Langevin approach to collisional bremsstrahlung in a magnetic field

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Using the appropriate Langevin equations describing the stochastic motion of test electrons, we investigate the problem of the changes of kinetic energy and velocity distribution of electrons colliding with ions in the presence of a moderately strong radiation field and a steady, homogeneous magnetic field **B**. The cases where the electric field of the wave is either linearly polarized along **B**, or left- or right-hand circularly polarized on a plane perpendicular to **B**, are explicitly considered. The results concerning the kinetic-energy changes extend similar results obtained by the same authors using a different approach. The results concerning the changes of the velocity distribution may be summarized as follows. For a linearly polarized wave, collisional bremsstrahlung forces the slow absorbing electrons and all emitting electrons to align on average their velocities along the electric-field direction. Velocity randomization due to pure collisions mostly contrasts this process, and is dominant for fast electrons. For circularly polarized radiation, the electrons are forced by collisional bremsstrahlung to draw their velocity near to the polarization plane if their unperturbed velocity component perpendicular to **B** is larger than either their parallel component or the wave-induced velocity. We find that the regions of velocity space where both this process and emission take place widen upon increasing $|\mathbf{B}|$, whereas the rates at which the latter occur are, in general, decreasing functions of $|\mathbf{B}|$.

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I. INTRODUCTION

Direct and inverse bremsstrahlung plays an important role in a number of phenomena where radiation interacts with matter. An example is the heating of a plasma in a laser field. In these processes, electrons collide with ions in the presence of an electromagnetic wave, and the combined effects of both the wave and the Coulomb fields of the scattering centers on the electron result either in a loss or in a gain of kinetic energy of the electron itself.

Several contributions have been devoted to the subject, and most of them may be traced back starting from Refs. [1,2]. A common feature of these contributions is, as a rule, the effort to consider strong radiation, so that nonperturbative treatments are needed.

Also of interest is the situation where direct and inverse collisional bremsstrahlung takes place in the presence of a magnetic field. The present work is devoted to this case.

Previous accounts treated this problem as an elementary single-particle process [3–5]. A quantum-mechanical formulation has been provided by Seely [3], while Karapetyan and Fedorov [4] and the present authors [5] have reported on several results obtained using a simpler classical model. These treatments basically give the average change with time of the electron kinetic energy, but no information is obtained about the evolution in velocity space of a population of electrons undergoing the bremsstrahlung process in a medium such as a plasma.

This latter problem is usually addressed solving the corresponding Fokker-Planck equation for the electron distribution function. Without a magnetic field, interesting results have been reported concerning the case where the radiation field is weak [2,6–8]. In particular, in Refs. [2,6,7], where only the isotropic part of the electron distribution function was investigated, analytical self-similar solutions were obtained, yielding specific information on the evolution that the electron population undergoes in the velocity space.

When either the radiation is not weak, or a magnetic field is present, anisotropies of the electron distribution function are in general unlikely to be negligible. In these cases, detailed information about the shape of the electron distribution function may be obtained by means of difficult numerical analysis.

The principal aim of this paper is to get information on the modifications of the electron distribution in the velocity space as a result of collisional bremsstrahlung. We describe the basic mechanisms determining the (velocity-space) trajectories of test electrons undergoing collisions in the presence of a radiation field and a homogeneous steady magnetic field, in order to show the origin and the relevance of the anisotropies in the electron distribution function for some special cases. We do this without actually solving the corresponding Fokker-Planck equations, but rather considering the simpler scheme of the associated Langevin equations. The latter describe the temporal evolution of the test electrons' coordinates, in terms of which the Fokker-Planck equation itself is expressed, and represent a linkage between the kinetic and the single-particle approaches to the problem. The same approach has been successfully used in Ref. [9] to investigate the problem of wave-induced current in magnetized fusion plasmas.

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In Sec. II some information about Langevin equations and their connection to the Fokker-Planck equation is given. Under the hypothesis that electron-electron collisions are negligible, the Langevin equations for the special cases where the wave is either linearly polarized along the magnetic field (parallel geometry), or left- and right-hand circularly polarized in a plane perpendicular to it (perpendicular geometry), are derived in Sec. III. In the same section, the expressions of the rates of kinetic-energy change of colliding electrons are derived, whose domain of validity is not restricted to the case where the radiation frequency ω is much larger than the collision frequency ν; thus an extension and an independent check of previous results [5] is furnished.

For the case of a linearly polarized wave in the parallel geometry, the presence of a moderately strong magnetic field is expected not to be particularly relevant to the results in which we are interested. This case, accordingly, is to be considered as representative of the situation when no magnetic field is present.

Section IV is devoted to discussion and concluding remarks.

II. FOKKER-PLANCK AND LANGEVIN EQUATIONS

We start with a kinetic equation for the electron distribution function to describe the evolution of the electron population in a plasma. Here, ions are thought to be uniformly distributed in space; moreover, the presence of a steady and homogeneous magnetic field **B** is considered, and the radiation wavelength is supposed to be much larger than any other characteristic length of the system (dipole approximation). Under these assumptions, the electron distribution function does not depend upon the ordinary space variables, and its kinetic equation assumes the form (in Gaussian units):

$$\frac{\partial f}{\partial t}-\frac{e}{m}\left[\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{B}\right]\cdot\nabla_v f=\mathfrak{C}[f] , \tag{1}$$

where f is the electron distribution function, **E** is the electric field of the wave, e and m are the absolute values of the charge and the mass of the electron, c is the speed of light, and **v** is the electron velocity; the operator $\mathfrak{C}$ describes the stochastic (collisional) interaction of electrons with all the other species in the plasma.

For a completely ionized plasma it is possible to write $\mathfrak{C}$ in such a way that Eq. (1) assumes the form of a Fokker-Planck equation [10]:

$$\frac{\partial f}{\partial t}=-\frac{\partial}{\partial v_i}[A_i f]+\frac{1}{2}\frac{\partial^2}{\partial v_i \partial v_j}[B_{ij} f] , \tag{2a}$$

where

$$A_i \equiv \lim_{\Delta t\to 0}\frac{\langle \Delta v_i\rangle_s}{\Delta t} , \qquad B_{ij} \equiv \lim_{\Delta t\to 0}\frac{\langle \Delta v_i \Delta v_j\rangle_s}{\Delta t} , \tag{2b}$$

$\langle A\rangle_s$ indicating the ensemble average of the quantity A.

The basic assumptions adopted to obtain the kinetic equation in the form of Eq. (2) are that [10] (a) collisions are instantaneous and localized in space, (b) small-angle scattering events (remote collisions) dominate over large-angle ones, and (c) the binary collision approximation is reliable. Point (b) implies that for the Coulomb logarithms relative to each couple of colliding species, $\Lambda\equiv\ln(\xi_M/\xi_m)\gg 1$, where ξ_m and ξ_M are the minimum and maximum impact parameters. This, in turn, makes it possible to write the kinetic equation in the form of Eq. (2a) [i.e., to neglect terms involving higher-order derivatives in **v**, which are of order Λ^{-1} at least with respect to those appearing on the right-hand side of Eq. (2a) [10]].

