BRIEF REPORTS

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Noninterferometric reconstruction of optical-field correlations

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A noninterferometric method for reconstructing the second-order correlations of a partially coherent optical field is described. It is assumed that the field propagates close to one particular direction, so that the paraxial approximation may be used, and that the field correlations in some cross section may be well approximated by the quasihomogeneous model. Two reconstruction formulas are derived which uniquely determine the cross-spectral density function, characterizing the correlations, from knowledge of the spatial distribution of the spectral density in several parallel planes.

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Because direct measurements of the phase of a coherent optical field are very difficult to make, this information is usually obtained either by holography or by reconstructing the phase from knowledge of the intensity. Many methods for phase reconstruction have been proposed that make use of intensity data in one or more planes, and some have been quite successful [1]. It is of interest to examine whether an analogous approach might be feasible for obtaining the second-order correlations of partially coherent light.

Conventional techniques for determining such correlations are based on interferometric measurements [2]. In general, it is rather difficult to obtain a complete characterization of a partially coherent field by such methods. For example, in order to determine the degree of coherence of a field from a Young's-type interference experiment, one must sample the field at all pairs of points.

In this Brief Report, we propose a noninterferometric technique for reconstructing the cross-spectral density function, which characterizes the second-order correlations at frequency ω, from knowledge of the spatial distribution of the spectral density at the same frequency [3,4]. Because, in general, knowledge of the spectral density throughout all space is not sufficient to uniquely determine the cross-spectral density [5], we assume *a priori* that the correlations in a certain plane can be well approximated by the quasihomogeneous model [6]. As will be evident later, this model leads to a unique solution which, in principle, makes it possible to obtain a complete reconstruction.

Let us assume that the fluctuating optical field can be adequately represented by the complex scalar field $V(\mathbf{r},t)$ and that $V(\mathbf{r},t)$ is statistically stationary, at least in the wide sense. The second-order correlations of the field fluctuations at two points in space, $\mathbf{r}_1$ and $\mathbf{r}_2$, and at two instants of time, t and $t+\tau$, may be characterized by the mutual coherence function $\Gamma(\mathbf{r}_1,\mathbf{r}_2;\tau)$, defined by the expression

$$\Gamma(\mathbf{r}_1,\mathbf{r}_2;\tau)\equiv\langle V^*(\mathbf{r}_1,t)V(\mathbf{r}_2,t+\tau)\rangle \ . \tag{1}$$

Here the asterisk denotes the complex conjugate and the angular brackets indicate an average taken over the ensemble $\{V(\mathbf{r},t)\}$ of field realizations.

For our purpose, it is more convenient to describe the field correlations by the cross-spectral density function $W(\mathbf{r}_1,\mathbf{r}_2;\omega)$, which is the Fourier transform of the mutual coherence function:

$$W(\mathbf{r}_1,\mathbf{r}_2;\omega)\equiv\frac{1}{2\pi}\int_{-\infty}^{\infty}\Gamma(\mathbf{r}_1,\mathbf{r}_2;\tau)e^{i\omega\tau}d\tau \ . \tag{2}$$

The spectral density and the spectral degree of coherence are then given by the formulas

$$S(\mathbf{r};\omega)\equiv W(\mathbf{r},\mathbf{r};\omega) \tag{3}$$

and

$$\mu(\mathbf{r}_1,\mathbf{r}_2;\omega)\equiv\frac{\omega(\mathbf{r}_i,\mathbf{r}_2;\omega)}{\sqrt{S(\mathbf{r}_1;\omega)}\sqrt{S(\mathbf{r}_2;\omega)}} \ , \tag{4}$$

respectively.

