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Growth laws for phase ordering

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We determine the characteristic length scale, $L(t)$, in phase-ordering kinetics for both scalar and vector fields, with either short- or long-range interactions and with or without conservation laws. We obtain $L(t)$ consistently by comparing the global rate of energy change to the energy dissipation from the local evolution of the order parameter. We derive growth laws for O(n) and other models, including systems with topological textures.

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Systems quenched from a disordered phase into an ordered phase do not order instantaneously. Instead, the length scale of ordered regions grows at a characteristic rate as different broken symmetry phases compete to select the ordered phase. During this process, energy is dissipated and topological defects, if present, are destroyed. Traditionally, systems with scalar order parameters, such as binary alloys and Ising models, have been studied [1–3]. In such systems, domains of both phases grow and intervening domain walls, the characteristic topological defects of scalar systems, shrink in total area, dissipating energy. Recently, there has been a growing interest in systems with vector and more complex order parameters which have defect structures such as lines, points, and textures. A series of experiments [4,5] and simulations [6–10] have been performed to explore such systems. Of particular interest is the time dependence, $L(t)$, of the characteristic length scale. Knowing this growth law provides a fundamental test of approximation schemes and a basis for further analysis. Growth laws have been obtained in some individual cases [11–14], but, to our knowledge, no unified theoretical approach has existed before now. Note that while renormalization-group (RG) treatments [13,15] predict the growth laws in systems with conservation laws, they have not been extended to treat systems with no conservation laws.

In this Rapid Communication we develop an approach to determine the characteristic length scale, *if it exists*, for quenched systems with either scalar or vector fields, with either short- or long-range interactions and with or without conservation laws. We do this by considering the time dependence of the energy as the system relaxes towards its ground state. We first evaluate the dependence of the energy on the length scale by considering topological defects, which set the energy scale when they exist. Assuming an arbitrary growth law, we then compare the global rate of energy change to the energy dissipation from the local evolution of the order parameter. From this we self-consistently determine the growth law of the length scale, $L(t)$. We present our results with continuous parameters to facilitate setting up and testing approximation or RG schemes. However, we do not use any approximation schemes in our approach, relying only on simple scaling assumptions.

Our results are summarized in Tables I and II. We find power-law growth laws, independent of the spatial dimension of the system, in which long-range forces and conservation laws are relevant in certain regimes. For the marginal cases, logarithmic factors are introduced. The power-law factors of our results are consistent with existing work. Physical arguments yield growth laws of $L \sim t^{1/2}$ for nonconserved [12] and $L \sim t^{1/3}$ for conserved [11] scalar order parameters. For scalar systems with general conservation laws, simulations and local approaches [3] agree with our results. RG arguments [13] reproduce the $t^{1/3}$ law for conserved scalar fields, predict $L \sim t^{1/4}$ for a conserved vector field, and also treat conserved systems with long-range interactions [15]. Simulations [6] suggest a $L \sim t^{1/2}$ growth law for nonconserved vector fields, and simulation [7] and experiments [4] obtain the same law for the related tensor fields that describe nematic liquid crystals. This result has also been obtained by approximate treatments for general n [16] and verified in the large-n limit [14]. Our approach provides a simple theoretical basis for these results, and also determines any additional logarithmic factors.

A generic energy functional for such a system with an n-component order parameter, $\vec{\phi}(\vec{x})$, and short-range interactions, is

$$H[\vec{\phi}] = \int d^d x \, [(\nabla\vec{\phi})^2 + V(\vec{\phi})] \, , \tag{1}$$

where $V(\vec{\phi})$ is a "mexican-hat"-shaped potential such as $V(\vec{\phi}) = (\vec{\phi}^2 - 1)^2$. After a temperature quench into the ordered phase, the equation of motion for the ordering kinetics of the Fourier components $\vec{\phi}_{\mathbf{k}}$ [17] is

$$\partial_t \vec{\phi}_{\mathbf{k}} = -k^\mu \, (\partial H / \partial \vec{\phi}_{-\mathbf{k}}), \tag{2}$$

where we consider systems with purely dissipative dynamics. We work at temperature $T = 0$, with no thermal noise, since T appears to be an "irrelevant variable" for ordering kinetics within the ordered phase [13,14]. The conventional nonconserved (model A) and conserved (model B) dynamics are $\mu = 0$ and $\mu = 2$, respectively, but any $\mu > 0$ represents a global conservation law [3].