The Langevin equations associated with the Fokker-Planck equation, Eq. (2a), are the following:

$$\dot v_i = A_i + F_i(t) , \tag{3}$$

where the dot stands for time derivation and $F_i(t)$ are stochastic forces with zero ensemble average, representing instantaneous and completely uncorrelated collisions:

$$\langle F_i(t)\rangle_s = 0 , \qquad \langle F_i(t)F_j(t')\rangle_s = B_{ij}\delta(t-t') . \tag{4}$$

The "macroscopic" equations of motion can be obtained from Eqs. (3) by averaging over the ensemble.

In the present case the Langevin equations will be found to be nonlinear; to make them solvable we assume that $\langle A_i\rangle_s = A_i(\langle \mathbf{v}\rangle_s)$, and the problem is reduced to a set of ordinary differential equations involving the ensemble averaged variables. This implies that fluctuations of the stochastic variables around their mean values are neglected [11], and diffusional contributions to the flow in velocity space (i.e., the effects of the gradient of the ensemble density) are ignored. All of what follows is based on the ensemble averaged Langevin equations, and since the above approximation will be tacitly assumed, the averaged variables will be indicated without angular brackets.

III. DERIVATION OF THE AVERAGED LANGEVIN EQUATIONS FOR SPECIAL CASES

We make the assumption that electron-ion collisions dominate over electron-electron collisions. A sufficient condition for this is $Z^{-1}v_w \ll v_t \ll v_w$, with $Z\gg 1$ the effective ion charge in units of e, v_w the magnitude of the wave-induced electron velocity, and $v_t\equiv\sqrt{T/m}$ the thermal velocity, T being the electron temperature in energy units [8].

The operator $\mathfrak{C}$, describing the collisional relaxation of the electron population upon ions at rest, has the following form in Cartesian coordinates in velocity space [10]:

$$\mathfrak{C}=\frac{Z}{2}\frac{\partial}{\partial v_i}\left\{\frac{\Gamma}{v}\left[\frac{\partial}{\partial v_i}-\frac{v_i v_j}{v^2}\frac{\partial}{\partial v_j}\right]\right\} , \tag{5}$$

where $\Gamma\equiv\omega_p^2 e^2\Lambda/m$, with $\omega_p\equiv(4\pi N_e e^2/m)^{1/2}$ the (electron) plasma frequency and N_e the electron density; $v\equiv|\mathbf{v}|$, Λ depends on the latter quantity through the minimum impact parameter $\xi_m = Ze^2/(mv^2)$; the interaction range ξ_M can assume the value of the Debye length if collective effects are important, or otherwise it may be given a value ranging from several tens to a few hundreds of Bohr radii.

A. Parallel polarization

It is convenient in this case to express the Fokker-Planck equation in spherical coordinates, i.e., in terms of the variables (ϕ,v,μ), which are the azimuthal angle of the electron velocity $\mathbf{v}$ around the magnetic-field direction $\mathbf{n}\equiv\mathbf{B}/|\mathbf{B}|$, the velocity magnitude, and the quantity $\mu\equiv v_\parallel/v$, being $v_\parallel\equiv\mathbf{n}\cdot\mathbf{v}$. For an electric field $\mathbf{E}\equiv\mathbf{n}E_0\cos\omega t$, one obtains:

$$\frac{\partial f}{\partial t}=\frac{1}{v^2}\frac{\partial}{\partial v}v^2[f\mu a\cos\omega t]+\frac{\partial}{\partial\mu}\left\{f\left[\frac{\Gamma Z}{v^3}\mu+\frac{1-\mu^2}{v}a\cos\omega t\right]\right\}-\frac{\partial}{\partial\phi}[f\Omega]+\frac{\Gamma Z}{2v^3}\left\{\frac{\partial^2}{\partial\mu^2}[f(1-\mu^2)]+\frac{1}{1-\mu^2}\frac{\partial^2 f}{\partial\phi^2}\right\}, \tag{6}$$

where $a\equiv eE_0/m$ and $\Omega\equiv e|\mathbf{B}|/mc$.

From Eq. (6) the following set of ensemble averaged Langevin equations is found:

$$\dot v=-\mu a\cos\omega t\ ,\qquad \dot\mu=-\frac{\Gamma Z}{v^3}\mu-a\cos\omega t\frac{1-\mu^2}{v}\ ,\qquad \dot\phi=\Omega\ . \tag{7}$$

The term $-\mu\Gamma Z/v^3$ accounts for the fact that the (ensemble-) average velocity of a population of test electrons, initially moving along a given direction, tends to zero because collisions cause a randomization of the directions of the scattered electrons' velocities. In the second of Eqs. (7), $-a\cos\omega t/v=\dot u(\partial_\mu/\partial v_\parallel)_{v=\text{const}}$ and $\mu^2 a\cos\omega t/v=\dot u(\partial v/\partial v_\parallel)(\partial\mu/\partial v)_{v_\parallel=\text{const}}$, with $u\equiv-u_0\sin\omega t$, $u_0\equiv a/\omega$ the wave-induced velocity. It is important to note that the last of Eqs. (7) is not coupled with the previous two, so that $\mathbf{B}$ is not relevant to the bremsstrahlung process in this geometry.

Here, we confine ourselves to the case where the time between subsequent collisions, $\nu^{-1}\equiv(\Gamma Z/v^3)^{-1}$, is much larger than any other characteristic time of the process. Moreover, to account for the fact that the instant at which any collision occurs is random, time averaging will be performed on relevant quantities, such as kinetic-energy changes and $\dot\mu$.

The time-averaged rate of change of the electron kinetic energy, $\overline{W}$, can be found immediately from the first of Eqs. (7):

$$\overline{W}=m\langle v_\parallel\dot u\rangle\ , \tag{8a}$$

where the angular brackets stand for time average. This equation states that the changes of electron kinetic energy are connected to the work performed by the radiation field along the parallel direction; in deriving it, no restriction is imposed on the ratio ω/ν.

Equation 8(a) can be integrated by parts; then, under the hypothesis that $\nu\ll\omega$ and for averaging time intervals T such that $T\gg\omega^{-1},\Omega^{-1},|\omega-\Omega|^{-1}$, we find

$$\overline{W}\simeq-m\langle u\dot v_\parallel\rangle \tag{8b}$$

in which $\langle u\dot v_\parallel\rangle$ can be approximately evaluated by direct substitution of $\dot v_\parallel$ as given by

$$\dot v_\parallel=-\frac{\Gamma Z}{v^3}v_\parallel+\dot u \tag{9}$$

[see Eqs. (7)], the magnitude $\mathscr{v}$ and the parallel component $\mathscr{v}_\parallel\equiv v_\parallel-u$ of the electron velocity in absence of radiation can be held constant. With this procedure a form of $\overline{W}$ is recovered which has been already considered in the literature [5].

