induced by the magnetic field amounts to violating the Maxwell equations. Previous theories, which treated sound as a purely mechanical phenomenon without any relation to electrodynamics, are thus not reliable, cf. Sec. VII below.

The paper is organized as follows: In Sec. II, the magnetic fluid hydrodynamic equations of motion are introduced in their general form, and in Sec. III, they are solved for the quiescent equilibrium state. In Sec. IV, they are linearized around equilibrium, and the variation of the magnetic variables $\delta\mathbf{M}$, $\delta\mathbf{H}$, and $\delta\mathbf{B}$ is studied. The final equations and their solutions are to be found in Sec. V. In Sec. VI, these solutions are discussed. Order of magnitude estimates of the predicted effects are also given. The theory is compared to previous works in Sec. VII. Section VIII contains the conclusions.

II. GENERAL EQUATIONS

Despite the long history of magnetic fluids research, their equations of motion have still to be established in a generally accepted form. In most theoretical models, incompressibility is assumed [3,4]. As in other liquids, this is usually an excellent approximation, but of course not applicable to the study of sound. Some theoretical approaches [5,6] take the mass continuity equation in its general form, and add a viscous term proportional to div**v** to the Navier-Stokes equation. Although, in these works, the constraint of incompressibility is formally relaxed, they are still not good enough to deal with sound: In a compressible fluid, the pressure (whose gradient enters the Navier-Stokes equation) is a true dynamic variable, and needs to be determined. To do so in the presence of the magnetic field requires an appropriate treatment of the electromagnetic contributions to the internal energy—something that has been done just recently [1,2]. The application of this formalism to magnetic fluids [7] finally provides the equations of motion as needed for the study of sound.

Even when the proper definition of the pressure is included, the hydrodynamics of magnetic fluids still contains one more unsettled point, namely, how to account best for angular momentum conservation, and the inter-

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nal rotational degrees of freedom. But fortunately, a correct treatment of the angular momentum is not a necessary prerequisite for the study of sound, cf. the discussion at the end of this section, below.

Hence, the equations to be analyzed here can be taken in the following form [5–7].

(i) The continuity equation of the mass density:

$$\dot{\rho}=-\mathrm{div}(\rho\mathbf{v}) \ . \tag{1}$$

(ii) The Navier-Stokes equation:

$$\rho\frac{d\mathbf{v}}{dt}=-\nabla p+\eta\nabla^2\mathbf{v}+\zeta\nabla\,\mathrm{div}\mathbf{v}+\mu_0 M\nabla H+\mathrm{rot}\hat{\eta}_r\boldsymbol{\Omega} \ , \tag{2}$$

where p is defined as

$$p=Ts+\xi\rho-e+\tfrac{1}{2}(\mathbf{H}\cdot\mathbf{B}+\mu_0\mathbf{M}\cdot\mathbf{H}) \ . \tag{3}$$

Here, T is the temperature, s the entropy density, ξ the chemical potential, and e the density of the total energy, including the fields. The magnetic field variables **H**, **B**, **M** are defined as usual, in the SI system of units. η is the shear, ζ the volume, and $\hat{\eta}_r$ the rotational viscosity. $\boldsymbol{\Omega}$ is the vorticity.

(iii) The quasistationary magnetic field equations:

$$\mathrm{div}\mathbf{B}=0, \quad \mathrm{rot}\mathbf{H}=\mathbf{0} \ . \tag{4}$$

On the time scale of electrodynamics, sound is a static phenomenon: $u \ll c$, where c is the velocity of light.

(iv) The heat diffusion equation:

$$\dot{T}=\frac{\lambda}{c_V}\nabla^2 T+\frac{K}{c_V}\dot{B}-\frac{1}{\rho\alpha}\dot{\rho} \ . \tag{5}$$

In this equation, λ is the heat conductivity, c_V the specific heat at constant volume, α the coefficient of thermal expansion, and K the pyromagnetic coefficient: $K=-T(\partial M/\partial T)_{\rho,B}>0$.