The dynamic scaling hypothesis describes most phase-ordering systems in the late stages of growth [18]. Accordingly, the correlation function of the order parameter, $C(\mathbf{r},t) = \left\langle \vec{\phi}(\mathbf{x},t) \cdot \vec{\phi}(\mathbf{x}+\mathbf{r},t) \right\rangle$, with an average over initial conditions, should exhibit the scaling form $C(\mathbf{r},t) = f(r/L(t))$, with a single characteristic length

scale $L(t)$. Fourier transforming this, we obtain the scaling form for the structure factor $S(\mathbf{k},t) = \left\langle \vec{\phi}_{\mathbf{k}}(t) \cdot \vec{\phi}_{-\mathbf{k}}(t) \right\rangle$,

$$S(\mathbf{k},t) = [L(t)]^d \, g(kL(t)) \, , \tag{3}$$

where d is the spatial dimension.

Integrating the rate of energy dissipation from each Fourier mode, and then using the equation of motion (2), we find

$$\begin{aligned} d\varepsilon/dt &= \int_{\mathbf{k}} \left\langle (\partial H/\partial \vec{\phi}_{\mathbf{k}}) \cdot \partial_t \vec{\phi}_{\mathbf{k}} \right\rangle , \\ &= - \int_{\mathbf{k}} k^{-\mu} \left\langle \partial_t \vec{\phi}_{\mathbf{k}} \cdot \partial_t \vec{\phi}_{-\mathbf{k}} \right\rangle \, , \end{aligned} \tag{4}$$

where $\varepsilon = \langle H \rangle / V$ is the mean energy density, and $\int_{\mathbf{k}}$ is the momentum integral $\int d^d k/(2\pi)^d$. We will calculate the scaling behavior of $d\varepsilon/dt$ using the integral on the right-hand side of (4) and also by calculating ε directly as an appropriate integral over the structure factor, $S(\mathbf{k},t)$. For each approach, either the integral converges in the ultraviolet (uv) and the dependence on the scale $L(t)$ can be extracted using scaling forms (3) or (8), or momenta on the order of the uv cutoff dominate the integral. This small-scale structure will only come from topological defects. For instance, $S(\mathbf{k},t)$ is proportional to the density of defect core, $\rho_{\rm def} \sim L^{d-n}/L^d \sim L^{-n}$ for $kL \gg 1$ [19]. The scaling form (3) then implies

$$S(\mathbf{k},t) \sim L^{-n} \, k^{-(d+n)} \, , \qquad kL \gg 1 \tag{5}$$

which is a generalized Porod's law valid for $n \leq d$, where singular topological defects exist [20,21]. We limit our approach to these cases, unless otherwise mentioned (see below).

We first calculate the scaling behavior of the energy density, ε, which is captured by that of the gradient term in (1):

$$\begin{aligned} \varepsilon &\sim \left\langle (\nabla \vec{\phi})^2 \right\rangle , \\ &= \int_k k^2 L^d g(kL), \end{aligned} \tag{6}$$

where we have used the scaling form (3) of the structure factor. We then use (5) and impose a cutoff at $k \sim 1/\xi$, where ξ is the defect core size, to obtain [21]

$$\varepsilon \sim \begin{cases} L^{-n} \, \xi^{n-2} \, , & n < 2 \\ L^{-2} \, \ln(L/\xi) \, , & n = 2 \\ L^{-2} \, , & n > 2 \, . \end{cases} \tag{7}$$

We see that the energy is dominated by the defect core density, $\rho_{\rm def}$, for $n < 2$, by the defect field at all length scales for $n = 2$, and by variations of the order parameter at scale $L(t)$ for $n > 2$.

