# Minimum energy dissipation model for river basin geometry

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A simple minimum energy dissipation model has been used to simulate the distribution of rivers and river basins in a region with a square boundary. The cumulative distribution of basin areas $N(A > A^*)$ was found to have a power law form $N(A > A^*) \sim A^{*-(\tau-1)}$ with an exponent $(\tau-1)$ that is close to the value of $\frac{1}{2}$ obtained from a simple scaling theory. The boundaries of the river basins were found to be self-similar with a fractal dimension of about 1.10. In many respects (distribution of stream internal link lengths, distribution of energy dissipation, basin boundary geometry, etc.) the results obtained from this model are similar to data obtained from natural channel networks. In other respects significant differences remain. In particular, the main channels of the individual rivers are much straighter than those of real rivers.

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## I. INTRODUCTION

River networks have been of interest to both geologist and physicist for many years. River networks exhibit a rich scaling structure, and the effects of this scaling structure on the behavior (hydrological response) are of considerable practical importance both in terms of present day phenomena (flooding and drought) and past events (the formation of oil reservoirs, for example). Rivers have frequently been cited as familiar examples of natural fractal structures [1], but the nature of the scaling geometry and the manner in which it is formed and evolves remain poorly understood. A variety of models has been studied including simple statistical models [2–8] and erosion models [9,10]. Rinaldo *et al.* [11] have proposed a model to describe the structure of river networks based on a minimum energy dissipation principle. Given the position of an outlet and the outer boundary of a basin, the structure of the minimum energy dissipation river network that drains the basin is obtained. It was demonstrated by Rinaldo *et al.* [11] that the structure of the river networks obtained from this model is similar in many respects to that of natural river networks obtained from digital elevation maps. For example, the structural characteristics of the minimum energy dissipation drainage network, such as Horton's law of stream lengths, the stream bifurcation ratio, and the multifractal spectrum of the width function, were found to be similar to those of natural drainage networks. The model presented here is based on that of Rinaldo *et al.* We demonstrate that as a consequence of the minimum energy dissipation principle, the drainage area self-organizes into a spatial structure, which has a power law distribution of drainage basin areas, self-similar fractal basin boundaries, and a self-similar network structure.

The idea that patterns formed by natural process are organized to minimize energy consumption is not new. A number of examples were collected by Stevens [12] in order to illustrate this idea. Murray [13] derived a relationship between the radius of a parent blood vessel and the radii of its daughter vessels by minimizing the power needed to maintain the blood flow. In this case the total power is the sum of the power required to overcome friction as described by Poiseuille's law and the metabolic energy involved in the maintenance of the blood volume and vessel tissue. Arterial branching and junctions have also been studied in the same context by Zamir and Bigelow [14,15] and Woldenberg and Horsfield [16]. Based on equilibrium thermodynamic considerations, Leopold and Langbein [3] studied landscape evolution by maximizing the "entropy" of the landscape. Yang [17] and Howard [18] also used the concept of minimum work in their studies of the development of stream networks. Recent studies indicate that the spatial distribution of earthquakes also tends to minimize the stress potential on the earth [19].

The work of Rinaldo *et al.* [11] on optimized river channel networks is based on three assumptions that were postulated by Rodriguez-Iturbe *et al.* [20]. These three postulates are the following:

(i) Equal energy dissipation per unit area of channel anywhere in the network.
(ii) Minimum energy expenditure in any link of the river network.
(iii) Minimum total energy dissipation in the whole network.

The first postulate is equivalent to the statement that the velocity of water flow is constant throughout the network [20], which was confirmed by field data [21]. The second postulate explains [20] other relationships observed in nature [22], that is, the depth $d$ and the width $w$ of channels scale with flow $Q$ as $d \sim Q^{0.5}$ and $w \sim Q^{0.5}$. Following Rodriguez-Iturbe *et al.* [20], it can be shown from the first and second postulates that the optimized energy dissipation at any link is proportional to the product of the square root of the flow in that link and the length of that

 

link. If $P_i$ is the energy dissipation in link $i$, $Q_i$ is the mean annual flow in link $i$, and $L_i$ is the link length, then $P_i = \kappa Q_i^{0.5} L_i$. The third postulate requires that the total energy dissipation of the whole network $\sum_i P_i = \sum_i \kappa Q_i^{0.5} L_i$ be the smallest possible. This postulate is a global optimum requirement for the whole drainage channel network.