As the set of independent variables, the mass density ρ, the temperature T, and the magnetic induction **B** are chosen throughout.

The balance of angular momentum is already incorporated into the Navier-Stokes equation (2), hence its last term [8]. This term is sometimes also stated as [5,6] $\mathrm{rot}\eta_r(\boldsymbol{\omega}-\boldsymbol{\Omega})$, where $\boldsymbol{\omega}$ is the rotational velocity of the suspended magnetic particles, or as [4] $\mathrm{rot}(\mathbf{s}-\Theta\boldsymbol{\Omega})/\tau_s$, where **s** is the density of the internal angular momentum, and Θ some density of moment of inertia. But obviously, in any case the term in question reads "rot[$\cdots$]." As only the divergence of the Navier-Stokes equation will be used in the subsequent analysis [cf. Eq. (10), below], this term does not contribute. The difficulties associated with the concept of an internal angular momentum [9] do not affect longitudinal excitations.

Sometimes it is argued that the sound damping should increase simply proportional to the well-known effect on the viscosity η [4]. This increase in the effective viscosity, as observed in shear experiments, is due just to the rotational degrees of freedom, i.e., to the rot[$\cdots$] term. In a compressional motion, the same term also contributes to the effective volume viscosity, and they cancel: ∇ rot[$\cdots$] stays zero.

III. EQUILIBRIUM STATE

As the equilibrium reference point, the quiescent liquid at rest is taken: $\mathbf{v}_0=\mathbf{0}$. This leads immediately to $\mathbf{M}_0 \| \mathbf{H}_0$. However, the magnetic field $\mathbf{H}_0$ is allowed to be nonuniform, as this will nearly always be the case in practice. The arising ponderomotive force density is balanced by a nonzero pressure gradient [7]:

$$\nabla p_0=\mu_0 M_0\nabla H_0\neq 0 \ . \tag{6}$$

The pressure p has the differential [2] $dp=s dT+\rho\, d\xi+\mu_0\mathbf{M}\cdot d\mathbf{H}$, or, in terms of the chosen independent variables ρ, T, and **B**:

$$dp=\left[s+\rho\left[\frac{\partial\xi}{\partial T}\right]\right]dT+\rho\left[\frac{\partial\xi}{\partial\rho}\right]d\rho+\rho\left[\frac{\partial\xi}{\partial B}\right]dB +\mu_0\mathbf{M}\cdot d\mathbf{H} \ . \tag{7}$$

With the help of Eq. (7), Eq. (6) can be rewritten as

$$\mathbf{0}=\nabla p_0-\mu_0 M_0\nabla H_0 =\left[s+\rho\left[\frac{\partial\xi}{\partial T}\right]\right]\nabla T_0+\rho\left[\frac{\partial\xi}{\partial\rho}\right]\nabla\rho_0+\rho\left[\frac{\partial\xi}{\partial B}\right]\nabla B_0 \ . \tag{8}$$

Of course, in equilibrium $\nabla T_0=\mathbf{0}$. Note that the (∇H_0) terms cancel in Eq. (8), whereas B as one of the chosen independent variables comes into play. At this point, the pressure has already dropped out of the theory, and so have all problems concerning uniqueness, or measurability [2]: p is a function of the density, the temperature, and the magnetic field, and can always be expressed in terms of these variables, cf. Eq. (3). On the other hand, this function is more or less arbitrary; variations in the pressure as such are not independently measurable.

Employing now the Maxwell relations for the second derivatives of the free energy, Eq. (8) is brought in its final form:

$$u_0^2\nabla\rho_0=\mathcal{M}\nabla B_0 \ . \tag{9}$$

Here, the abbreviation $\mathcal{M}=\rho(\partial M/\partial\rho)_{T,B_0}$ is introduced, and $u_0^2=\rho(\partial\xi/\partial\rho)_{T,B_0}$ is put in analogy to the velocity of sound u in normal liquids. Equation (9) is the static equilibrium solution for a magnetic fluid in an arbitrary magnetic field.