In the weak-field limit, i.e., when $u_0\ll|\mathscr{v}_\parallel|$, Eq. (8b) reduces to

$$\overline{W}\simeq m\frac{\Gamma Z}{\mathscr{v}^3}\frac{u_0^2}{2}\left[1-3\frac{\mathscr{v}_\parallel^2}{\mathscr{v}^2}\right], \tag{10}$$

which is consistent with the results of [4,5].

In conclusion, Eq. (8a) extends the treatment given in [4,5] in that it removes the restriction $\nu\ll\omega$.

Information about the time-averaged scattering direction can be obtained from the second of Eqs. (7). The quantity $\langle\dot\mu\rangle$ may be written as a sum of two distinct contributions:

$$\langle\dot\mu\rangle\equiv\langle\dot\mu\rangle_c+\langle\dot\mu\rangle_r\ ,\qquad \langle\dot\mu\rangle_c\equiv-Z\left\langle\frac{\Gamma}{v^3}\mu\right\rangle\ ,\qquad \langle\dot\mu\rangle_r\equiv\left\langle\dot u\frac{1-\mu^2}{v}\right\rangle\ . \tag{11}$$

The term $\langle\dot\mu\rangle_r$, which henceforth we call *radiative*, describes the effect of collisional bremsstrahlung; of course, it depends on collisions and it can be integrated by parts to render its dependence on collisions explicit:

$$\langle\dot\mu\rangle_r\simeq-2Z\left\langle\frac{\Gamma}{v^4}\mu^2 u\right\rangle\ . \tag{12}$$

Pure collisional friction is described instead by the term $\langle\dot\mu\rangle_c$, which we call *collisional* from now on.

Consider the weak-field limit of Eq. (12); straightforward algebra leads, up to order $u_0^2/\mathscr{v}^2$, to the following expression:

$$\langle\dot\mu\rangle_r\simeq-2\frac{\Gamma Z}{\mathscr{v}^6}u_0^2\mathscr{v}_\parallel\left[1-3\frac{\mathscr{v}_\parallel^2}{\mathscr{v}^2}\right]\simeq-\left[\frac{4\mathscr{v}_\parallel}{m\mathscr{v}^3}\right]\overline{W}\ , \tag{13}$$

where Eq. (10) has been used as well. Equation (13) states that in this limit the signs of $\langle\dot\mu\rangle_r$ and $\overline{W}$ are opposite. It means that the velocity of a test electron initially within the Marcuse emission cone defined by $1-3(\mathscr{v}_\parallel/\mathscr{v})^2<0$ is forced by the bremsstrahlung process to rotate closer to the magnetic-field direction while decreasing in magnitude. On the other hand, if it lies out of the cone the reverse is true.

When the radiation field is not weak, the exact Eqs.

(11) must be used. Figure 1 shows the level curves of $\langle \dot{\mu} \rangle$ as a function of $v_\parallel/c$ and $v_\perp/c \equiv [v^2 - v_\parallel^2]^{1/2}/c$ for a moderately strong radiation field. The dashed line in Fig. 1 represents, for the same parameters, $\overline{W}=0$. Figures 2(a) and 2(b) show the separate contributions $\langle \dot{\mu} \rangle_c$ and $\langle \dot{\mu} \rangle_r$. The weak-field level curves of $\langle \dot{\mu} \rangle_r$ are reported in Fig. 3.

B. Perpendicular polarization

In this subsection the case when the radiation field is either right- or left-hand circularly polarized on a plane perpendicular to **B** is considered; in Cartesian coordinates:

$$\mathbf{E} = E_0[\mathbf{e}_x \cos\omega t \pm \mathbf{e}_y \sin\omega t] \,, \tag{14}$$

with $\mathbf{e}_x$ and $\mathbf{e}_y$ orthonormal unit vectors perpendicular to **B**. For both cases, a transformation to cylindrical coordinates is appropriate. In fact, the ensemble averaged Langevin equations describing explicitly the relaxation process within the plane perpendicular to **B** must now be considered. In terms of the coordinates ϕ, $v_\parallel$, and $v_\perp \equiv |\mathbf{v}\times\mathbf{n}|$, the Fokker-Planck equation reads

$$\begin{aligned}\frac{\partial f}{\partial t} = {} & \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp} v_\perp \left\{ \left[\frac{\Gamma Z}{2v^3} v_\perp [1+2\Lambda^{-1}] + a\cos(\omega t \mp \phi) \right] f \right\} + \frac{\partial}{\partial v_\parallel}\left\{ \frac{\Gamma Z}{v^3} v_\parallel [1+\Lambda^{-1}] f \right\} \\ & + \frac{1}{v_\perp}\frac{\partial}{\partial \phi}\{(\pm a \sin[\omega t \mp \phi] - v_\perp \Omega) f\} + \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp} v_\perp \frac{\partial}{\partial v_\perp}\left\{ \frac{\Gamma Z}{2v^3} v_\parallel^2 f \right\} \\ & + \frac{\Gamma Z}{2 v v_\perp^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2}{\partial v_\parallel^2}\left\{ \frac{\Gamma Z}{2v^3} v_\perp^2 f \right\} - \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp} v_\perp \frac{\partial}{\partial v_\parallel}\left\{ \frac{\Gamma Z}{v^3} v_\parallel v_\perp f \right\} ,\end{aligned} \tag{15}$$

where the upper (lower) sign stands for left- (right-) hand circular polarization.

After neglecting $O(\Lambda^{-1})$ terms, as is usually done to obtain a kinetic equation in the Fokker-Planck form, the corresponding set of ensemble averaged Langevin equations takes the form

$$\begin{aligned} \dot{v}_\perp &= -\frac{\Gamma Z}{2v^3} v_\perp - a\cos[\omega t \mp \phi] \,, \\ \dot{v}_\parallel &= -\frac{\Gamma Z}{v^3} v_\parallel \,, \\ \dot{\phi} &= \mp \frac{a}{v_\perp}\sin[\omega t \mp \phi] + \Omega \,. \end{aligned} \tag{16}$$

Note that the collisional relaxation rate for $v_\perp$ is one-half

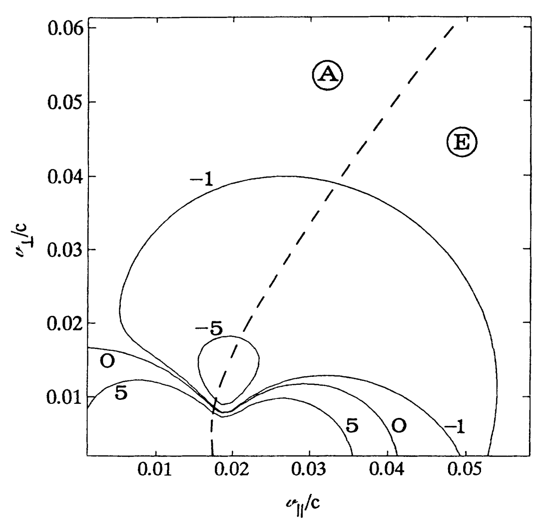

FIG. 1. Level curves of $\langle \dot{\mu} \rangle$ in units of 10^4 sec^{-1} as a function of the unperturbed velocities $v_\parallel$ and $v_\perp$. $\hbar\omega/2\pi = 10^{-5}$ eV, $E_0 = 5\times10^3$ V/cm, $N_e = 10^{13}$ cm^{-3}, and ξ_M 50 times the Bohr radius. The dashed line represents the solution of the equation $\overline{W}=0$; to the right (left) of it the test electron loses (gains) energy, which is indicated by the letter E (A) in the graph.

