Electron-ion relaxation in a plasma interacting with an intense laser field

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The effect of an external high-frequency electromagnetic field on the electron-ion energy exchange rate in a plasma is considered. Particular consideration is given to very high intensities of the laser field and explicit analytical formulas are obtained for this case. The implication of the present approach for the more simple case of electron heating is also briefly considered. For practical use, simple interpolation formulas for the variation rate of the ion and electron temperatures are suggested, describing the entire region from zero intensity to extremely high intensities below the relativistic threshold.

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I. INTRODUCTION

The problem of the electron-ion energy exchange in plasma [1–3] as well as the electron-phonon energy exchange in metals [4] has attracted much attention during the past three decades. The interest in these problems is renewed in connection with the investigation of superintense ultrashort laser pulse interaction with materials [5–8], since the characteristic time of the electron-ion energy exchange is comparable to a pulse duration, and therefore a strongly nonisothermal plasma can be generated. At such conditions, the mechanism of electron transport considerably changes [9].

Although the kinetic properties of electrons in plasma exposed to a strong electromagnetic field has been intensively studied recently [10–15], the problem of the electron-ion relaxation at such conditions was not adequately considered. The heating of ions via electron-ion collisions in the laser field has been recently studied in a weak-field limit [15]. The aim of the present paper is to consider the electron-ion energy exchange in an electromagnetic field of arbitrary intensity. The limit of zero-field strength is also recovered for completeness. Particular attention is given to the case of very intense fields, however the limit of zero-field strength is also recovered for completeness, and a simple interpolation formula covering the intermediate region of field intensities is finally suggested.

In the present paper we shall consider the electron-ion scattering problem using quantum consideration [16] since electrons are accelerated to large velocities in an intense field. However, we restrict ourselves to a nonrelativistic consideration. In the following, the Planck constant $\hbar$ is assumed to be unity.

II. ELECTRON-ION INELASTIC COLLISIONS IN AN EXTERNAL FIELD

We consider the electron-ion plasma in the high-frequency field $\mathbf{E}\cos\omega t$. The wave function of the free electron with a canonical momentum $\mathbf{k}$ can be written as

$$\psi_{\mathbf{k}}=\frac{1}{\sqrt{\Omega}}\exp\left[i\mathbf{k}\cdot\mathbf{r}-\frac{i}{2m}\int^{t}\left[\mathbf{k}+\frac{e\mathbf{E}}{\omega}\sin\omega t'\right]^{2}dt'\right], \tag{1}$$

where Ω is the volume of the system and e is the electron charge. The influence of the electric field on the ion motion can be neglected due to the small ratio m/M of electron and ion masses.

Let the colliding electron and the ion have initial momenta $\mathbf{k}$ and $\mathbf{p}$ and final momenta $\mathbf{k}'$ and $\mathbf{p}'$, respectively. Then, using the conventional perturbation theory approach for the electron-ion scattering problem, we can calculate the corresponding transition probability per unit time,

$$W(\mathbf{p}',\mathbf{k}'|\mathbf{p},\mathbf{k})=\sum_{n=-\infty}^{\infty}J_n^2(\rho)\frac{(2\pi)^4}{\Omega^3}\times\delta(\mathbf{p}'+\mathbf{k}'-\mathbf{p}-\mathbf{k})|V(\mathbf{k}-\mathbf{k}')|^2\times\delta\left[\frac{k'^2-k^2}{2m}-\frac{p'^2-p^2}{2M}+n\omega\right]. \tag{2}$$

Here, J_n are Bessel functions, $\rho=(\mathbf{k}-\mathbf{k}')\cdot\mathbf{E}e/m\omega^2$, $V_q=4\pi e^2Z/q^2$, and Z is the ion charge. The δ functions in Eq. (2) describe n-photon emission and absorption processes accompanying the electron-ion scattering event.