Our reconstruction problem is to determine the cross-spectral density $W(\mathbf{r}_1,\mathbf{r}_2;\omega)$ at frequency ω from knowledge of the spectral density $S(\mathbf{r};\omega)$ at the same frequency, for a field propagating close to the positive z axis. The cross-spectral density at two points $\mathbf{r}_1=(\rho_1,z_1)$ and $\mathbf{r}_2=(\rho_2,z_2)$ in the half-space $z>0$ may be expressed in the following form, which is valid in the paraxial approximation [7]:

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$$W(\boldsymbol{\rho}_1,z_1,\boldsymbol{\rho}_2,z_2)=\left[\frac{k}{2\pi}\right]^2\frac{e^{ik(z_2-z_1)}}{z_1z_2}\int\int W^{(0)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2')e^{-ik(\boldsymbol{\rho}_1-\boldsymbol{\rho}_1')^2/2z_1}e^{ik(\boldsymbol{\rho}_2-\boldsymbol{\rho}_2')^2/2z_2}d^2\rho_1'd^2\rho_2' , \tag{5}$$

where the domains of integration for both integrals extend over the entire $z=0$ plane, $W^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\equiv W(\boldsymbol{\rho}_1,0,\boldsymbol{\rho}_2,0)$ and $k=\omega/c$ (c being the speed of light in vacuum). $W^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)$ can be considered to be the cross-spectral density of a planar secondary source that generates the cross-spectral density $W(\boldsymbol{\rho}_1,z_1,\boldsymbol{\rho}_2,z_2)$ on propagation. For convenience, we have omitted the frequency argument ω in W and we shall continue to do so. From Eq. (5), it is clear that any method that determines $W^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)$ can also be used to determine $W(\boldsymbol{\rho}_1,z_1,\boldsymbol{\rho}_2,z_2)$. Therefore we will now restrict our discussion to the reconstruction of $W^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)$ from knowledge of the spectral density $S(\mathbf{r})$.

On substituting from Eq. (5) into Eq. (3), we obtain the following expression for the spectral density of the field throughout the half-space $z>0$:

$$S(\boldsymbol{\rho},z)=\left[\frac{k}{2\pi}\right]^2\frac{1}{z^2}\int\int W^{(0)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2')e^{-ik[2\boldsymbol{\rho}\cdot(\boldsymbol{\rho}_2'-\boldsymbol{\rho}_1')+\rho_1'^2-\rho_2'^2]/2z}d^2\rho_1'd^2\rho_2' . \tag{6}$$

It will be useful to rewrite this expression in an alternate form. If we take the spatial Fourier transform of Eq. (6) with respect to $\boldsymbol{\rho}$ and change the variables of integration to the average and difference coordinates, $\mathbf{R}\equiv(\boldsymbol{\rho}_1'+\boldsymbol{\rho}_2')/2$ and $\boldsymbol{\Delta}\equiv\boldsymbol{\rho}_2'-\boldsymbol{\rho}_1'$, we obtain the formula

$$\tilde{S}(\mathbf{f},z)=\int \mathcal{W}^{(0)}(\mathbf{R},-\mathbf{f}z/k)e^{-i\mathbf{f}\cdot\mathbf{R}}d^2R , \tag{7}$$

where the Fourier transform $\tilde{S}(\mathbf{f},z)$ of the spectral density is defined as

$$\tilde{S}(\mathbf{f},z)\equiv\int S(\boldsymbol{\rho},z)e^{-i\mathbf{f}\cdot\boldsymbol{\rho}}d^2\rho \tag{8}$$

and

$$\mathcal{W}^{(0)}(\mathbf{R},\boldsymbol{\Delta})\equiv W^{(0)}(\mathbf{R}-\boldsymbol{\Delta}/2,\mathbf{R}+\boldsymbol{\Delta}/2) . \tag{9}$$

Equation (6), or alternatively Eq. (7), is the basis of the mathematical formulation of our reconstruction problem: with $S(\boldsymbol{\rho},z)$ assumed to be known throughout the half-space $z>0$, we wish to determine $W^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)$. However, as already mentioned, there are many different $W^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)$'s that yield the same $S(\boldsymbol{\rho},z)$. Therefore, in general, the problem does not have a unique solution.