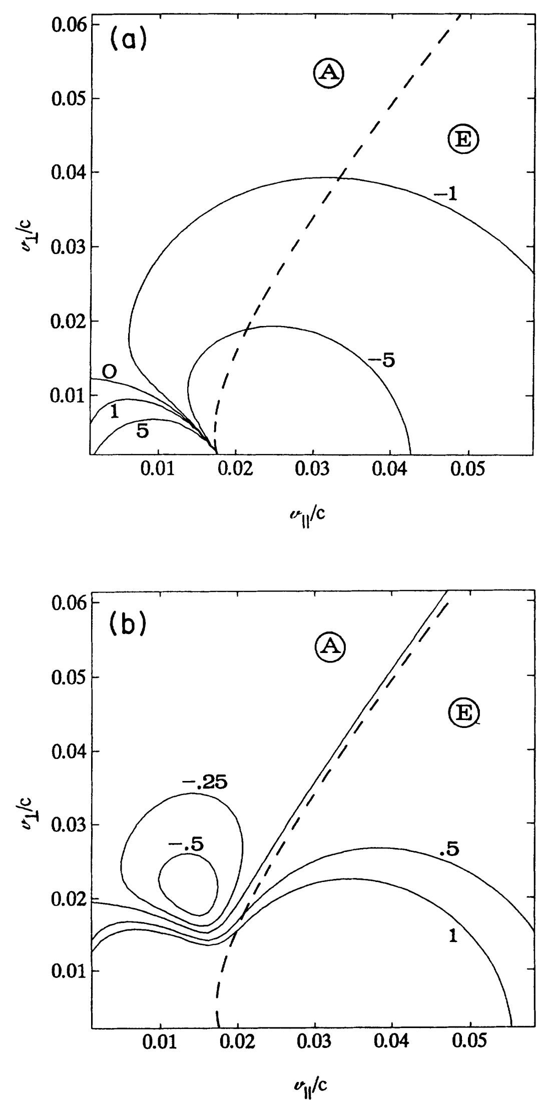

FIG. 2. Same as Fig. 1, but the collisional (a) and the radiative (b) contributions to $\langle \dot{\mu} \rangle$ are separately plotted.

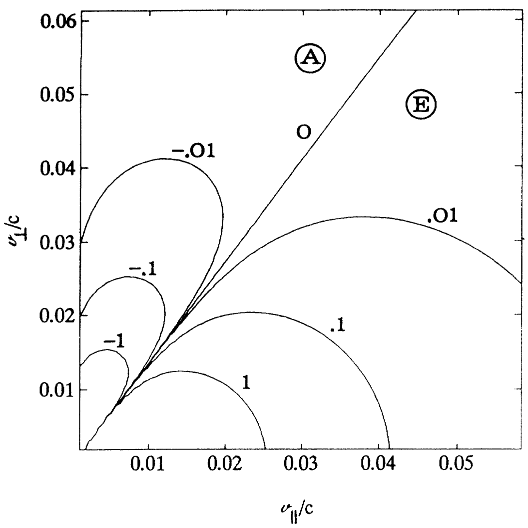

FIG. 3. Same as Fig. 1 for $\langle\dot{\mu}\rangle_r$ in units of 10^{-6} sec^{-1} in the case of the weak field: $E_0=10^{-2}$ V/cm (all other parameters are unchanged).

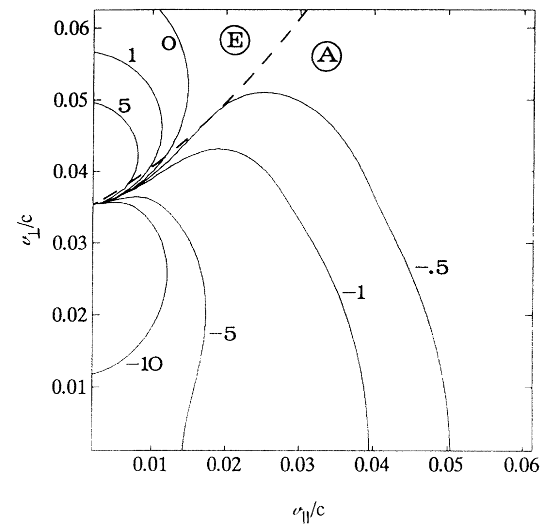

FIG. 4. Level curves of $\langle\dot{\eta}\rangle$ in units of 10^4 sec^{-1} as a function of the unperturbed velocities $\upsilon_\parallel$ and $\upsilon_\perp$. $h\Omega/2\pi=10^{-4}$ eV, $h\omega/2\pi=10^{-5}$ eV, $E_0=10^5$ V/cm (right-hand polarization), $N_e=10^{13}$ cm^{-3} and ξ_M 50 times the Bohr radius. The dashed line represents the solution of the equation $\overline{W}=0$; to the left (right) of it the test electron loses (gains) energy, which is indicated by the letter E (A) in the graph.

of the corresponding one for $v_\parallel$. This is connected with the fact that the first equation describes an evolution dynamics into the (two-dimensional) plane perpendicular to **B**, whereas the second relates to the (one-dimensional) dynamics along **B**.

The rate of change of v is given by the equation

$$\dot{v}=-a\eta\cos[\omega t\mp\phi]\ , \tag{17}$$

where $\eta\equiv v_\perp/v$ is the cosine of the angle formed by the electron velocity with the polarization plane.

In this case, the equation for ϕ is coupled with those for $v_\perp$ and v, and the magnetic field now plays an important role in determining $\overline{W}$ and the rate of change of η.

To find the time-averaged power $\overline{W}$ emitted or absorbed by a test electron, Eq. (17) is profitably changed to a form which explicitly contains both the perpendicular wave-induced velocity $\mathbf{w}^\pm$ and the unperturbed one $\boldsymbol{\upsilon}_\perp$, which in Cartesian coordinates read

$$\begin{aligned}\mathbf{w}^\pm&=w_0^\pm[\mathbf{e}_x\sin\omega t\pm\mathbf{e}_y\cos\omega t]\ ,\\ \boldsymbol{\upsilon}_\perp&=\upsilon_\perp[\mathbf{e}_x\cos\Omega t+\mathbf{e}_y\sin\Omega t]\ ,\end{aligned}\tag{18}$$

where $w_o^\pm\equiv-a/(\omega\pm\Omega)$. The perpendicular electron velocity is, of course, $\mathbf{v}_\perp=\mathbf{w}^\pm+\boldsymbol{\upsilon}_\perp$ in the plane perpendicular to **B**.