In the model of Rinaldo *et al.* [11], the boundary of the lattice used in the simulation was also the boundary of a single river network. The strong dependence of the river network's structure on its boundary has been shown by Rigon *et al.* [23]. In this work, the third postulate is extended from minimum energy dissipation of the whole river network in a single drainage basin to minimum energy dissipation of all river basins that cover a drainage area. Thus the boundary of each basin is determined in a natural way by the competition and cooperation between the basins that share common boundaries. In the model described here the lattice boundary is used as the boundary for the drainage area and the rivers are allowed to exit at any position on this boundary. The model might be further improved by using a more realistic, possibly fractal, boundary for the entire system.

A river basin $B_j$ is defined as the area drained by a single river network that leads to a single outlet on the boundary of the area $A$. Suppose $A$ is covered by the set of basin $B_j$, $j = 1,2,3,\ldots,n$. The model requires that $\sum_j(\sum_i P_i)$ have a minimum value, where the index $i$ sums over all the links that belong to the basin $B_j$ and the index $j$ sums from 1 to $n$. For any particular partition of the area $A$ into $n$ river basins (defined by the basin boundaries) a minimum value for $\sum_j(\sum_i P_i)$ also implies that $\sum_i P_i$ in each basin should be the minimum. This is exactly the case in the model of Rinaldo *et al.* With a known boundary, the structure of the river network is determined by minimizing the energy dissipation $\sum_i P_i$ inside the basin defined by the given boundary.

In other words, in the model of Rinaldo *et al.*, $n = 1$ and $B_1 = A$, while in the model presented here, the partition into $n$ basins is also a result of the optimized (minimum) energy dissipation for the whole system. The boundary of each basin $B_j$ is obtained by requiring that the structure of all the river basins draining the area $A$ and the structure of each channel network inside each basin must be organized in such a way that $\sum_j B_j = A$ and $\sum_j(\sum_i P_i)$ has the smallest possible value.

## II. COMPUTER MODELS

In the computer simulation a triangular lattice (grid) is superimposed on the area $A$. The size of the grid is $256 \times 256$. An open boundary condition has been used. Each site on the boundary of the lattice has a possibility of being a channel network outlet. The lattice can be regarded as a large island or continent surrounded by the sea and the river networks on the island transport the precipitation from the island to the sea. It is assumed that for any site $i$ on the lattice, the mean annual flow on that site $Q_i$ can be expressed as $Q_i = \sum_{i'} Q_{i'} + 1$. Here $i'$ is a nearest neighbor site of $i$ from which there is a flow $Q_{i'}$ into site $i$. The addition of a unit flow to $\sum_{i'} Q_{i'}$ represents the precipitation on the $i$th site. The precipitation is assumed to be uniform over the entire area $A$. We assume no evaporation and no subsurface transport.

The optimization method used in this study is simulated annealing [24]. It was first introduced by Kirkpatrick, Gelatt, and Vecchi in 1983. If $f(\mathbf{x})$ is the energy dissipation function of the system and $\mathbf{x}$ is a vector in the configuration space $X$ of that system, it can be optimized by assigning a probability [25] $p$ for changing from one configuration $\mathbf{x}$ to a randomly selected "neighbor" configuration $\mathbf{x}'$. The probability is 1 when $f(\mathbf{x}') < f(\mathbf{x})$ and $p = \exp\{-[f(\mathbf{x}') - f(\mathbf{x})]/T\}$ when $f(\mathbf{x}') > f(\mathbf{x})$. $T$ is a parameter that is controlled during the optimization procedure. In some applications $T$ can be thought of as the temperature during the process of simulated annealing. For the river network model $f(\mathbf{x}) = \sum_j(\sum_i P_i)$ where the energy dissipation $P_i$ is summed over all the sites that belong to the basin $B_j$, the index $j$ sums from 1 to $n$ where $n$ is the total number of river basins draining the area $A$. A neighbor configuration $\mathbf{x}'$ of $\mathbf{x}$ is defined as a configuration that differs from $\mathbf{x}$ in the direction of flow in only a single link on the lattice, with the constraint that loops are not allowed in the channel network.

The algorithm can be summarized as follows:

(i) An initial configuration is generated by performing a branched Eden growth [26] simulation from the whole boundary of the lattice until the lattice is filled. This generates a loopless initial configuration that covers the entire lattice and in which all the "rivers" drain to the border of the lattice.