IV. LINEARIZATION AROUND THE EQUILIBRIUM STATE

The equations of motion are linearized around the equilibrium solution, and the time derivative of Eq. (1) is combined with the divergence of Eq. (2), with the result

$$\ddot{\rho}-u_0^2\nabla^2\delta\rho-\left[s-\rho\left[\frac{\partial s}{\partial\rho}\right]\right]\nabla^2\delta T+\mathcal{M}\nabla^2\delta B+\frac{(\eta+\zeta)}{\rho_0}\left[-\nabla^2\dot{\rho}+\frac{\dot{\rho}}{\rho_0}\nabla^2\rho_0-\nabla^2\left[\frac{1}{\rho_0}(\mathbf{v}\cdot\nabla)\rho_0\right]\right]-(\nabla u_0^2)(\nabla\delta\rho)+(\nabla\mathcal{M})(\nabla\delta B)=0\ . \quad (10)$$

In linear order, only the field-parallel component of $\delta\mathbf{B}$ contributes in Eq. (10): $\delta B=\delta\mathbf{B}\cdot\hat{\mathbf{b}}$, where $\hat{\mathbf{b}}$ is the direction of the undisturbed field.

The last four terms of Eq. (10) are nonzero, since in magnetized materials the density is not necessarily uniform in equilibrium. However, given the generally low compressibility (i.e., high velocity of sound) of liquids, Eq. (9) is now used to assume that the equilibrium variation is small compared to the one in the sound wave:

$$|\nabla\rho_0|=\frac{1}{u_0^2}|\mathcal{M}\nabla B_0|\ll k\rho_0\ , \quad (11)$$

where k is the wave vector. From this single assumption it follows that

$$\left|\frac{\dot{\rho}}{\rho_0}\nabla^2\rho_0\right|\ll|\nabla^2\dot{\rho}|,\quad \left|\nabla^2\left[\frac{1}{\rho_0}(\mathbf{v}\cdot\nabla)\rho_0\right]\right|\ll|\nabla^2\dot{\rho}|\ ,$$

$$|(\nabla u_0^2)(\nabla\delta\rho)|\ll|u_0^2\nabla^2\delta\rho|,\quad |(\nabla\mathcal{M})(\nabla\delta B)|\ll|\mathcal{M}\nabla^2\delta B|\ .$$

Hence, the last four terms of Eq. (10) are all negligible. As a result, arbitrary magnetic field gradients do not affect the propagation of sound, even when they induce strong force densities and strong pressure gradients (cf. the numerical estimate in Sec. VI, below). The analysis presented here is therefore generally applicable; it is not restricted to homogeneous fields.

To finally close the set of two equations, Eq. (10) and Eq. (5), δB has to be determined as a function of $\delta\rho$ and δT. To do so, the Maxwell equations (4) are used. We observe that, when oscillating with the density, the magnetization is still in equilibrium. The magnetic relaxation time τ is only about $\tau\approx10^{-6}$ s, and therefore $\omega\tau\ll1$ for the whole acoustic frequency range up to 20 kHz. This leads to the ansatz

$$\delta\mathbf{M}=\left[\left[\frac{\partial M}{\partial\rho}\right]\delta\rho+\left[\frac{\partial M}{\partial T}\right]\delta T+\left[\frac{\partial M}{\partial B}\right]\delta B\right]\hat{\mathbf{b}}+\frac{M}{B}\hat{\mathbf{b}}\times(\delta\mathbf{B}\times\hat{\mathbf{b}})\ . \quad (12)$$