We shall concentrate on the calculation of the variation rate of the ion subsystem energy E_i' caused by electron-ion collisions. We can therefore write

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$$E_i'=\sum_{\mathbf{k},\mathbf{k}',\mathbf{p},\mathbf{p}'} F(\mathbf{p})f(\mathbf{k})W(\mathbf{p}',\mathbf{k}'|\mathbf{p},\mathbf{k})\frac{p'^2-p^2}{2M} , \quad (3)$$

where F and f are ion and electron distribution functions, respectively. The summation in Eq. (3) goes over all possible scattering events. Substituting Eq. (2) into Eq. (3) we can write the expression for E_i' in the following form:

$$E_i'=\sum_{n=0}^{\infty} E_i'(n) ,$$

$$E_i'(n)=\frac{N_i}{8(2\pi)^6}\int d\mathbf{Q}\int d\mathbf{q}|V_q|^2(2\pi)\left[\delta\left[\frac{\mathbf{q}\cdot\mathbf{Q}}{2m}-n\omega\right]+\delta\left[\frac{\mathbf{q}\cdot\mathbf{Q}}{2m}+n\omega\right]\right]$$
$$\times J_n^2(\rho)f\left[\frac{1}{2}\left[\mathbf{Q}+\frac{2m}{M}\mathbf{p}-\frac{m}{M}\mathbf{q}-\mathbf{q}\right]\right]\left[\frac{\mathbf{p}\cdot\mathbf{q}}{M}-\frac{q^2}{2M}\right] . \quad (4)$$

Here, $\rho=\mathbf{q}\cdot\mathbf{E}e/m\omega^2$, N_i is the total number of ions, and averaging over ion momenta $\mathbf{p}$ is assumed. When $n=0$ only one δ function is assumed in Eq. (4).

For the particular analysis in the present paper we assume Maxwellian distribution functions for ions and electrons with temperatures T_i and T_e, respectively, in particular, $f(k)=A_T\exp(-k^2/2mT_e)$, $A_T=n_e(2\pi/mT_e)^{3/2}$, and n_e is the electron number density.

Let us now take into account the existence of the small parameter m/M and consider the expansion of Eq. (4) in power series of $1/M$. It is clear, that after averaging over directions of $\mathbf{p}$, the main contribution $\sim 1/\sqrt{M}$ determined by the term $\mathbf{p}\cdot\mathbf{q}/M\sim\sqrt{T_i}q/\sqrt{M}$ in the last parentheses in Eq. (4) vanishes, but the remaining term $\sim 1/M$ in the same parentheses survives. Therefore to take all terms $\sim 1/M$ into account one needs to also consider the contribution of the respective order resulting from the expansion of the distribution function f in Eq. (4). Retaining the necessary terms we get after the integration with respect to $\mathbf{Q}$ in Eq. (4)

$$E_i'(n)=\frac{N_i m^2 A_T}{(2\pi)^4}\int d\mathbf{q}|V_q|^2\frac{1}{q}\exp\left[-\frac{m\omega^2n^2}{2q^2T_e}-\frac{q^2}{8mT_e}\right]J_n^2(\rho)$$
$$\times\left[(e^{n\omega/2T_e}+e^{-n\omega/2T_e})\frac{q^2}{2M}(T_e-T_i)+T_i\frac{n\omega m}{M}(e^{n\omega/2T_e}-e^{-n\omega/2T_e})\right] . \quad (5)$$

We are reminded that for $n=0$, in accordance with the remark given after Eq. (4), only one exponent is assumed in the term proportional to (T_e-T_i) in Eq. (5). In the limit of weak electric fields one may retain only the term with $n=0$ in Eq. (4) and put $J_0(\rho)=1$ in Eq. (5). The integration in (5) then gives

$$\frac{dT_i}{dt}=n_eC_0(T_e-T_i)\ln\Lambda\frac{1}{v_e^3} ,$$
$$C_0=\frac{8\sqrt{2\pi}}{3mM}e^4Z^2 . \quad (6)$$

In deriving Eq. (6) we took into account the relation $E_i=3N_iT_i/2$ and introduced the notations $v_e=\sqrt{T_e/m}$ and $\Lambda=mv_e^2/\omega_p$, where the ω_p is the electron plasma frequency. The quantity ω_p appears in the expression only for $n=0$, as the result of the cutoff due to screening effects. ω_p does not enter results for $n\neq 0$ if the relation $n\omega>\omega_p$ is valid, and therefore screening effects are not important.