(ii) The flow on each site is calculated according to the formula $Q_i = \sum_{i'} Q_{i'} + 1$, where $i'$ is a neighbor site of $i$ from which there is a flow $Q_{i'}$ into site $i$. Thus the total energy dissipation $P$ of $A$ is obtained by calculating $\sum_i P_i$, where the index $i$ sums over all the sites on the lattice.

(iii) One site in the lattice is randomly selected and a change in the direction of the flow from that site is also randomly selected.

(iv) The new configuration is checked to see if it contains any loops in the channel network. If it does, the new configuration is rejected and step (iii) is repeated.

(v) The flow on each site is recalculated for the new "proposed" configuration, and the total energy dissipation $P'$ of that configuration is evaluated.

(vi) If $P' < P$, then the new configuration is accepted. If $P' > P$, the probability $p = \exp[-(P' - P)/T]$ is calculated. A random number uniformly distributed over the range 0–1 is generated and compared with $p$. If this random number is less than $p$, the new configuration is accepted. Otherwise, the system is returned to the "old" configuration.

(vii) Steps (iii)–(vi) are repeated many times and the temperature $T$ is lowered gradually, according a "cooling schedule."

The cooling schedule used here is similar to the exponential temperature reduction schedule suggested by Kirkpatrick, Gelatt, and Vecchi [24]. The temperature $T(n)$ during the $n$th stage is related to the initial temperature $T(0)$ by $T(n) = \alpha^n T(0)$ with $0 < \alpha < 1$. Each stage contains $\Lambda$ steps, during which an attempt is made to change the network configurations. The temperature only changes after each stage ($\Lambda$ steps) has been complet-

ed. In the simulations presented here, $T(0)$ is set to 12 800, $\alpha$ is chosen as 0.982, and $\Lambda$ is set to 65 536 ($256\times256$). In the simulations the annealing process was stopped when $T(n)$ became less than 0.0004. Each run of the simulation takes approximately 50 h computational time (user time) on a Hewlett Packard 700 computer workstation. The simulated annealing procedure has been repeated more than ten times with different initial configurations. Here we report the result obtained from the run that gave the smallest energy dissipation. The results obtained from the other runs were indistinguishable from those reported for all of the statistical properties that were measured.

The total energy dissipation is monitored during the simulated annealing procedure. In Fig. 1(a) the total energy dissipation in the whole area is plotted as a function of the number of "annealing" stages. Figure 1(b) shows the dependence of the total energy dissipation on the temperature $T$. The small window in Fig. 1(b) shows, in more detail, the dependence of the total energy dissipation on $T$ at the lower temperatures. It can be seen from the figure that the total energy dissipation $P$ drops sharply when the temperature changes from about $10^{1}$ to about $10^{-1}$.

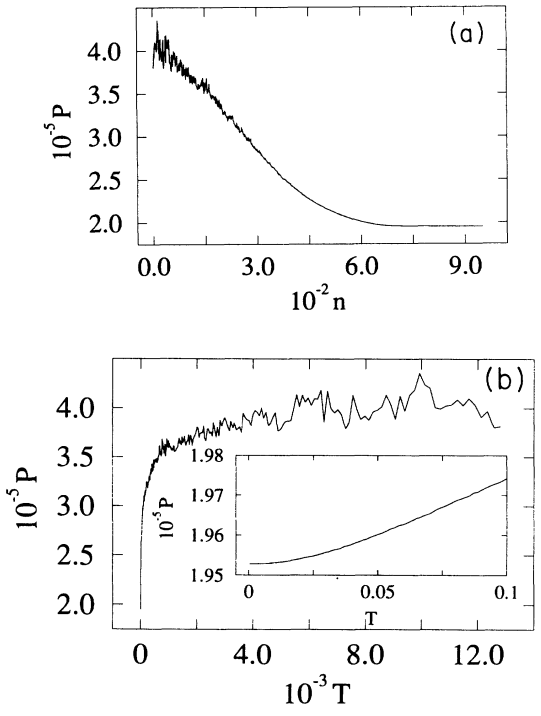


FIG. 1. (a) Total energy dissipation $P$ as a function of the number of "annealing" stages $n$. In each stage, there are 65 536 steps, during which an attempt is made to change the network configurations. The "temperature" was kept unchanged during each stage but was changed between successive stages. (b) Changes of $P$ with the "temperature" $T$. The units of $T$ are the same as the units of the total energy dissipation. The "temperature" was decreased using an exponential temperature reduction schedule. The "temperature" $T(n)$ during the $n$th stage is equal to $\alpha^{n}T(0)$, where $T(0)$ is the initial "temperature." Here $T(0)=12\,800$ and $\alpha=0.982$.