The static Maxwell equations demand $\mathrm{div}\delta\mathbf{B}=0$, and $\mathrm{rot}\delta\mathbf{H}=0$. These equations have the plane wave solution

$$\delta\mathbf{H}=-(\delta\mathbf{M}\cdot\hat{\mathbf{k}})\hat{\mathbf{k}}\ ,$$
$$\delta\mathbf{B}=-\mu_0(\delta\mathbf{M}\cdot\hat{\mathbf{k}})\hat{\mathbf{k}}+\mu_0\delta\mathbf{M}=-\mu_0(\delta\mathbf{M}\times\hat{\mathbf{k}})\times\hat{\mathbf{k}}\ , \quad (13)$$

with the direction of the wave vector $\hat{\mathbf{k}}=\mathbf{k}/k$. The insertion of Eq. (13) into Eq. (12) yields

$$\delta B=\frac{\mu\left[\left[\frac{\partial M}{\partial\rho}\right]\delta\rho+\left[\frac{\partial M}{\partial T}\right]\delta T\right](1-\cos^2\psi)}{1+\left[\chi-\mu\frac{M}{B}\right]\cos^2\psi}\ , \quad (14)$$

where the susceptibility $\chi/\mu=(\partial M/\partial B)_{\rho,T}$ has to be taken at the point B_0. ψ is the angle between the undisturbed magnetic field $\mathbf{B}_0$ and the wave vector $\mathbf{k}$. The fact that a nonzero $\delta\mathbf{M}$ is accompanied by field variations $\delta\mathbf{H}$ and $\delta\mathbf{B}$, and Eq. (13) relating these three quantities, has never been noted before when discussing waves in magnetic fluids. The structure of Eq. (14), due to the Maxwell equations rather than a microscopic model, is thus a completely new result.

V. FINAL EQUATIONS AND THEIR SOLUTIONS

With Eq. (14), the derivation of the linearized equations is complete: δB is expressed in terms of $\delta\rho$ and δT, and the set of two coupled differential equations (10) and (5) is closed:

$$\ddot{\rho}-u_\gamma^2\nabla^2\delta\rho-a\nabla^2\delta T-\frac{(\eta+\zeta)}{\rho_0}\nabla^2\dot{\rho}=0\ , \quad (15)$$

$$\dot{T}-\frac{\lambda}{c_V\kappa}\nabla^2\delta T+b\dot{\rho}=0\ . \quad (16)$$

The constants are

$$a=s-\rho(\partial s/\partial\rho)+\gamma\mu\mathcal{M}K/T_0\ ,$$
$$b=[1-\gamma\mu\mathcal{M}K\alpha/c_V]/(\alpha\rho_0\kappa)\ ,$$
$$\kappa=1+\gamma\mu K^2/(T_0c_V)\ ,$$

and

$$u_\gamma^2=u_0^2-\gamma\frac{\mu\mathcal{M}^2}{\rho_0}\ . \quad (17)$$

γ denotes the anisotropy

$$\gamma=\frac{(1-\cos^2\psi)}{1+\left[\chi-\mu\frac{M}{B}\right]\cos^2\psi}\ . \quad (18)$$

Equations (15) and (16) have exactly the same structure as in normal, nonmagnetic fluids. They have therefore the two following familiar solutions.

(i) *A pair of sound modes:* The propagating variable is $v/u=\delta\rho/\rho_0=-\alpha\delta T$. It has the dispersion relation

$$\omega=\pm uk-i\frac{(\eta+\zeta)}{2\rho_0}k^2\ . \quad (19)$$

u is the velocity of sound,

$$u^2=u_\gamma^2-ab\ . \quad (20)$$

(ii) *Heat diffusion:* The diffusing variable is $\delta T=-\delta\rho\,u_\gamma^2/a$. It has the dispersion relation

$$\omega=-iDk^2\ . \quad (21)$$

D is the diffusion constant

$$D = \frac{\lambda}{c_V \kappa} \frac{u_\gamma^2}{u^2} \,. \tag{22}$$

In these equations, the effect of the magnetic field is felt only through the contributions proportional to the parameter γ. Being anisotropic, they are interesting in principle. But they are usually very small. This is discussed in the next section.

VI. DISCUSSION AND ORDER OF MAGNITUDE ESTIMATES

The longitudinal collective modes in magnetic fluids differ only very slightly from the respective ones in normal liquids; their structure is not changed at all. The dispersion relation $\omega = \pm uk - iD_s k^2$, and also the sound damping $D_s = (\eta + \zeta)/2\rho_0$ are exactly as in a nonmagnetic normal fluid. The physical reason, of course, is that sound is a wave in the density, rather than in the field-dependent pressure. Due to the low compressibility of liquids, a nonzero ponderomotive force density—rendering the pressure in the Navier-Stokes equation essentially inhomogeneous—does not considerably affect the equilibrium density distribution, cf. the estimate below.