As it is shown in Appendix A, from Eqs. (17) and (18) one finds

$$\overline{W}\equiv-am[\omega w_o^\pm]^{-1}\langle\mathbf{v}_\perp\cdot\dot{\mathbf{w}}^\pm\rangle\ . \tag{19a}$$

After integration by parts, for $\nu\ll\omega,\Omega,|\omega-\Omega|$, the following approximate expression can be used for $\overline{W}$:

$$\overline{W}=\frac{mZ}{2}\left\langle\frac{\Gamma}{v^3}\mathbf{v}_\perp\cdot\mathbf{w}^\pm\right\rangle\ . \tag{19b}$$

In the weak-field limit ($|w_0^\pm|\ll\upsilon_\perp$):

$$\overline{W}\simeq-m\frac{\Gamma Z}{\upsilon^3}\frac{(w_0^\pm)^2}{4}\left[1-3\frac{\upsilon_\parallel^2}{\upsilon^2}\right]\ . \tag{20}$$

From Eqs. (16) and (17) the following equation for η can be derived:

$$\dot{\eta}=-\frac{\Gamma Z}{2v^3}\eta-a\cos[\omega t\mp\phi]\frac{1-\eta^2}{v} \tag{21}$$

[compare with Eqs. (7)].

With a procedure analogous to that followed for the case of parallel polarization, the contribution $\langle\dot{\eta}\rangle_r\equiv-a(1-\eta^2)v^{-1}\cos(\omega t\pm\phi)$ can be put in the form:

$$\langle\dot{\eta}\rangle_r=-Z\left\langle\frac{\Gamma}{v^4}\eta^2\mathbf{e}_\perp\cdot\mathbf{w}^\pm\right\rangle\ , \tag{22}$$

where $\mathbf{e}_\perp\equiv\mathbf{v}_\perp/v_\perp$. The weak-field limit of Eq. (22) is obtained by substituting Eqs. (18) into Eq. (22) and letting $|w_0^\pm|\ll\upsilon_\perp$:

$$\langle\dot{\eta}\rangle_r\simeq\frac{3}{2}\frac{\Gamma Z}{\upsilon^6}(w_0^\pm)^2\upsilon_\perp\left[1-2\frac{\upsilon_\parallel^2}{\upsilon^2}\right]\ . \tag{23}$$

In Fig. 4 level curves of $\langle\dot{\eta}\rangle$ and $\overline{W}=0$ for a moderately strong (right-hand circularly polarized) field are reported. Figure 5 shows the corresponding level curves for $\langle\dot{\eta}\rangle_r$; 5(a) and 5(b) represent the situations of strong- and weak-field regimes, respectively.

IV. COMMENTS AND CONCLUSIONS

We have reported calculations on two quantities: (i) on $\langle\dot{\mu}\rangle$ and $\langle\dot{\eta}\rangle$, the time-averaged time derivatives of the cosines of the angle the test-particle velocity forms, respectively, with the radiation-field polarization direction (parallel polarization), and with the field polarization plane (perpendicular polarization); (ii) on $\overline{W}$, the time-averaged change of the electron kinetic energy, for both polarization cases.

The calculations on $\langle \dot{\mu} \rangle$ and $\langle \dot{\eta} \rangle$ aimed to obtain information on the changes occurring on the electron velocity distribution as a result of collisional bremsstrahlung; the calculations on $\overline{W}$ provide information about the regions in velocity space where prevailing of emission (field amplification) or of absorption (particle heating) takes place during breamsstrahlung.

Concerning $\overline{W}$, our derivation extends to arbitrary ratios ν/ω of previous derivations [4,5], though, for the sake of comparison and simplicity, we confine calculations to the high radiation frequencies ($\nu \ll \omega$). Concerning, instead, $\langle \dot{\mu} \rangle$ and $\langle \dot{\eta} \rangle$, as far as we can judge, both derivation and calculations are new.

The results of the calculations are reported in the figures in such a way to allow for a correlation (if any) between emission (absorption) zones and increasing/decreasing of either $\langle \dot{\mu} \rangle$ or $\langle \dot{\eta} \rangle$. Moreover, the analysis is far more informative when we disentangle the contributions to $\langle \dot{\mu} \rangle$ and $\langle \dot{\eta} \rangle$ due to collisions only ($\langle \dot{\mu} \rangle_c$ and $\langle \dot{\eta} \rangle_c$) and to the collisional interaction with the radiation field ($\langle \dot{\mu} \rangle_r$ and $\langle \dot{\eta} \rangle_r$). Of course, these contributions cannot be entirely separated, as in our treatment there is no bremsstrahlung without collisions. Nevertheless, the splitting of $\langle \dot{\mu} \rangle$ and $\langle \dot{\eta} \rangle$ in two terms provides additional insight into the results. In particular, the radiative contributions to $\langle \dot{\mu} \rangle$ and $\langle \dot{\eta} \rangle$ are of the same nature of the radiation-induced power $\overline{W}$ [see Eqs. (8), (12), (19), and (22)]. In fact,

$$\overline{W} = \left\langle \nu u \left\{ \frac{\partial}{\partial v_\parallel} \left[\frac{m}{2} v^2 \right] \right\} \right\rangle , \qquad \langle \dot{\mu} \rangle_r = \left\langle \nu u \left[\frac{\partial \mu^2}{\partial v} \right]_{v_\parallel = \text{const}} \right\rangle \tag{24}$$

for linear polarization along **B**, and

$$\overline{W} = \frac{1}{2} \left\langle \nu \mathbf{w}^{\pm} \cdot \mathbf{e}_\perp \left\{ \frac{\partial}{\partial v_\perp} \left[\frac{m}{2} v^2 \right] \right\} \right\rangle , \qquad \langle \dot{\eta} \rangle_r = \frac{1}{2} \left\langle \nu \mathbf{w}^{\pm} \cdot \mathbf{e}_\perp \left[\frac{\partial \eta^2}{\partial v} \right]_{v_\perp = \text{const}} \right\rangle \tag{25}$$

for circular polarization perpendicular to **B**.

The above equations point out that (a) $\overline{W}$, $\langle \dot{\mu} \rangle_r$, and $\langle \dot{\eta} \rangle_r$ can be nonzero only if both collisions and radiation are simultaneously present ($\nu u_0 \neq 0$) and (b) $\overline{W}$ and either $\langle \dot{\mu} \rangle_r$ or $\langle \dot{\eta} \rangle_r$ are correlated; namely, radiation-induced changes of $\langle \mu \rangle$ ($\langle \eta \rangle$) are caused by radiation-induced variations of v at constant $v_\parallel$ ($v_\perp$).