## III. RESULTS

The networks generated by this algorithm are displayed in Figs. 2(a) and 2(b). Figure 2(a) shows the structures of the minimum energy dissipation river networks that cover the area $A$. Since the total energy dissipation of all the rivers is a minimum, each river network

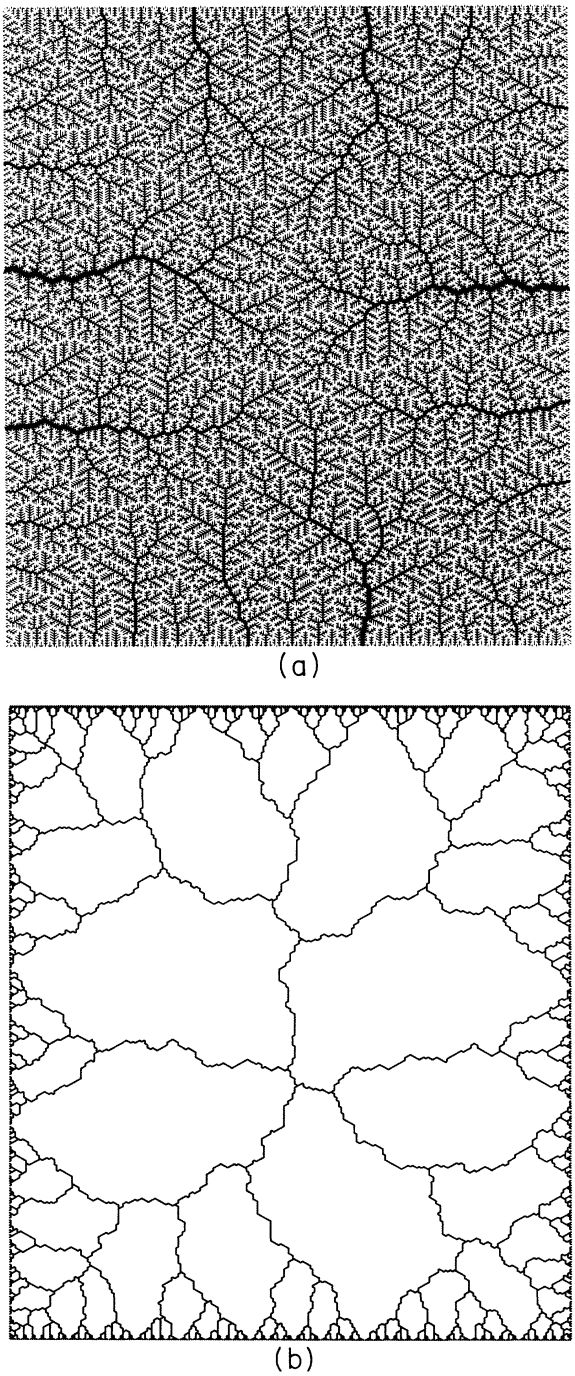


FIG. 2. (a) Structures of the minimum energy dissipation drainage networks that cover the area $A$. The thickness of each segment in the figure is proportional to the square root of the amount of flow it carries. The arrows in the figure (not readily visible on the scale of this figure) indicate the direction of the flow on the sites. (b) Boundaries of all the drainage basins that cover the drainage area $A$.

displayed here also has an optimal structure within its own boundary. The boundary of each river basin is shown in Fig. 2(b). It can be seen from this figure that the optimal drainage structure of an area tends to have a few large river networks together with a distribution of smaller networks covering the whole area. Figure 3 shows the dependence of $\ln[N(A > A^*)]$ on $\ln(A^*)$, the cumulative size distribution of the basins covering the whole drainage area. This figure indicates that the river basin areas follow a power law distribution $N(A) \sim A^{-\tau}$ with $\tau \approx 1+0.51=1.51$. Meakin, Feder, and Jøssang [8] have shown that one version of Leopold and Langbein's simple statistic river network model [3] in which the river network is represented as a tree of coalescing self-avoiding random walks has a similar power law distribution of river basin areas with $\tau \approx 1.39$. Recent studies by Inaoka and Takayasu [10] using an erosion model give a cumulative distribution of the drainage-basin area with the form $P(\geq A) \sim A^{-0.42}$, which means that the distribution of the drainage-basin areas can be represented by the power law $P \sim A^{-1.42}$. However, we are not aware of field measurements of drainage-basin area distributions that can be compared with the simulations.