Let us briefly discuss the structure of Eq. (5). For $n=0$ only the first term in parentheses contributes and the energy transmission between electrons and ions would stop at $T_i=T_e$, provided the electric field is weak enough and, therefore, terms with $n>0$ are negligibly small. However, at higher-field intensities, photon absorption and emission processes take place ($n\neq 0$) and the energy exchange between electrons and ions does not stop even at $T_i=T_e$. The physical explanation of this effect is quite simple: electrons continuously absorb energy from the electromagnetic wave due to collisions and transmit, roughly speaking, the m/M fraction of the absorbed energy to ions.

Equation (5) can be considerably simplified in the limit of strong fields. Furthermore, it will be clear that in this case the sum over n in Eq. (4) is determined by large n. Therefore, the well-known conventional procedure [17] can be applied. The asymptotic expression for $J_n^2(\rho)$ at $\rho>n\gg 1$ should be used [18]:

$$J_n^2(\rho)=\frac{2}{\pi\rho}\cos^2\rho \quad (7)$$

and the cutoff of the integral in Eq. (5) at $|\mathbf{q}\cdot\mathbf{E}|/E=q_1=m\omega^2n/|eE|$ should be introduced, because asymptotic Eq. (7) is not valid in the region $|\mathbf{q}\cdot\mathbf{E}|/E<q_1$ and the latter contributes negligibly in the integral over $\mathbf{q}$. Because the argument ρ is large and $\cos\rho$ in Eq. (7) rapidly oscillates, we can put $\cos^2\rho=\frac{1}{2}$ when calculating the integral with respect to the directions of $\mathbf{q}$ in Eq. (5), keeping in mind the above cutoff. Instead of Eq. (5), we then obtain

$$E_i'(n)=\frac{N_i m^2 A_T}{(2\pi)^4}\int_{q_1}^{\infty}dq|V_q|^2 4\frac{\omega}{v_E}\ln\frac{qv_E}{\omega}\exp\left[-\frac{m\omega^2 n^2}{2q^2 T_e}-\frac{q^2}{8mT_e}\right]$$
$$\times\left[(e^{n\omega/2T_e}+e^{-n\omega/2T_e})\frac{q^2}{2M}(T_e-T_i)+T_i\frac{n\omega m}{M}(e^{n\omega/2T_e}-e^{-n\omega/2T_e})\right]. \quad (8)$$

Here, we introduced the electron quiver velocity $v_E=|eE|/m\omega$. In the following we assume the relation $mv_E^2 \gg mv_e^2, \omega$.

Let us consider the integral (7) without the cutoff at $q=q_1$. If $n\omega \ll T_e$ then the main contribution is given by $q<q_2=mn\omega/\sqrt{mT_e}$. Thus, we have $q_2 \gg q_1$. If $n\omega \gg T_e$ then the main contribution is determined by the region of q in the vicinity of $q_3 \sim \sqrt{nm\omega}$ with the width $\sim\sqrt{mT_e} \ll q_3$. Provided $n \ll mv_E^2/\omega$ we also have $q_3 \gg q_1$. Therefore, we can set $q_1=0$ while calculating the integral (5) for $n<mv_E^2/\omega$. If $n>mv_E^2/\omega$, then $n\omega \gg T_e$, $q_3<q_1$, and the series in Eq. (4) is interrupted.