From the calculations reported above we find that, in the chosen geometries, the velocity of all test electrons undergoing emission during stimulated bremsstrahlung is forced either to align along the polarization direction or to draw near to the polarization plane, depending on whether linearly [Fig. 2(b)] or circularly polarized waves [Fig. 5(a)] are considered.

The effect of this mechanism on the shape of the electron distribution function is further explored in Appendix B for the case of a weak, linearly polarized (along **B**) radiation field. We consider the situation where a quasistationary state is attained, i.e., where the equilibrium electron distribution slightly deviates from the Maxwellian form in such a way that collisional relaxation balances the distorting effect of radiation (via stimulated bremsstrahlung). In this situation we find that the deviation f_a of the time-averaged (over the wave period) distribution function f from its isotropic part is positive within the Marcuse cone. In the light of the result reported above, we interpret this deviation as a combination of two main processes: a crowding of electrons along the axis of the Marcuse cone, due to $\langle \dot{\mu} \rangle_r$ [Fig. 2(b)], and a spreading in the whole velocity space due to pure collisions. It is important to note that in this limit $|f_a/f| \sim |\langle \dot{\mu} \rangle_r / \langle \dot{\mu} \rangle_c| \sim O(u_0^2/v^2)$.

On this basis, again in the weak-field limit, it is expect-

A)

B)

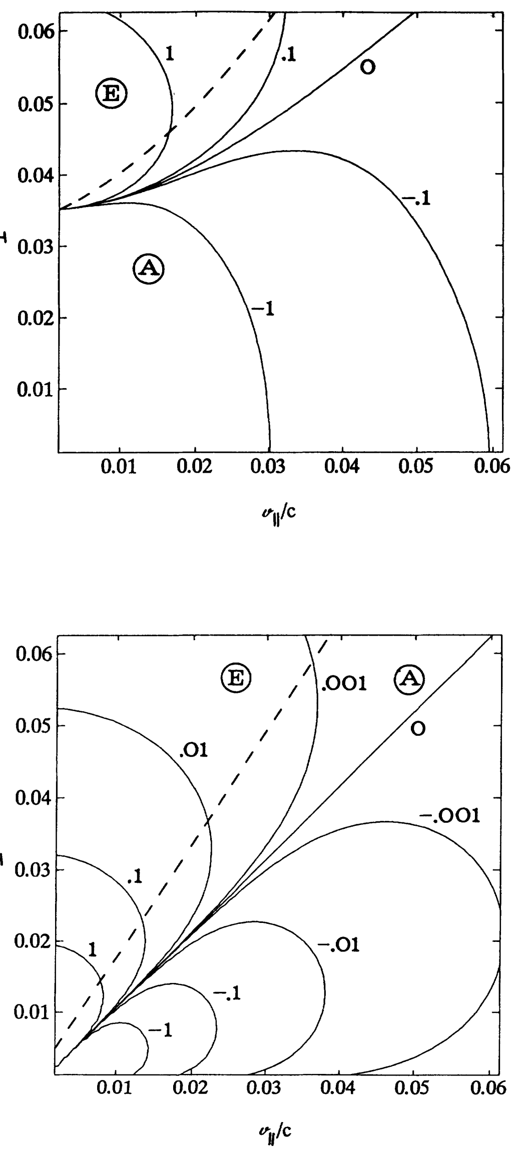

FIG. 5. Level curves for $\langle \dot{\eta} \rangle_r$: (a) in units of 10^4 sec^{-1} and in the same parameter range of Fig. 4; (b) in units of 10^{-8} sec^{-1} for an electric field $E_0 = 10^{-2}$ V/cm (all other parameters are the same); note that the dashed line (defining the Marcuse cone) is steeper than the curve $\langle \dot{\eta} \rangle_r = 0$.

ed that for a perpendicular (with respect to **B**) circularly polarized wave, the quasistationary electron distribution deviates from the Maxwellian form in such a way to be depleted in the cone $v_\perp < |v_\parallel|$ [Fig. 5(b)].

We now turn to the more interesting case where the wave field is not weak. Consider first the case of linear polarization. In a large domain of velocity space $\langle \dot{\mu} \rangle_c$ and $\langle \dot{\mu} \rangle_r$ are in competition, the latter prevailing on the former only for relatively low $v_\perp$, and $v_\parallel$ around u_0 (Fig. 1). An interesting result is that the unperturbed velocity of the test electrons for which $\langle \dot{\mu} \rangle_r$ dominates is forced by bremsstrahlung to oscillate around u_0 while keeping aligned with the polarization direction. In fact, in these conditions, transition from absorption to emission occurs at $v_\parallel \simeq u_0$, as shown in Fig. 1. These oscillations can lead to a flattening of the distribution in the direction of the electric field of the wave at $|v_\parallel| \simeq u_0$ and relatively small $v_\perp$.

In the case of (perpendicular) circularly polarized radiation, the magnetic field is important, as it can significantly influence the perpendicular electron motion. We first note that **B** enters Eqs. (25) through the quantity $w_0^\mp \equiv -eE_0/m(\omega \pm \Omega)$; then, the following statements may briefly summarize the results of the calculations when a magnetic field is present:

(a) The magnetic field is relevant when Ω is larger than or comparable with ω.

(b) The case where $|w_0^\pm| \to 0$ is indicative of either very weak radiation field or large $|\mathbf{B}|$ or both.

(c) $\langle \dot{\eta} \rangle_r$ (Fig. 5) and $\overline{W}$ [5] tend to an overall decrease on increasing $|\mathbf{B}|$.

(d) Both the domains where $\langle \dot{\eta} \rangle_r > 0$ and $\overline{W} < 0$ widen on increasing $|\mathbf{B}|$ (Fig. 5).

(e) When $\Omega \gg \omega$, for a moderately strong radiation field the transition from absorption to emission, as well as the transition from positive to negative $\langle \dot{\eta} \rangle_r$, occurs at $v_\perp \approx |w_0^\pm| \simeq cE_0/|\mathbf{B}|$ if $|v_\parallel| \ll |v_\perp|$. In other words, the condition $\overline{W}=0$ as well as $\langle \dot{\eta} \rangle_r = 0$ depends, for low unperturbed parallel velocities, on the ratio of the electric-field to the magnetic-field amplitudes (see also [5]).

In conclusion, the adopted approach leads to a better and more complete understanding of the main processes involved in collisional bremsstrahlung. Although restricted to selected geometries, the expressions derived for the relevant quantities are more reliable than the corresponding ones reported in the literature. As previously indicated in the Introduction, averaged Langevin equations are able to give information about the origin of anisotropies of the electron distribution function when collisional bremsstrahlung is active. The picture they give may well serve as a support for the interpretation of outcomes from numerical evaluations of solutions of the Fokker-Planck equation.

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APPENDIX A

Here, the explicit derivation of Eq. (20) from Eqs. (17)–(19) in the text is presented for the particular case of right-hand circular polarization; for the other one (left-hand) the procedure is quite the same.