The length of a basin can be defined as the longest distance from any point on the boundary of that basin to its outlet. If the basins' shapes are similar to each other, it can be anticipated that the basin length $l$ and the area of the basin are related by $A(l) \sim l^2$, since the networks fill the whole drainage area. Figure 4 shows the dependence of the logarithm of the drainage-basin area on the logarithm of its length. A straight line with a slope of 2 has been drawn in the figure to compare with the data. Except for the basins with a very small drainage area, most of the data are represented quite well by the straight line. The deviation for small basins may be caused by finite size effects.

The dependence of the drainage-basin area on the drainage length indicates that the drainage basins are not fractal. However, recent field observations by Breyer and Snow [27] on 12 river basins whose areas range from 150

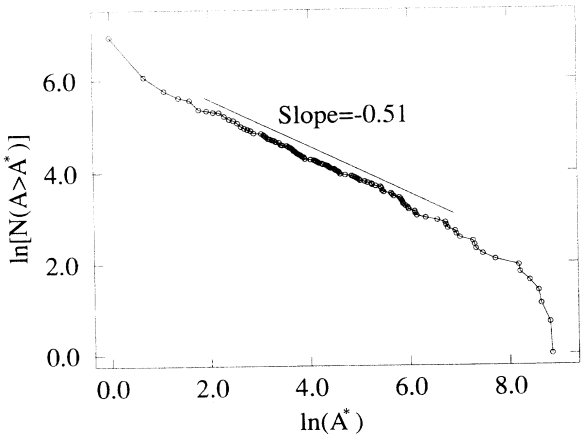


FIG. 3. Log-log plot of the cumulative distribution of drainage basin areas $N(A > A^*)$. $N(A > A^*)$ is the number of the basins that have areas larger than $A^*$. As indicated on the figure, this data set can be fitted to a straight line with slope of about $-0.51$.

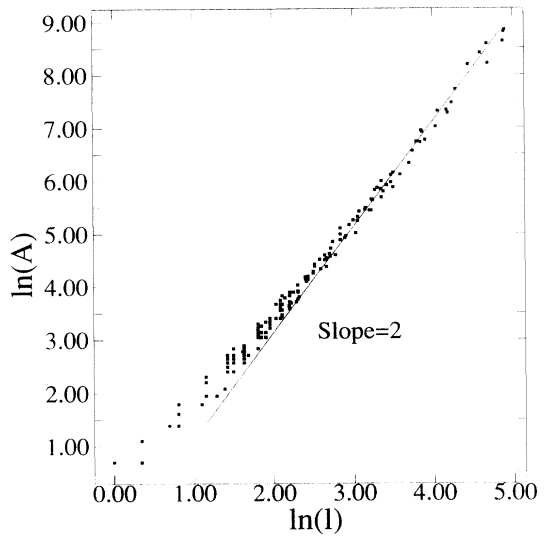


FIG. 4. Dependence of the area $A$ of a drainage basin on its length $l$. A straight line of slope 2 was used to fit the data.

to 50 000 km$^2$, in the United States, indicate that the *boundaries* of river basins are fractal with dimensions between 1.06 and 1.12. In Fig. 5 the dependence of drainage-basin perimeters on their lengths is shown. It indicates that the perimeters of minimum energy dissipation drainage basins have an effective fractal dimension of about 1.10. A least squares fit to the data gave a value of $1.093 \pm 0.008$ for $D$. The true uncertainty, due to finite size and other effects, is almost certainly larger than the 95% confidence range given here. This value for $D$ agrees well with field observations. The occurrence of fractal boundaries for the optimal drainage basins, as a result of the area filling requirement and optimization of their energy dissipations, suggests that fractal structures may be a natural consequence of the requirement of least energy dissipation in natural systems.