After this analysis, the integral in Eq. (7) can be easily evaluated:

$$E_i'(n)=\frac{3N_i n_e}{2\sqrt{2\pi}}\frac{C_0}{v_E v_e^2}\frac{1}{n}\left[(T_e-T_i)(1+e^{-(n\omega/T_e)})+T_i\left[1+\frac{2T_e}{n\omega}\right](1-e^{-(n\omega/T_e)})\right]\times\ln\frac{\tilde{q}v_E}{n\omega}. \quad (9)$$

Here, $\tilde{q}\sim q_2$ if $n\omega<T_e$ and $\tilde{q}\sim q_3$ if $n\omega>T_e$.

It is clear now that the sum E_i' in Eq. (4) is determined by large values of n. Thus, summing up of the series in Eqs. (4) and (9) can be done by replacing the sum by the integral (let us be reminded that the cutoff at $n\sim mv_E^2/\omega$ exists).

If $\omega \gg T_e$, we simply obtain, retaining only large logarithmic terms,

$$\frac{dT_i}{dt}=n_e\frac{C_0}{4\sqrt{2\pi}}\frac{T_e}{v_e^2 v_E}\left[\ln\frac{mv_E^2}{\omega}\right]^2. \quad (10)$$

If $\omega \ll T_e$ we separate the region of the integration over n into two domains $(1, T_e/\omega)$ and $(T_e/\omega, mv_E^2/\omega)$. Then taking into account that large numbers n contribute into the sum over both domains we can expand exponential terms in Eq. (8) in the first domain and neglect them in the second one. The integration with retaining only large logarithmic terms results in

$$\frac{dT_i}{dt}=n_e\frac{C_0}{\sqrt{2\pi}}\frac{T_e}{v_e^2 v_E}\left[\frac{1}{4}\left[\ln\frac{mv_E^2}{T_e}\right]^2+\ln\frac{mv_E^2}{T_e}\ln\frac{T_e}{\omega}\right]. \quad (11)$$

III. DISCUSSION

Formulas (10) and (11) are represented in the form simplifying comparison with the low-field limit (6). The main distinctions are (1) the appearance of the small factor v_e/v_E and (2) the substantial modification of the logarithmic factor—a double logarithmic factor appeared and the arguments under the logarithm sign changed.

It is seen that the energy exchange rate is strongly suppressed by an external high-frequency field. At high intensities, it does not depend on the temperature T_e, provided $\omega \gg T_e$ [Eq. (10)], and weakly depends on the latter, if $T_e \gg \omega$ [Eq. (11)]. A substantial distinction between Eqs. (10) and (11) is due to the different role of light emission and absorption processes in e-i collisions. At $\omega \gg T_e$ the stimulated emission of radiation is evidently strongly suppressed, the thermal motion has no significance and the characteristic momentum transfer $\tilde{q}$ is determined by the absorbed energy $\tilde{q}\sim\sqrt{n\omega m}$. If $\omega \ll T_e$, then absorption and emission processes are equally significant. The absorption or the emission n quanta with $n<T_e/\omega$ does not change the electron energy considerably and the characteristic momentum transfer is $\tilde{q}\sim n\omega/v_e$. For $n>T_e/\omega$, the influence of the electromagnetic field is like that of $\omega \gg T_e$, which has already been described. From either Eq. (10) or Eq. (11), it is seen again that at $T_e=T_i$ ion heating does not stop (T_i simply drops out).