Referring to Fig. 6, where the vectors $\mathbf{u}^+ \equiv \mathbf{w}^+/w_0^+$, $v_\perp$, and $\mathbf{v}_\perp = v_\perp + \mathbf{w}^+$ are explicitly indicated, one can check that

$$v_\perp \cos(\omega t + \phi) = \omega^{-1} \mathbf{v}_\perp \cdot \dot{\mathbf{u}}^+ \ . \tag{A1}$$

From Eq. (18):

$$\overline{W} = m \langle v\dot{v} \rangle = -\frac{ma}{\omega} \langle \mathbf{v}_\perp \cdot \dot{\mathbf{u}}^+ \rangle \ . \tag{A2}$$

Upon integrating by parts, and neglecting the term $(ma/\omega)[\mathbf{v}_\perp \cdot \mathbf{u}^+]_0^T$,

$$\overline{W} \simeq \frac{ma}{\omega} \langle \dot{\mathbf{v}}_\perp \cdot \mathbf{u}^+ \rangle \ . \tag{A3}$$

Now, in cylindrical coordinates, $\mathbf{v}_\perp = v_\perp \mathbf{e}_\perp$ and $\dot{\mathbf{v}}_\perp = \dot{v}_\perp \mathbf{e}_\perp + v_\perp \dot{\phi} \mathbf{e}_\phi$, with $\mathbf{e}_\perp$ and $\mathbf{e}_\phi$ the radial and azimuthal unit vectors, respectively. Since

$$\mathbf{u}^+ \cdot \mathbf{e}_\perp = \cos\left(\frac{\pi}{2} - \omega t - \phi\right) ,$$
$$\mathbf{u}^+ \cdot \mathbf{e}_\phi = \cos(\omega t + \phi) , \tag{A4}$$

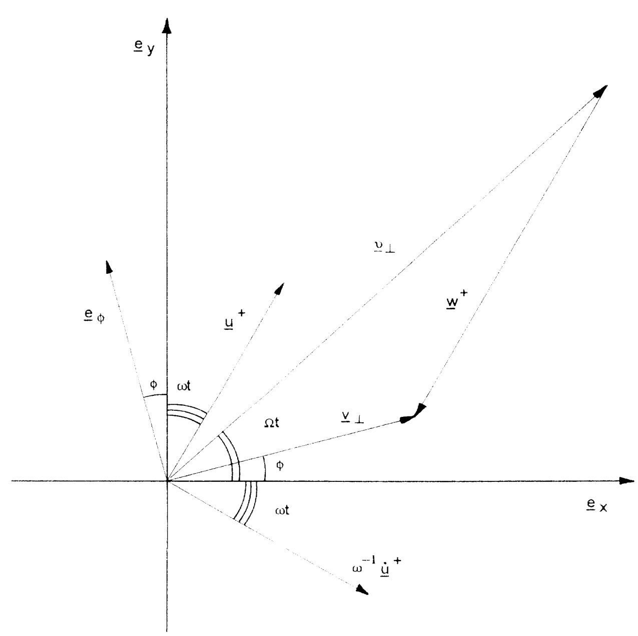

FIG. 6. Vector representation of the unperturbed and wave-induced velocities for the case of right-hand circular polarization perpendicular to the magnetic field.

from Eqs. (17)–(19) one finds:

$$\dot{\mathbf{v}}_\perp\cdot\mathbf{u}^+=-\frac{Z\Gamma}{2v^3}v_\perp\cos\left[\frac{\pi}{2}-\omega t-\phi\right]+\Omega v_\perp\cos(\omega t+\phi)\ . \tag{A5}$$

Equation (20) then follows immediately.

APPENDIX B

In this Appendix we report for a special case a derivation of the modifications of the electron distribution function due to the radiation field, which lends support to some results reported in the main body of the paper.

Let us consider now an equilibrium (Maxwellian) electron distribution function. We suppose that a suitably weak radiation field is present, so that the time over which the temperature changes is much longer than any other characteristic time of the process. Thus a quasistationary state is attained, where the shape of the electron distribution function deviates from the Maxwellian form, and collisional relaxation is in competition with the distortions induced by radiation. Below, this problem is explicitly formulated and an asymptotic solution is found for electrons whose velocity suitably exceeds the thermal one.

The Fokker-Planck equation for the electron distribution function f in the presence of a wave whose electric field is linearly polarized along $\mathbf{B}$ is, in the dipole approximation,

$$\frac{\partial f}{\partial t}=-\dot{u}\left\{\frac{\mu}{v^2}\frac{\partial}{\partial v}[v^2 f]+\frac{1}{v}\frac{\partial}{\partial\mu}[(1-\mu^2)f]\right\}+\mathfrak{C}[f]+\mathfrak{C}_e[f,f]\ , \tag{B1}$$

where $u\equiv -u_0\sin\omega t$, $u_0\equiv eE_0/m\omega$; $\mathfrak{C}$ is defined in Sec. III, and $\mathfrak{C}_e[f,f]$ is the electron-electron collisional operator. The bilinear form $\mathfrak{C}_e[h,g]$ is defined as

$$\mathfrak{C}_e[h,g]\equiv\frac{4\pi}{N_e}\Gamma\Lambda\left\{hg-\frac{\partial^2 h}{\partial v_i\partial v_j}\frac{\partial^2\psi\{g\}}{\partial v_i\partial v_j}\right\}\ , \tag{B2}$$

with

$$\psi\{g\}\equiv-\frac{1}{8\pi}\int d^3\mathbf{w}|\mathbf{v}-\mathbf{w}|g(\mathbf{w})\ , \tag{B3}$$

a Rosenbluth potential [10,12]; N_e is the electron density and Γ is defined in Sec. III. The electron distribution function is now split into two contributions: $f=f_0+f_1$, with $f_0\equiv\langle f\rangle$ the slowly evolving part (angular brackets stand for time average), and f_1 is the component evolving over a time scale of the order of the wave period, i.e., over a much shorter time scale.

The evolution equation for f_0 is obtained by time-averaging Eq. (B1); after reordering of terms:

$$\frac{\partial f_0}{\partial t}-\mathfrak{C}[f_0]-\mathfrak{C}_e[f_0,f_0]=-\left\langle\dot{u}\left\{\frac{\mu}{v^2}\frac{\partial}{\partial v}[v^2 f_1]+\frac{1}{v}\frac{\partial}{\partial\mu}[(1-\mu^2)f_1]\right\}\right\rangle+\langle\mathfrak{C}_e[f_1,f_1]\rangle \tag{B4}$$

where the conditions $\langle\dot{u}f_0\rangle=\langle\mathfrak{C}_e[f_0,f_1]\rangle=\langle\mathfrak{C}_e[f_1,f_0]\rangle=0$ have been imposed.

