Stationary convection in a plasma of inhomogeneous density

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Stationary convection in a current-carrying cylindrical incompressible plasma of inhomogeneous density is studied. It is shown that nonideal effects such as viscosity, resistivity, and thermal conductivity give stationary convection in a plasma of inhomogeneous density. However, when the Hall effect is considered, convective cells are not allowed to be formed. It is proposed that convective cells stay mainly in the center of the cylinder.

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Large-scale stationary convection, or well-organized plasma motion, has been considered by several authors. The first studies were by Timofeev [1] and Simon [2] for a partially ionized plasma. They showed that the convection can be driven by the toroidal curvature with the neutral species providing the required dissipation. Kadomtsev and Pogutse [3] showed that the resistive rippling modes lead to convection. The existence of thermally excited convection caused by driftlike modes is showed by Okuda and Dawson [4]. Roberts and Taylor [5] proposed the formation of quasimodes of large spatial extent. Wobig and Maschke and Paris [6,7] showed that convective cells are produced when the plasma is unstable and nonideal effects are considered. The existence of stationary convection modes in a plasma slab with magnetic shear is shown by Dagazian and Paris [8]. Gomberoff *et al.*, in several papers [9–15], showed the existence of stationary convection in a cylindrical incompressible plasma of constant density when nonideal effects are considered. More recently, Santos and Galvão [16,17] studied the influence of the Hall effect on convection in plasmas. They showed that when the Hall effect, viscosity, resistivity, and thermal conductivity are simultaneously considered, it is not possible for convective cells to be formed. However, any other combination of these effects gives convective modes in an incompressible plasma of constant density. In this paper the combined viscosity, resistivity, thermal conductivity, and Hall effects are studied in an incompressible plasma of inhomogeneous density.

The basic equations that describe the system are

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right]=\mathbf{J}\times\mathbf{B}-\nabla P-\mu\nabla\times(\nabla\times\mathbf{v})\ , \qquad (1)$$

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\mathbf{v})=0\ , \qquad (2)$$

$$\frac{\partial P}{\partial t}+\mathbf{v}\cdot\nabla P-\tfrac{2}{3}K\,\nabla^2 P-S_0=-\gamma P\nabla\cdot\mathbf{v}+\tfrac{2}{3}\eta|\mathbf{J}|^2\ , \qquad (3)$$

$$\mathbf{E}+\mathbf{v}\times\mathbf{B}=\eta\mathbf{J}+\frac{\epsilon_H}{\rho}\mathbf{J}\times\mathbf{B}\ , \qquad (4)$$

$$\frac{\partial B}{\partial t}=-\nabla\times\mathbf{E}\ , \qquad (5)$$

$$\nabla\times\mathbf{B}=\mu_0\mathbf{J}\ , \qquad (6)$$

and

$$\nabla\cdot\mathbf{B}=0\ , \qquad (7)$$

where μ is the perpendicular part of the viscosity tensor [18], K is the thermal conductivity, η is the resistivity, ϵ_H is the Hall parameter (mass/charge ratio for the ions), and S_0 is a constant heat source that maintains the equilibrium pressure profile,

$$S_0=\tfrac{8}{3}[K-\eta/\mu_0]\frac{B_i}{\mu_0 r_0^2}\ . \qquad (8)$$

The equilibrium magnetohydrodynamic is characterized by

$$\mathbf{B}_0=B_i(r/r_0)\hat{\mathbf{e}}_r+B_0\hat{\mathbf{e}}_\theta \qquad (9)$$

and

$$P_0=\mathcal{P}_0+\frac{B_i^2}{\mu_0}(r/r_0)^2\ , \qquad (10)$$

where B_0 is the equilibrium magnetic field; P_0 is the equilibrium pressure; r_0 is the radius of the cylinder; and B_0, B_i, and $\mathcal{P}_0$ are constants.

Assuming an incompressible plasma of inhomogeneous density and force-free conditions $\nabla\times\mathbf{v}_1=\beta\mathbf{v}_1$ and $\nabla\times\mathbf{B}_1=\beta^*\mathbf{B}_1$ [19], we obtain from the basic equations the following relation for the displacement vector ξ:

$$\Big[\rho_0\hat{\omega}^2+\{\rho_0\hat{\eta}\beta^{*2}r_0^2+\hat{\mu}\beta^2r_0^2+i\hat{\epsilon}_H[(m-nq)\beta^*-2K]\}\hat{\omega}+\hat{\mu}\eta\beta^2\beta^{*2}r_0^4+i\frac{\epsilon_H}{\rho_0}\hat{\mu}\beta^2r_0^2[(m-nq)\beta^*-2K]+(m-nq)^2\Big]\xi=-r_0^2\nabla\hat{P}_1+2i(m-nq)(\hat{\mathbf{e}}_r\times\xi)\ , \qquad (11)$$

where the perturbed parameters are given by Fourier components $f_1(r,\theta,z,t)=f_1(r)\exp(im\theta+ikz+\omega t)$; m and n are, respectively, the poloidal and toroidal modes; q is the inverse rotational transform; and