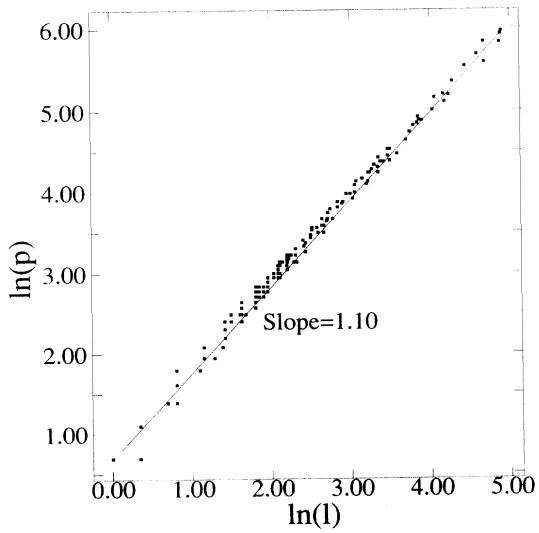


FIG. 5. Relationship between the perimeter length ($p$) and the basin length ($l$). The data were fitted by a straight line with slope of 1.10.

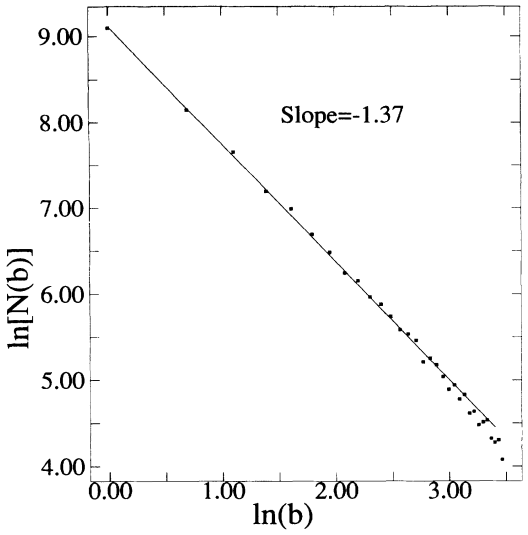


FIG. 6. Result of a box counting analysis of the boundaries of a minimum energy dissipation drainage network. $N(b)$ is the number of the boxes with sides of length $b$ that cover the whole pattern. The line used to fit the data has a slope of $-1.37$.

The pattern consisting of all the boundaries of river basins that drain the area $A$ is also fractal. The box counting method was used to measure the fractal dimensions of this pattern. Figure 6 shows a log-log plot of the dependence of the number of boxes needed to cover the whole pattern on the size of the box. The result can be fitted by a straight line with a slope of 1.37. So the pattern has an effective fractal dimension of 1.37.

The main stream is defined as the longest nonreentrant path in a drainage basin. Its length $L$ is measured along the stream. The "Pythagorean" length $\lambda$ is the distance between the starting point of a stream and its outlet measured in the two-dimensional embedding space.

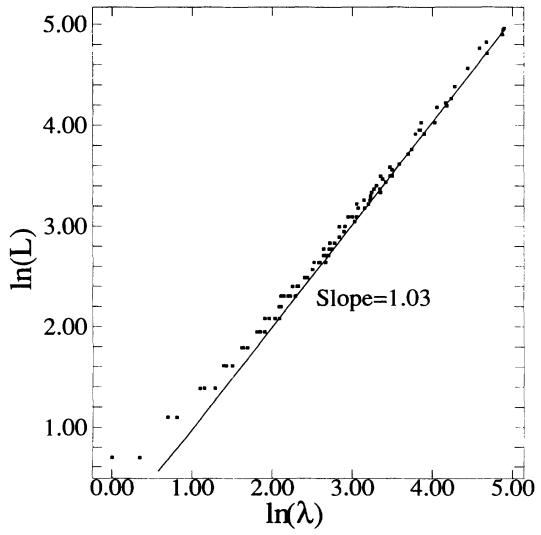


FIG. 7. Dependence of the main stream length $L$ on the Pythagorean distance $\lambda$ between the starting point of the main stream and its outlet. The straight line used to fit the data has a slope of 1.03.