For weak field, an approximate evolution equation for f_1 can be obtained by first subtracting Eq. (B4) from Eq. (B1), and then retaining in the resulting equation only linear terms in the field amplitude E_0. Thus f_1 is linear in E_0, and the lowest-order deviation of f_0 from the equilibrium Maxwellian $f_m\equiv N\exp\{-v^2/2v_t^2\}$, with $N\equiv N_e(2\pi v_t^2)^{-3/2}$, is quadratic in E_0. The limit $Z\gg1$ (Lorentz gas) will be taken in the following, so that in the equation for f_1 only the electron-ion collision term will be retained [by the way, one may show that $\mathfrak{C}_{ee}[f_m,f_1]=0$ for f_1 given by Eq. (A6) below]. The perturbation f_1 is then found from

$$\frac{\partial f_1}{\partial t}=-\dot{u}\left\{\frac{\mu}{v^2}\frac{\partial}{\partial v}[v^2 f_m]+\frac{1}{v}\frac{\partial}{\partial\mu}[(1-\mu^2)f_m]\right\}+\mathfrak{C}[f_1]\ . \tag{B5}$$

Under the hypothesis that $\lambda\equiv\Gamma Z v^{-3}\omega^{-1}\ll1$, the following solution is found [apart from $O(\lambda^2)$ corrections]:

$$f_1\simeq u_0\mu\frac{\partial f_m}{\partial v}[\lambda\cos\omega t+\sin\omega t]\ . \tag{B6}$$

The term $\langle\mathfrak{C}_e[f_1,f_1]\rangle$ can be evaluated, apart from $O(\lambda^2)$ terms, taking only the contribution proportional to $\sin\omega t$ in Eq. (B6). For the potential ψ one finds

$$\psi\{f_1\}\simeq-\sqrt{\pi/2}\frac{Nu_0}{2}v_t^3\mu\sin\omega t,\quad v\gg v_t\ , \tag{B7}$$

so that

$$\langle\mathfrak{C}_e[f_1,f_1]\rangle\simeq\frac{\Gamma}{v_t^4}\frac{u_0^2}{v}P_2(\mu)f_m,\quad v\gg v_t\ , \tag{B8}$$

where $P_2(\mu)$ is the Legendre polynomial of order 2. For the first term on the right-hand side of Eq. (B4):

$$\left\langle\dot{u}\left\{\frac{\mu}{v^2}\frac{\partial}{\partial v}[v^2 f_m]+\frac{1}{v}\frac{\partial}{\partial\mu}[(1-\mu^2)f_m]\right\}\right\rangle\simeq-\frac{\Gamma Z}{3v_t^4}\frac{u_0^2}{v}[P_2(\mu)+\tfrac{1}{2}]f_m,\quad v\gg v_t\ . \tag{B9}$$

Consider now Eq. (B4); a quasistationary solution is obtained by setting $\partial f_0/\partial t=0$. The time-independent solution is now put in the form:

$$f_0(v,\mu)=f_m(v)+f_i(v)+f_a(v)P_2(\mu)\ , \tag{B10}$$

where f_i and f_a are the isotropic corrections to f_m, respectively, and proportional to u_0^2.

The Maxwellian function Af_m, with A an arbitrary constant, is a solution of the homogeneous equation associated to the quasistationary problem, since $\mathfrak{C}_e[f_m,f_m]=\mathfrak{C}[f_m]=0$.

For the collisional operator in the inhomogeneous quasistationary equation, to lowest order in E_0,

$$\mathfrak{C}_e[f_0,f_0]\simeq\mathfrak{C}_e[f_m,f_i]+\mathfrak{C}_e[f_i,f_m]+\mathfrak{C}_e[f_m,P_2f_a]+\mathfrak{C}_e[P_2f_a,f_m] \tag{B11}$$

and for the potentials ψ:

$$\psi\{f_m\} \simeq -\frac{Nv_t^3}{2}\sqrt{\pi/2}, \quad v \gg v_t ,$$

$$\psi\{f_i\} \simeq -\frac{v}{2}\int_0^{+\infty} dx\, x^2 f_i(x), \quad v \gg v_t . \qquad \text{(B12)}$$

$$\psi\{f_a\} \simeq \frac{P_2(\mu)}{30v}\int_0^{+\infty} dx\, x^4 f_a(x), \quad v \gg v_t .$$

Since $\mathfrak{C}_e[f_m, P_n h(v)]$ and $\mathfrak{C}_e[P_n h(v), f_m]$ are proportional to P_n [12], the solution to the quasistationary problem can be found for f_i and f_a separately. Under the hypothesis that f_i is Maxwellian-like, the term proportional to $f_i f_m$ is neglected in the corresponding equation ($v \gg v_t$), which takes the form

$$f_i' + 2\left[\frac{2}{\pi}\right]^{1/2}\frac{f_m}{N}\frac{v}{v_t^5}\psi'\{f_i\} = -\frac{Zvu_0^2}{6v_t^4}f_m , \qquad \text{(B13)}$$

where primes stand for the derivative with respect to v. One sees immediately that $f_i = Bf_m$ is a solution; the constant B can be found by direct substitution into Eq. (B13), and the final result is

$$f_i \simeq \frac{Zu_0^2}{12v_t^2}f_m . \qquad \text{(B14)}$$

For the anisotropic correction, let $f_a = yf_m$; then (using the condition $v \gg v_t$, and again neglecting the term proportional to $f_a f_m$) the following equation for y is found:

$$vy' - \left[3(Z+1) + \frac{v^2}{v_t^2}\right]y = \Phi\frac{v^2}{v_t^2} , \qquad \text{(B15)}$$

where

$$\Phi \equiv \left[\frac{2}{\pi}\right]^{1/2}\frac{I_a}{15Nv_t^5} - \frac{Z+3}{3}\frac{u_0^2}{v_t^2} ,$$

$$I_a \equiv \int_0^{+\infty} dx\, x^4 y f_m(x) . \qquad \text{(B16)}$$

Since the coefficient of y is large, an approximate solution to Eq. (B15) can be obtained by neglecting the term vy' (and one is led to a Fredholm equation of the first kind):

$$y \simeq C\frac{v^2}{v_t^2}\left[3(Z+1) + \frac{v^2}{v_t^2}\right]^{-1} , \qquad \text{(B17)}$$

with C a constant to be determined by introducing Eq. (B17) into Eq. (B15) [it may be checked that the term vy' in Eq. (B15) is, in fact, negligible].

After straightforward algebra,

$$C = \frac{Z+3}{3}\frac{u_0^2}{v_t^2}\left\{1 + \frac{[3(Z+1)]^{5/2}}{30}e^{3(Z+1)/2}\Gamma\left[\frac{7}{2}\right] \times \Gamma\left[-\frac{5}{2}, \frac{3(Z+1)}{2}\right]\right\}^{-1} . \qquad \text{(B18)}$$

Finally, letting Z be large, the following expression for f_a is found:

$$f_a \simeq \frac{Z}{3}\frac{u_0^2}{v_t^2}\frac{v^2}{v_t^2}\left[3Z + \frac{v^2}{v_t^2}\right]^{-1} P_2(\mu) f_m . \qquad \text{(B19)}$$

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