The velocity of sound u has some anisotropic magnetic contributions though. Rather than from p, they stem from the dependence of the magnetization on the temperature and the density (and vice versa) and are thus given in terms of various thermodynamic coefficients:

$$\begin{aligned} u^2 &= u_\gamma^2 - ab \\ &\approx u_0^2 - \frac{a_0}{\alpha\rho_0} - \gamma \left[\frac{\mu \mathcal{M} K}{T_0 \alpha \rho_0} + \frac{\mu \mathcal{M}^2}{\rho_0} \right. \\ &\quad \left. - \frac{a_0 \mu K^2}{T_0 c_V \alpha \rho_0} - \frac{a_0 \mu \mathcal{M} K}{c_V \rho_0} \right] , \end{aligned}$$

where $a_0 = s - \rho(\partial s/\partial \rho)$; γ from Eq. (18) is the anisotropy parameter.

Let us approximate $\mathcal{M}$ by $M = \chi H_0$, and take [10]

$$\chi \approx 0.1, \quad H_0 \approx 10^6 \text{ A/m} ,$$

$$\rho_0 \approx 1.5 \times 10^3 \text{ kg/m}^3, \quad \alpha \approx 3 \times 10^{-4} \text{ 1/K} ,$$

$$K \approx 5 \times 10^3 \text{ A/m}, \quad T_0 \approx 300 \text{ K} ,$$

$$a_0 \approx c_V \approx 3 \times 10^6 \text{ J/K m}^3 .$$

Then we have $\mu \mathcal{M} K/(T_0 \alpha \rho_0) \approx 10^1$ (m/s)^2, $\mu \mathcal{M}^2/\rho_0 \approx 10^1$ (m/s)^2, $a_0 \mu \mathcal{M} K/(c_V \rho_0) \approx 10^0$ (m/s)^2, and $a_0 \mu K^2/(T_0 c_V \alpha \rho_0) \approx 10^{-1}$ (m/s)^2. As the sound velocity in magnetic fluids has approximately the same value as that in the carrier fluid ($u \approx 1400$ m/s), all these contributions yield only a very small velocity shift: $\delta u \approx 10^{-5} u$—the same order of magnitude as the dependence of the thermodynamic coefficients on the magnetic field (cf. Appendix), which is usually neglected. The propagation of sound waves in magnetized liquids is thus essentially unaffected by the magnetic field.

The anisotropic magnetic shift in the velocity of sound might be observable, however, if one searches for it with the help of a resonance chamber. A quality factor of 10^5 should be achievable in a sound resonance experiment. Then, a change in the direction of the applied magnetic field will result in a detuning of the resonance. This method allows us to obtain information on the magnetization from sound experiments.

The second collective mode in magnetic fluids studied here, heat diffusion (which is, due to thermal expansion, also a density diffusion), is also slightly altered in the presence of a magnetization. The ratio of temperature to density variation of the diffusion mode is $\delta T/\delta\rho = -u_\gamma^2/a = -[1 - \gamma\delta_1] u_0^2/a_0$, and the diffusion constant is $D = \lambda u_0^2 [1 - \gamma \delta_2]/[c_V(u_0^2 - a_0/\alpha\rho_0)]$, where $\delta_{1,2}$ stand for a number of terms of a magnitude of about 10^{-5}. As diffusion cannot be brought into resonance, these contributions will probably never be observable.

Finally, the inequality Eq. (11) has to be checked. Let us assume the field gradient to be $|\nabla B| \approx 10$ T/m. This yields a force density $|\mu_0 M \nabla H| \approx 10^6$ N/m^3 that far exceeds gravity. But, with Eq. (9), $|\nabla \rho_0| \approx 1$ kg/m^4 and $|\nabla \rho_0| \ll k\rho_0$ is well fulfilled for all acoustic frequencies.