$$\hat{P}_1\equiv(\mu_0/B_i^2)[P_1+(\mathbf{B}_0\cdot\mathbf{B}_1)/\mu_0] ,$$

$$\hat{\omega}\equiv(r_0^2\mu_0/B^2)^{1/2}\omega ,$$

$$\hat{\mu}\equiv(\mu_0/r_0B)^{1/2}\mu ,$$

$$\hat{\eta}\equiv(r_0^2\mu_0B_i^2)^{-1/2}\eta .$$

Defining

$$A\equiv\rho_0\hat{\omega}^2+\{\rho_0\hat{\eta}\beta^{*2}r_0^2+\hat{\mu}\beta^2r_0^2+i\hat{\epsilon}_H[(m-nq)\beta^*-2K]\}\hat{\omega}+\hat{\mu}\hat{\eta}\beta^2\beta^{*2}r_0^4+i\frac{\hat{\epsilon}_H}{\rho_0}\hat{\mu}\beta^2r_0^2[(m-nq)\beta^*-2K]+(m-nq)^2 ,$$

$$B\equiv-2i(m-nq) ,$$

and making the inner products $\hat{e}_r\cdot$, $\hat{e}_\theta\cdot$, and $\hat{e}_z\cdot$ of Eq. (11), we find that, respectively,

$$A\xi_r-B\xi_\theta=-r_0^2\frac{\partial\hat{P}_1}{\partial r} , \tag{12}$$

$$A\xi_\theta+B\xi_r=-i\frac{m}{r}r_0^2\hat{P}_1 , \tag{13}$$

and

$$A\xi_z=-ikr_0^2\hat{P}_1 \tag{14}$$

give the components of the displacement vector

$$\xi_r=-\frac{ir_0^2B}{A^2+B^2}\left[\tau(r)\frac{\partial\hat{P}_1}{\partial r}+\frac{m}{r}\hat{P}_1\right] , \tag{15}$$

$$\xi_\theta=\frac{ir_0^2B}{A^2+B^2}\left[\frac{\partial\hat{P}_1}{\partial r}+\tau(r)\frac{m}{r}\hat{P}_1\right] , \tag{16}$$

and

$$\xi_z=-ir_0^2\frac{k}{A}\hat{P}_1 , \tag{17}$$

where

$$\tau(r)=-i\frac{A}{B} .$$

Due to boundary conditions of the perfectly conducting cylindrical walls $\xi_r(r=r_0)=0$, and perpendicular viscosity $\xi_\theta(r=r_0)=0$, $\tau(r=r_0)\simeq1$, although no restriction can be made at $\tau(r)$ for $r<r_0$.

From the definition of (r) we can write

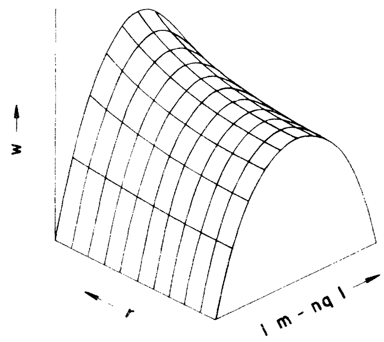

FIG. 1. Schematic spatial view of the dependence of ω with $|m-nq|$ and r for systems with $\mu\neq0$, $\eta=0$, and $\epsilon_H=0$.

$$\rho_0\hat{\omega}^2+\{\rho_0\hat{\eta}\beta^{*2}r_0^2+\hat{\mu}\beta^2r_0^2+i\hat{\epsilon}_H[(m-nq)\beta^*-2K]\}\hat{\omega}+i\frac{\epsilon_H}{\rho_0}\hat{\mu}\beta^2r_0^2[(m-nq)\beta^*-2K]+(m-nq)^2-2|m-nq|\tau=0 . \tag{18}$$

The conditions for the formation of the convective modes are given by $\mathrm{Re}(\omega)=\mathrm{Im}(\omega)=0$ [20]. Applying these conditions to Eq. (18) we obtain

$$\hat{\mu}\hat{\eta}\beta^2\beta^{*2}r_0^4+(m-nq)^2-2|m-nq|\tau=0$$

and (19)

$$\frac{\epsilon_H}{\rho_0}\hat{\mu}\beta^2r_0^2[(m-nq)\beta^*-2K]=0 .$$

For $\hat{\epsilon}_H=0$, convective cells are formed in the cases where: (i) $\mu\neq0$, $\eta=0$ and $|m-nq|=0$ or 2τ; and (ii) $\mu\neq0$, $\eta\neq0$ and $|m-nq|=\tau\{1\pm[1-(\lambda/\tau)^2]^{1/2}\}$, where $\lambda^2=\hat{\mu}\hat{\eta}\beta^4r_0^4$.

The results obtained previously by Gomberoff and Hernandez [9–11] can be reproduced if we impose the condition $\tau\simeq1$.

In the presence of the Hall effect, convective modes cannot be obtained for any combination with other nonideal effects, such as can be seen from Eqs. (19).

Figure 1 shows a schematic spatial view of the dependence of the growth rate of instabilities with convective modes and radial location for systems with $\mu\neq0$, $\eta=0$ and $\epsilon_H=0$. The figure for systems with $\mu\neq0$, $\eta\neq0$, and $\epsilon_H=0$ would be very similar to Fig. 1. To get a better idea of how it would look, see, for example, Ref. [11].

We can conclude that for a plasma having inhomogeneous density, this presents a large-scale stationary convection when effects such as viscosity, resistivity, and thermal conductivity are considered. The growth rate of instabilities ω now depends on radial position, increasing with the increase of radial position. The result can impose that the convective cells stay mainly in the center of the cylinder.

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