We now evaluate the right-hand side of the energy-dissipation equation (4) in a similar way. Using the scaling assumption for the two-time function, $\left\langle \vec{\phi}_{\mathbf{k}}(t) \cdot \vec{\phi}_{-\mathbf{k}}(t') \right\rangle = k^{-d} \tilde{g}(kL(t), kL(t'))$, we find

$$\begin{aligned} \left\langle \partial_t \vec{\phi}_{\mathbf{k}} \cdot \partial_t \vec{\phi}_{-\mathbf{k}} \right\rangle &= \left. \frac{\partial^2}{\partial t \partial t'} \right|_{t=t'} \left\langle \vec{\phi}_{\mathbf{k}}(t) \cdot \vec{\phi}_{-\mathbf{k}}(t') \right\rangle , \\ &= (dL/dt)^2 L^{d-2} h(kL). \end{aligned} \tag{8}$$

If the integral in (4) converges we use (8) and change variables to obtain $d\varepsilon/dt \sim L^{\mu-2}(dL/dt)^2$. When, *and only when* [25], the momentum integral in (4) diverges in the uv, we evaluate (8) in the $kL \gg 1$ limit by expressing the derivatives in terms of the length scales and changing to sum and difference variables, $L \equiv [L(t) + L(t')]/2$ and $\Delta \equiv [L(t) - L(t')]/2$:

$$\begin{aligned} \left\langle \partial_t \vec{\phi}_{\mathbf{k}} \cdot \partial_t \vec{\phi}_{-\mathbf{k}} \right\rangle &= \frac{1}{4} \left(\frac{dL}{dt} \right)^2 \left(\frac{\partial^2}{\partial L^2} - \frac{\partial^2}{\partial \Delta^2} \right) \Big|_{\Delta=0} \left\langle \vec{\phi}_{\mathbf{k}} \cdot \vec{\phi}_{-\mathbf{k}} \right\rangle_{L,\Delta} , \\ &\sim \left(\frac{dL}{dt} \right)^2 \left(\frac{\partial^2}{\partial L^2} - \frac{\partial^2}{\partial \Delta^2} \right) \Big|_{\Delta=0} \frac{1}{L^n k^{d+n}} b(k\Delta), \qquad kL \gg 1 \\ &\sim L^{-n} \, k^{-(d+n-2)} \, (dL/dt)^2 \, , \qquad kL \gg 1 \, . \end{aligned} \tag{9}$$

To get the second line we recognize that if the $kL \gg 1$ limit is probing isolated defects [19], i.e., defects which are locally flat or well separated with respect to their core size, then the significant part of the two-time function is just proportional to the defect core density, $\rho_{\rm def} \sim L^{-n}$.

In which cases is it appropriate to use the comoving field of an isolated defect, as we have effectively done in Eq. (9)? Certainly if $n > 2$ the energy density (7) and hence dissipation are dominated by variations at scale $L(t)$, the scaling form (8) can be used, and the question is irrelevant for our argument. For $n \leq 2$, the rate of energy dissipation can be written as the rate of change of the energy of all the defect features,

$$\begin{aligned} d\varepsilon/dt &\sim \frac{\partial}{\partial t} \int_\xi^\infty dl \, n(l,t) \, \varepsilon(l) \, , \\ &\sim - \int_\xi^\infty dl \frac{\partial j(l,t)}{\partial l} \, \varepsilon(l) \, , \\ &\sim j(\xi)\varepsilon(\xi) + \int_\xi^\infty dl \, j(l,t) \, \frac{\partial \varepsilon(l)}{\partial l} \, , \end{aligned} \tag{10}$$

where $n(l,t)$ is the number density of defect features of scale l, $\varepsilon(l) \sim l^{d-n}$ is the energy of the defect feature [with a $\ln(l/\xi)$ factor for $n = 2$], and $j(l)$ is the number flux of defect features. We have used the continuity equation, $\partial n/\partial t + \partial j/\partial l = 0$, to obtain the second line in (10). The total number density of defects, N, scales

as the inverse volume, $N \sim 1/L^d$, and hence $\dot{N}$ is slowly varying for times of order $L/\dot{L}$. Since defects only vanish at the core scale ξ, we have $\dot{N} \sim j(\xi)$. This implies that $j(l)$ is independent of l for $l \ll L$, in order to provide a constant rate of defect extinction.

For $d > n$, the integral in (10) is well behaved at $l \ll L$ and the integral dominates the $j(\xi)\,\epsilon(\xi)$ term. Hence structures with scales and separations $l \sim L(t)$ dominate the energy dissipation, and looking at isolated defects in the $kL \gg 1$ limit is appropriate. Note that the uv divergence of the integral in (4) in these cases is due to the internal structure of an isolated stable defect–dissipation is dominated by the motion of defect cores, not by the annihilation of small features.