VII. COMPARISON TO PREVIOUS WORKS

Up to now, only a few authors have attempted to investigate the propagation of density waves in magnetic fluids. In 1975, Parsons [11] published a much-cited study which, however, seems to not capture the relevant physics. He treats the domain spin of the suspended magnetic particles in the spirit of the director in the theory of nematic liquid crystals, i.e., as a broken-symmetry variable. Undoubtedly, the magnetic relaxation times in magnetic fluids are long enough to allow deviations of the magnetization from its equilibrium value to be experimentally observable. But nevertheless, under symmetry transformations, magnetic fluids are isotropic liquids in a magnetic field, not liquid crystals [8]. Parsons's ansatz is therefore not *a priori* convincing.

On the other hand, Parsons ignores all features discussed in this work. He assumes the existence of a purely mechanical pressure and does not pay attention to electrodynamics, or the Maxwell equations. Parsons's predictions have proven difficult to verify in experiments.

Tarapov [12] studies a variety of small-amplitude waves in isotropic magnetic fluids. His ansatz is quite general; the dynamic Maxwell equations are included in the set of equations of motion. But there are a number of ambiguities in his treatment of the thermodynamics. The entropy is taken as one of the independent variables, but the magnetization is assumed to be a function of T, not of s; the relation between T and s, with and without the magnetic field, respectively, is not clarified. In defining the thermodynamic coefficients such as the compressibility or the specific heat, the independent magnetic variable is not specified. The formula for the sound velocity depends crucially on these details, cf. Appendix. Therefore, Tarapov's work gives much insight into the complexity of the problem, but is not very useful for obtaining explicit results.

The same conclusions have been reached by Gotoh, Isler, and Chung, who carried out a series of ultrasound experiments in magnetic fluids [13–17]. They find Parsons's theory inapplicable [13], and take Tarapov's work as the starting point for their own theoretical model [14,15]. Unfortunately, they do not resolve the uncertainties in Tarapov's derivation and retain a large number of adjustable parameters.

They also report a strong dependence on the fluid history [13,16] and unusually long response times, comparable to the formation of aggregates [17]. These results indicate that ultrasound in magnetic fluids probes the inhomogeneities associated with the formation of chains or clusters. Anisotropies recorded with ultrasound therefore seem to depend on a completely different mechanism than the anisotropy in the hydrodynamic velocity of sound predicted here; they cannot be compared.

This remark also applies to the work of Taketomi [18]. His ideas on cluster dynamics are interesting and valuable for a discussion of ultrasound damping. But he neglects any dependence of the thermodynamic variables on the magnetic field, which is wrong on a hydrodynamic scale. His approach is thus a microscopic model rather than a macroscopic theory; it cannot be applied to sound.

VIII. CONCLUSIONS

Sound propagation and heat diffusion in magnetic fluids are studied theoretically, starting from the hydrodynamic equations of motion. The field is treated as a homogeneous, isotropic liquid, exposed to an arbitrary magnetic field. The longitudinal collective modes do not change their structure in the presence of a magnetic field, even if it is inhomogeneous. For the sound velocity, the sound damping, and the heat diffusion constant, explicit expressions are derived. The velocity of sound is shifted slightly from its simple-liquid value, due to the dependence of the magnetization on the density and on the temperature. The correction to the sound velocity is only about 10^{-5} u, but might be observable in a resonance experiment. It depends on the angle between $\mathbf{B}$ and $\mathbf{k}$, cf. Eq. (18). This anisotropy is derived from the Maxwell equations; it reflects the demagnetization factors of the wave.