The dependence of the main stream length $L$ on $\lambda$ is shown in Fig. 7. It can be seen from the figure that $L$ depends almost linearly on $\lambda$ (the effective fractal dimension of the main stream is about 1.03). Field data collected by Marani, Rigon, and Rinaldo [28] indicate that the fractal dimension of the main stream for real rivers ranges from 1.1 to 1.3. This indicates that the main stream in optimal river networks is much straighter than main streams in the real world.

Power law distributions of energy dissipation are not rare in nature [29]. It has been shown by Mandelbrot [30] that uniform energy input to a dissipative system often results in a power law distribution of energy dissi-

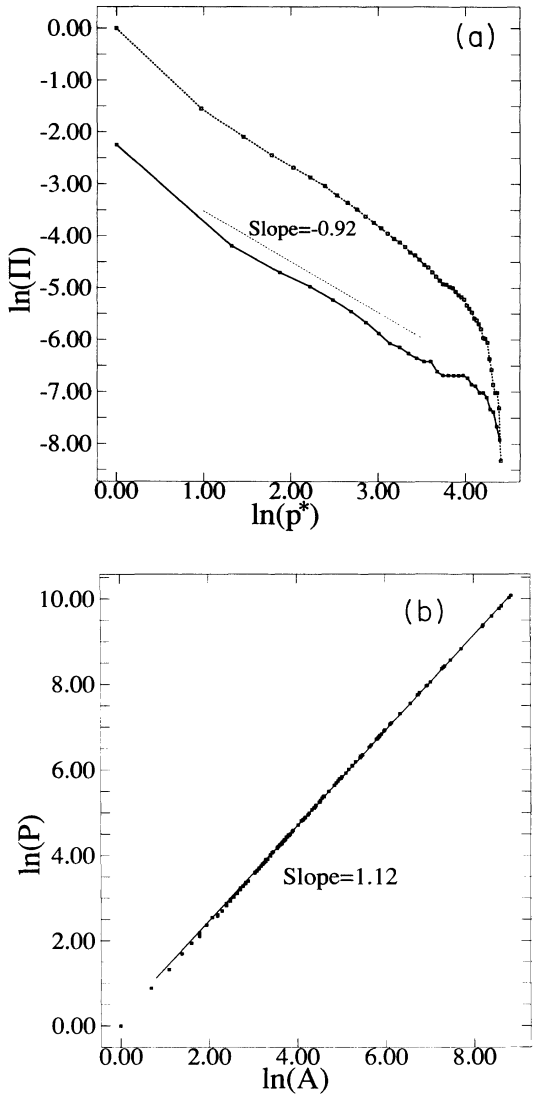


FIG. 8. (a) Cumulative probability distributions $\Pi(p > p^*)$ of energy dissipation $p$ per unit channel link length in the biggest river (indicated by the filled squares in the figure) and in the whole drainage area (indicated by the empty squares in the figure) of the minimum energy dissipation drainage networks. The dashed line has a slope of $-0.92$. (b) Total energy dissipation $P$ in drainage basins of area $A$ is shown as a function of $A$ in the form of a log-log plot. The fitted straight line has a slope of 1.12.

pation in the space in which the system extends. Drainage networks that are embedded in a drainage area and subject to a uniform energy input from precipitation may also exhibit a power law distribution of its energy dissipation. This power law distribution of energy dissipation is shown in Fig. 8. Figure 8(a) displays the cumulative distributions of energy dissipation per unit channel link length in the largest drainage network and in the whole drainage area of an optimal drainage network. Both of the two curves can be fitted by straight lines with a slope of about $-0.92$. This indicates that the cumulative distribution $\Pi(p>p^*)$ has the form $\Pi(p>p^*)\sim p^{*-v}$, with $v$ being about 0.92. This power law distribution of energy dissipation is very close to that observed in natural drainage basins [29]. The exponent $v$ for energy dissipation in five drainage basins in North America [29] falls in a narrow range between 0.90 and 0.93. Figure 8(b) shows that the total energy dissipation