Furthermore, in practice, it is unreasonable to expect that we would know $S(\boldsymbol{\rho},z)$ for all $\boldsymbol{\rho}$ and all $z>0$. More realistically, we could measure $S(\boldsymbol{\rho},z)$ in several planes, say, $z=\zeta_1$, $z=\zeta_2$, ..., $z=\zeta_m$ (see Fig. 1). For example, we could take several photographic exposures, or use a charge-coupled-device (CCD) camera, to determine $S(\boldsymbol{\rho},z)$ in these planes. As the number m of measurement planes is diminished, we would expect the problem to become more ill-posed, i.e., the number of possible solutions to increase further.

In order to make the problem well-posed, even for the case when $S(\boldsymbol{\rho},z)$ is only known in a small number of planes, we will introduce some additional information about $W^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)$. In particular, we will assume that the correlations of the field in the plane $z=0$ can be described well by the quasihomogeneous model [6], which frequently provides a good description of sources encountered in nature and in the laboratory. This model represents a secondary source whose spectral degree of coherence $\mu^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\equiv\mu(\boldsymbol{\rho}_1,0,\boldsymbol{\rho}_2,0)$ in the plane $z=0$ depends on the two position vectors $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ only through their difference $\boldsymbol{\rho}_2-\boldsymbol{\rho}_1$ and, further, whose spectral density $S^{(0)}(\boldsymbol{\rho})\equiv S(\boldsymbol{\rho},0)$ in the plane $z=0$ is approximately constant over the transverse correlation width [i.e., over the effective width of the absolute value of the spectral degree of coherence $\mu^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)$]. When the field satisfies these conditions, the cross-spectral density may be expressed in the approximate form

$$W^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\approx S^{(0)}\left[\frac{\boldsymbol{\rho}_1+\boldsymbol{\rho}_2}{2}\right]\mu^{(0)}(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1) . \tag{10}$$

We will now assume that we know the spectral density in two arbitrary planes $z=\zeta_1>0$ and $z=\zeta_2>\zeta_1$ and that we also know the location of the secondary source plane $z=0$, where the field distribution is quasihomogeneous. In order to completely determine $W^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)$, we need to reconstruct both $S^{(0)}(\boldsymbol{\rho})$ and $\mu^{(0)}(\boldsymbol{\Delta})$. We will later describe how knowledge of the spectral density in a third plane can be used to locate the source plane $z=0$.

Applying Eq. (7) to the two planes $z=\zeta_1$ and $z=\zeta_2$ and using Eqs. (8)–(10), we obtain the following equivalent expressions for $\mu^{(0)}(\boldsymbol{\Delta})$:

$$\mu^{(0)}(\boldsymbol{\Delta})=\frac{\tilde{S}(-k\boldsymbol{\Delta}/\zeta_1,\zeta_1)}{\tilde{S}^{(0)}(-k\boldsymbol{\Delta}/\zeta_1)} , \tag{11}$$

$$\mu^{(0)}(\boldsymbol{\Delta})=\frac{\tilde{S}(-k\boldsymbol{\Delta}/\zeta_2,\zeta_2)}{\tilde{S}^{(0)}(-k\boldsymbol{\Delta}/\zeta_2)} . \tag{12}$$

The solutions of these two equations for $\tilde{S}^{(0)}(\mathbf{f})$ and $\mu^{(0)}(\boldsymbol{\Delta})$ are readily found to be

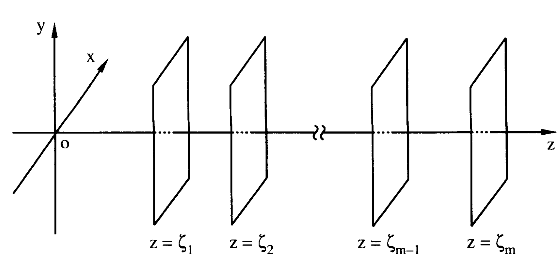

FIG. 1. Illustrating the m measurement planes in which the spectral density $S(\boldsymbol{\rho},z)$ is known.