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APPENDIX: H vs B

There seems to be no effect of the magnetic field for $\mathbf{k}\|\mathbf{B}$, since $\delta B=0$ in that case [Eq. (14), $\cos\psi=1$]. But this is not true. From Eq. (13), a wave parallel to the magnetic field ($\mathbf{k}\|\hat{\mathbf{b}}$) acts as an excitation at constant B, whereas a wave perpendicular to the magnetic field ($\mathbf{k}\perp\mathbf{b}$) is an excitation at constant H. All fluid parameters depend on that difference:

$$\begin{aligned} u_0^2 &= \rho\left(\frac{\partial\xi}{\partial\rho}\right)_B = \rho\left(\frac{\partial\xi}{\partial\rho}\right)_H + \frac{\mu\mathcal{M}^2}{\rho}\,, \\ c_V &= T\left(\frac{\partial s}{\partial T}\right)_B = T\left(\frac{\partial s}{\partial T}\right)_H - \frac{\mu K^2}{T}\,, \end{aligned} \tag{A1}$$

etc. Of course, the theory can also be formulated with H, instead of B, as the chosen independent variable. The thermodynamic coefficients are then redefined in terms of partial derivatives at fixed H. In that formulation, the anisotropy parameter γ will change to $(\gamma-1)$. Contrary to the equations above, the effect of the magnetic field then seemingly disappears for $\mathbf{k}\perp\mathbf{H}$, i.e., $\delta H=0$.

But a comparison of Eqs. (17) and (A1) shows clearly that the final results do not depend on the choice of formulation, and are exactly the same in both cases:

$$\begin{aligned} u_\gamma^2(B) &= \rho\left(\frac{\partial\xi}{\partial\rho}\right)_B - \gamma\frac{\mu\mathcal{M}^2}{\rho} \\ &= \rho\left(\frac{\partial\xi}{\partial\rho}\right)_H - (\gamma-1)\frac{\mu\mathcal{M}^2}{\rho} = u_\gamma^2(H)\,. \end{aligned}$$

In principle, *all* thermodynamic coefficients are functions of the magnetic field and dependent on whether the partial derivatives are defined at fixed H or fixed B. Therefore, the question of the field-induced effects has to be studied very carefully; it cannot be answered by discussing single terms added to the simple-liquid result.

[1] K. Henjes and M. Liu, Ann. Phys. (N.Y.) **223**, 243 (1993).
[2] K. Henjes, Ann. Phys. (N.Y.) **223**, 277 (1993).
[3] R. A. Curtis, Phys. Fluids **14**, 2096 (1971).
[4] M. I. Shliomis, Usp. Fiz. Nauk **112**, 427 (1974) [Sov. Phys. Usp. **17**, 153 (1974)].
[5] R. E. Rosensweig, *Ferrohydrodynamics* (Cambridge University Press, Cambridge, 1985).
[6] K. Shizawa and T. Tanahashi, Bull. JSME **29** (255), 32 (1986).
[7] K. Henjes, Z. Phys. B **92**, 113 (1993).
[8] K. Henjes, Physica A (to be published).
[9] K. Henjes, J. Magn. Magn. Mater. **117**, L311 (1992).
[10] T. Sawada, H. Kikura, A. Saito, and T. Tanahashi, in *Experimental Heat Transfer, Fluid Mechanics, and Thermodynamics 1991,* edited by J. F. Keffer, R. K. Shah, and E. N. Ganić (Elsevier, Amsterdam, 1991), p. 1433.
[11] J. D. Parsons, J. Phys. D **8**, 1219 (1975).
[12] I. E. Tarapov, Appl. Math. Mech. **37**, 770 (1973).
[13] W. E. Isler and D. Y. Chung, J. Appl. Phys. **49**, 1812 (1978).
[14] K. Gotoh, W. E. Isler, and D. Y. Chung, IEEE Trans. Magn. **16**, 211 (1980).
[15] K. Gotoh and D. Y. Chung, J. Phys. Soc. Jpn. **53**, 2521 (1984).
[16] D. Y. Chung and W. E. Isler, J. Appl. Phys. **49**, 1809 (1978).
[17] D. Y. Chung and W. E. Isler, IEEE Trans. Magn. **14**, 984 (1978).
[18] S. Taketomi, J. Phys. Soc. Jpn. **55**, 838 (1986).