However, for $n < 2$ with $n = d$, $\varepsilon(l) \sim$ const and dissipation is dominated by the $j(\xi)\,\varepsilon(\xi)$ term in (10) which describes defect pairs annihilating. Since the dissipation occurs at separations $l \sim \xi \ll L$ we cannot use a single defect description of dissipation as in (9). In fact, since the energy of a defect pair does not depend on the separation l for $l \gg \xi$, we expect the system to be disordered, with an equilibrium density of defects, at any nonzero temperature. At $T = 0$ we expect slow growth laws that depend on the details of the potential $V(\vec{\phi})$. These cases, including the one-dimensional (1D) Ising model, are at their lower critical dimension and are outside the scope of our approach.

The 2D XY model ($n = d = 2$) is a special case. The logarithm in the energy of a vortex pair, $\varepsilon(l) \sim \ln(l/\xi)$, leads to a logarithmically divergent integral in (10) for $l \ll L$, and we see that structures at all scales between ξ and $L(t)$ are significant for the energy dissipation. In this case, we doubt the validity of (9) which depends on the $kL \gg 1$ limit being a single defect property. As a result we cannot address the 2D XY model. It is interesting that numerical work by Blundell and Bray [22] for the nonconserved case shows a lack of scaling with respect to the defect density. In addition, there are indications of a systematic scaling violation for the conserved ($\mu = 2$) case [8].

With the exception of the above cases, $n = d \leq 2$, we find

$$\int_{\mathbf{k}} k^{-\mu} \left\langle \partial_t \vec{\phi}_{\mathbf{k}} \cdot \partial_t \vec{\phi}_{-\mathbf{k}} \right\rangle \sim \begin{cases} L^{-n}\,\xi^{n+\mu-2}\,(dL/dt)^2\,, & n+\mu < 2 \\ L^{-n}\ln(L/\xi)\,(dL/dt)^2\,, & n+\mu = 2 \\ L^{\mu-2}\,(dL/dt)^2\,, & n+\mu > 2 \end{cases} \tag{11}$$

provided $n \leq d$ so that singular topological defects exist. For the first two cases the integral is uv divergent and we have imposed a cutoff at $k \sim 1/\xi$ and used the $kL \gg 1$ form (9), and for the remaining case the integral is uv convergent and we use the scaling form (8).

We compare the rate of energy dissipation between (11) and the time derivative of (7) to obtain dL/dt and hence $L(t)$. The results are summarized in Table I.

For nonconserved fields ($\mu = 0$), we find $L \sim t^{1/2}$ for all systems (with $d > n$ or $n > 2$). Leading corrections in the $n = 2$ case are interesting: the $\ln L$ factors in (7) and (11) will in general have different effective cutoffs, of the order of the core size ξ. This leads to a logarithmic correction to scaling, $L \sim t^{1/2}[1 + \mathrm{O}(1/\ln t)]$, and may account for the smaller exponent (~ 0.45) seen in simulations of O(2) systems [6,7].

For conserved fields ($\mu > 0$) our results agree with the the RG analysis [13], with additional logarithmic factors for the marginal cases. We see that the conservation law is only relevant for $n + \mu \geq 2$. Simulations by Siegert and Rao [9] for $n = 2$, with $d = 3$ and $\mu = 2$, obtain growth exponents slightly over 1/4, which is not inconsistent with our predicted $L \sim (t \ln t)^{1/4}$ behavior.

The above discussion covers systems with purely short-ranged interactions. We can also add long-ranged interactions to our energy functional (1) [15]:

$$H_{\mathrm{LR}} \sim \int d^d x \int d^d r \frac{[\vec{\phi}(\mathbf{x}+\mathbf{r}) - \vec{\phi}(\mathbf{x})]^2}{r^{d+\sigma}}, \tag{12}$$

which apparently dominate when $\sigma < 2$. The calculation (11) of the right-hand side of (4) is unchanged, but the mean energy has a new contribution,

$$\varepsilon_{\mathrm{LR}} \sim \int_{\mathbf{k}} k^{\sigma}\, S(\mathbf{k}, t)\,, \qquad \sigma < 2\,. \tag{13}$$