For practical aims the case $T_e \gg \omega, T_i$ is the most important one and the following interpolation formula is suggested (here we have returned to Gaussian units and introduced the Planck constant $\hbar$ and the Boltzmann constant κ):

$$\frac{dT_i}{dt}=\frac{8n_e e^4 Z^2(v_E^2+v_e^2)}{3M(v_E^2+v_e^2)^{3/2}}\left[\frac{1}{4}\ln^2\left[1+\frac{mv_E^2}{\kappa T_e}\right]+\ln\left[\frac{mv_E^2}{\kappa T_e}+\exp(\sqrt{2\pi})\right]\times\ln\left[\frac{\kappa T_e(v_E^2+v_e^2)}{\hbar\omega v_E^2+\hbar\omega_p v_e^2}\right]\right]. \quad (12)$$

It is seen from formula (12) as well as from the direct comparison of Eq. (6) with Eqs. (10), (11) that the effect of an intense external field is essentially a substitution of the electron thermal velocity v_e by its quiver velocity v_E. Of course, the numerical factors as well as more complicated logarithmic terms cannot be deduced from such a qualitative argument alone.

In conclusion, let us briefly discuss the implication of the above theory for the less complicated case of the electron heating in an intense laser field. Using the above technique, one can readily obtain the following expression for the variation rate of the electron subsystem energy E_e':

$$E_e'=\sum_{n=1}^{\infty} E_e'(n) ,$$
$$E_e'(n)=4N_e n_i e^4 Z^2 \frac{1}{mv_E}(1-e^{-n\omega/T_e})\left[1+\frac{2T_e}{n\omega}\right]\times\ln\frac{\tilde{q}vE}{n\omega} . \quad (13)$$

This expression is valid for $mv_E^2 \gg \omega, T_e$, $\tilde{q}$ has the same meaning as in Eq. (9). Let us compare Eq. (13) and Eq. (9) in more detail. Since the ion temperature does not enter the final result [see Eqs. (10) and (11)], only the term proportional to T_e is important in Eq. (9). It is clear that multiphoton emission and absorption processes play considerably different roles in the electron and ion heating. In the former, they are competitive, but in the latter, both contribute to the ion heating. Nevertheless, due to the accidental cancellation of numerical factors, the final results for the rate of temperature variation appear to be quite similar. Using the approach of the preceding section we immediately get

$$\frac{dT_e}{dt}=\frac{8n_i e^4 Z^2}{3mv_E}\ln\Lambda_1 ,$$
$$\ln\Lambda_1=\begin{cases}\frac{1}{4}\left[\ln\frac{mv_E^2}{\omega}\right]^2, & \omega\gg T_e\\ \frac{1}{4}\left[\ln\frac{mv_E^2}{T_e}\right]^2+\ln\frac{mv_E^2}{T_e}\ln\frac{T_e}{\omega}, & \omega\ll T_e .\end{cases} \quad (14)$$

For the sake of completeness the following interpolation formula, the counterpart of formula (12) (again written in Gaussian units) is suggested:

$$\frac{dT_e}{dt}=\frac{8n_i e^4 Z^2 v_E^2}{3m(v_E^2+v_e^2)^{3/2}}\left[\frac{1}{4}\ln^2\left[1+\frac{mv_E^2}{\kappa T_e}\right]+\ln\left[\frac{mv_E^2}{\kappa T_e}+\exp(\tfrac{1}{3}\sqrt{\pi/2})\right]\times\ln\left[\frac{\kappa T_e}{\hbar\omega}\right]\right] . \quad (15)$$

The interpolation formulas (12) and (15) describe the entire region from zero laser intensity to extremely high intensities below relativistic threshold, provided $T_e \gg \hbar\omega$. Although both the analogy and distinction between Eq. (12) and Eq. (15) are clear after attentive comparison, let us emphasize some points. The formulas are much more similar at high-field intensities rather than in the low-field limit. In the latter case, dT_i/dt does not depend on the field intensity, but dT_e/dt is obviously proportional to E^2. Besides, Coulomb logarithms (Gaunt factors) are considerably different: $\ln(T_e/\omega_p)$ for ions, and $\ln(T_e/\omega)$ for electrons, although the inequality $\omega\gg\omega_p$ was assumed in the present paper. The physics of this distinction is clear, since a weak laser field does not affect the electron-ion energy transfer at all [summation in (4) starts from $n=0$], but it does course the electron heating.

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