$$\tilde{S}^{(0)}(\mathbf{f})=\tilde{S}^{(0)}(\mathbf{f}/\alpha)H_S(\mathbf{f}) \tag{13}$$

and

$$\mu^{(0)}(\mathbf{\Delta})=\mu^{(0)}(\mathbf{\Delta}/\alpha)H_\mu(\mathbf{\Delta}) , \tag{14}$$

where

$$\alpha \equiv \zeta_2/\zeta_1 > 1 , \tag{15}$$

$$H_S(\mathbf{f}) \equiv \frac{\tilde{S}(\mathbf{f},\zeta_1)}{\tilde{S}(\mathbf{f}\zeta_1/\zeta_2,\zeta_2)} , \tag{16}$$

$$H_\mu(\mathbf{\Delta}) \equiv \frac{\tilde{S}(-k\mathbf{\Delta}/\zeta_2,\zeta_2)}{\tilde{S}(-k\mathbf{\Delta}/\zeta_2,\zeta_1)} . \tag{17}$$

We can now eliminate $\tilde{S}^{(0)}$ and $\mu^{(0)}$ from the right-hand sides of Eqs. (13) and (14) as follows. We first form N new pairs of equations by replacing $\mathbf{f}$ by $\mathbf{f}/\alpha^m$ in Eq. (13) and $\mathbf{\Delta}$ by $\mathbf{\Delta}/\alpha^m$ in Eq. (14), where m takes on the values $1,2,\ldots,N$. These new equations are then combined, yielding the formulas

$$\tilde{S}^{(0)}(\mathbf{f})=\tilde{S}^{(0)}(\alpha^{-(N+1)}\mathbf{f}) \prod_{n=0}^{N} H_S(\alpha^{-n}\mathbf{f}) \tag{18}$$

and

$$\mu^{(0)}(\mathbf{\Delta})=\mu^{(0)}(\alpha^{-(N+1)}\mathbf{\Delta}) \prod_{n=0}^{N} H_\mu(\alpha^{-n}\mathbf{\Delta}) . \tag{19}$$

If we proceed to the limit as $N\to\infty$ of the above equations and make use of the limiting forms

$$\lim_{N\to\infty} \tilde{S}^{(0)}(\alpha^{-(N+1)}\mathbf{f})=\int S(\boldsymbol{\rho},\zeta_1)d^2\rho \tag{20}$$

and

$$\lim_{N\to\infty} \mu^{(0)}(\alpha^{-(N+1)}\mathbf{\Delta})=1 , \tag{21}$$

we obtain the reconstruction formulas [8]

$$\tilde{S}^{(0)}(\mathbf{f})=\left\{\int S(\boldsymbol{\rho},\zeta_1)d^2\rho\right\} \prod_{n=0}^{\infty} H_S(\alpha^{-n}\mathbf{f}) \tag{22}$$

and

$$\mu^{(0)}(\mathbf{\Delta})=\prod_{n=0}^{\infty} H_\mu(\alpha^{-n}\mathbf{\Delta}) . \tag{23}$$

It should be noted that the conservation law [9]

$$\int S^{(0)}(\boldsymbol{\rho})d^2\rho=\int S(\boldsymbol{\rho},\zeta_1)d^2\rho \tag{24}$$

was used in deriving Eq. (20). To determine the inverse spatial density $S^{(0)}(\boldsymbol{\rho})$ itself, one only needs to take the inverse spatial Fourier transform of expression (22).

Formulas (22) and (23) allow us to reconstruct $S^{(0)}(\boldsymbol{\rho})$ and $\mu^{(0)}(\mathbf{\Delta})$ from knowledge of the spectral density in any two planes parallel to the source plane $z=0$. If the spectral density were known in three (or more) planes, we would expect to obtain the same reconstructions for any pair of planes. Therefore, if the location of the source plane $z=0$ were not known, we could, in principle, determine the location of the source plane $z=0$ by requiring that the reconstructions from different pairs of planes be identical. However, the uniqueness of such a procedure still needs further study.