This leads to, using (3) and (5),

$$\varepsilon_{\mathrm{LR}} \sim \begin{cases} L^{-n}\,\xi^{n-\sigma}\,, & n < \sigma \\ L^{-\sigma}\ln(L/\xi)\,, & n = \sigma \\ L^{-\sigma}\,, & n > \sigma \end{cases} \tag{14}$$

which actually dominates (7) for $\sigma \leq n$ when $n < 2$. We then compare the rate of energy dissipation, still given by (11), and the time derivative of (14), to find dL/dt and determine Table II. Our discussion about the validity of the $kL \gg 1$ limit in (9) carries over. Our results agree with the leading power laws from an RG treatment by Bray [15] of the conserved case ($\mu > 0$), but for $n \leq 2$ we disagree with the approximate calculation by Hayakawa *et al.* [23]. It is also interesting to note that simulations suggest that a new length scale may break scaling for $\sigma < 1$ [24]. However, preliminary numerical work [25] indicates that the scaling regime may only be reached in

TABLE I. The growth law of the length scale $L(t)$ for various number of components, n, and conservation laws, μ [see Eq. (2)]. Note that $n = d$ is excluded for $n \leq 2$.

$L(t)$	$n < 2$	$n = 2$	$n > 2$
$n + \mu < 2$	$t^{1/2}$		
$n + \mu = 2$	$(t/\ln t)^{1/2}$	$t^{1/2}$	
$n + \mu > 2$	$t^{1/(n+\mu)}$	$(t \ln t)^{1/(2+\mu)}$	$t^{1/(2+\mu)}$

TABLE II. The growth law of the length scale $L(t)$ for long-range forces, $0 < \sigma < 2$ [see Eq. (12)], with various number of components, n, and conservation laws, μ. Note that $n = d$ is excluded for $n \leq \sigma$.

$L(t)$	$n < \sigma$	$n = \sigma$	$n > \sigma$
$n + \mu < 2$	$t^{1/2}$	$(t \ln t)^{1/2}$	$t^{1/(2+\sigma-n)}$
$n + \mu = 2$	$(t/\ln t)^{1/2}$	$t^{1/2}$	$(t/\ln t)^{1/(\sigma+\mu)}$
$n + \mu > 2$	$t^{1/(n+\mu)}$	$(t \ln t)^{1/(n+\mu)}$	$t^{1/(\sigma+\mu)}$

larger systems than have been treated.

We can also address systems with topological textures ($n = d + 1$), even though the appropriate Porod's law is not known. Since defects with $n > d$ must be spatially extended and without a singular core, they will have a smaller large-k tail to their structure factor $S(\mathbf{k}, t)$ than any defects with cores. So for $n > 2$ (or more precisely when $n > \sigma$ and $n + \mu > 2$), when the momentum integrals for defects with cores converge for $kL \gg 1$, the growth law for systems with textures will be given by using the scaling forms (3) and (8) in (4) to obtain $L \sim t^{1/(\sigma+\mu)}$, where $\sigma = 2$ for short-ranged forces. We can also apply this result to systems without topological defects ($n > d + 1$) which similarly have convergent momentum integrals for $kL \gg 1$. As a result, Tables I and II will apply to *all* systems of physical interest, except perhaps for the case of $d = 1$, $n = 2$. Indeed, this case has an anomalous growth law for $\mu = 0$ [26] related to the breakdown of the scaling form (8) [25]. Of course, whether the scaling assumptions hold, as in (3) and (8), in other cases remains an open question.

The generality of our results deserves emphasis. Since our derivation is independent of the initial conditions, it also applies equally to critical and off-critical quenches *as long as the system scales at late times.* Our approach can also be applied to systems with more complicated order parameters than n-component vectors. The details of the energy functional (1) are unimportant [27]; all we need is the existence of some short- or long-ranged "elastic" energy (σ), a conservation law (μ), and the defect structure if any. We can also treat systems with more than one defect type. We simply calculate both sides of (4) with the defect type that dominates the energy and energy dissipation, which is the one represented by the smallest n.

For example, in bulk nematic liquid crystals, the existence of string defects leads to (5) and (9) with $n = 2$, which with no conservation law implies a $L \sim t^{1/2}$ growth law, consistent with recent experiments [4] and simulations [7]. Similarly, in Potts models the existence of domain walls leads us to use our results for scalar ($n = 1$) systems, consistent with the $L \sim t^{1/2}$ growth seen in simulation [28].

In summary, by focusing on the total energy dissipation, rather than the detailed dynamics of the system, we obtain growth laws for phase ordering. Our method applies to systems satisfying simple scaling hypotheses.

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