The reconstruction formulas that we have derived contain infinite products. Let us examine the quality of the reconstructions when only a finite number of terms is retained, for the case of a Gaussian, quasihomogeneous, secondary source. The spectral density and the spectral degree of coherence in the source plane then have the form

$$S^{(0)}(\boldsymbol{\rho})=Ae^{-\rho^2/2\sigma_S^2} \tag{25}$$

and

$$\mu^{(0)}(\mathbf{\Delta})=e^{-\Delta^2/2\sigma_\mu^2} , \tag{26}$$

respectively. Figures 2(a) and 2(b) show the reconstructions of these functions from knowledge of the spectral density in the two measurement planes $\zeta_1=10$ cm and $\zeta_2=12$ cm. The parameters in Eqs. (25) and (26) were chosen to have the values $\sigma_S=1$ cm and $\sigma_\mu=1$ mm. For this example, the required calculations, i.e., first deter-

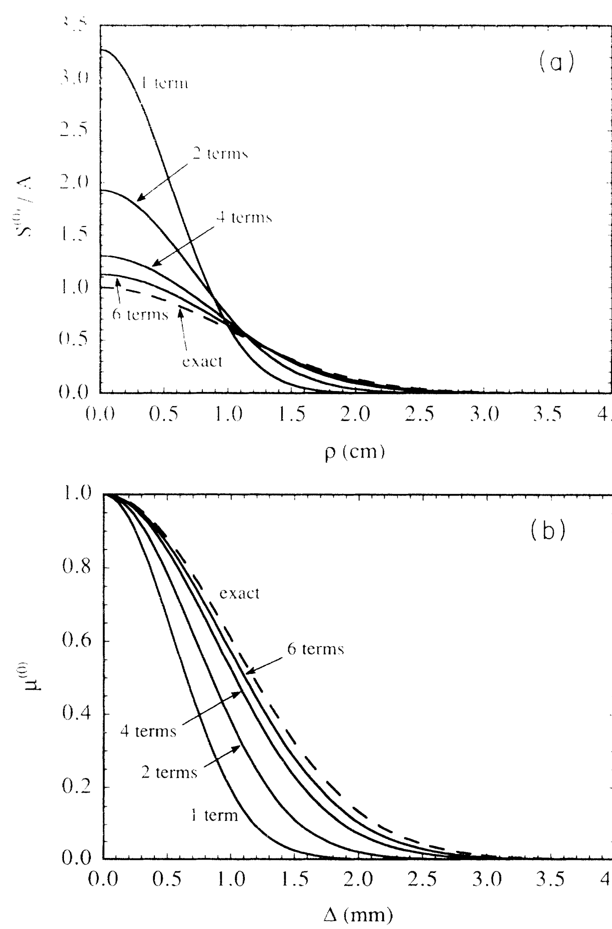

FIG. 2. Reconstructions of a Gaussian, quasihomogeneous, secondary source [Eqs. (25) and (26)], using Eqs. (22) and (23) and retaining only a finite number of terms in the products. (a) shows the spectrum $S^{(0)}(\boldsymbol{\rho})$ and (b) shows the spectral degree of coherence $\mu^{(0)}(\mathbf{\Delta})$. These reconstructions were performed using the spectral density in the planes $\zeta_1=10$ cm and $\zeta_2=12$ cm. The parameters characterizing the field in the source plane $z=0$ were taken to be $\sigma_S=1$ cm and $\sigma_\mu=1$ mm.

mining the spectral density in the measurement planes, then applying Eqs. (22) and (23) to reconstruct $\mu^{(0)}(\boldsymbol{\Delta})$ and $\tilde{S}^{(0)}(\mathbf{f})$, and finally taking the inverse spatial Fourier transform of $\tilde{S}^{(0)}(\mathbf{f})$ to determine $S^{(0)}(\boldsymbol{\rho})$, were all performed analytically. We see that excellent results are obtained even with only a few terms in the products appearing in Eqs. (22) and (23).

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[6] W. H. Carter and E. Wolf, J. Opt. Soc. Am. **67**, 785 (1977).

[7] See, for example, A. Friberg and R. J. Sudol, Opt. Acta **30**, 1075 (1983), Eq. (2.8).

[8] The convergence of the infinite products in these formulas will be discussed in a future presentation.

[9] A discussion of this conservation law, and related ones for the cross-spectral density, is given in M. W. Kowarz and E. Wolf, J. Opt. Soc. Am. A **10**, 88 (1993).