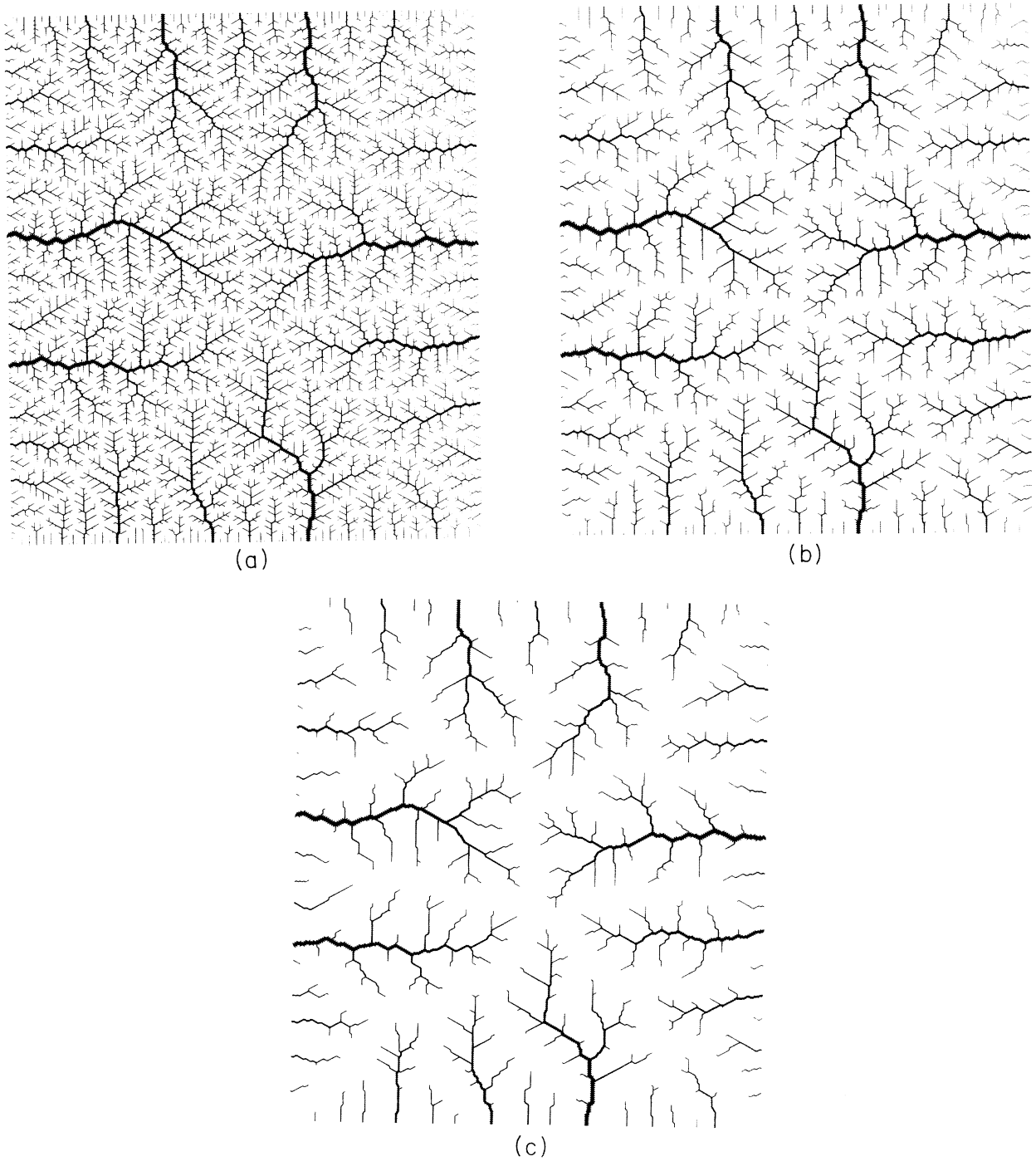


FIG. 9. Structure of an optimal channel network is shown with different flow thresholds. (a)–(c) show the river network with resolutions of 4, 16, and 64 pixels, respectively.

$P$ in a drainage basin of area $A$ is proportional to $A^{1.12}$. As has been shown above, the number of drainage basins that have a drainage area $A$ is proportional to $A^{-1.51}$, so the distribution of the total basins' energy dissipations in an entire drainage area is also a power law $[N(P)\sim P^{-\beta}]$ with an exponent $\beta$ of $-1.51/1.12=-1.35$.

The structure of the optimal channel networks in the drainage areas is self-similar. The definition of self-similarity of the channel network used here is that the internal link length distributions $p(l)$ of the channel network with different flow thresholds $t$ are invariant under the transform $l'_{\rm int}=l_{\rm int}/t^{0.5}$ and $p'=pt^{0.5}$, where $l_{\rm int}$ is the internal link length, $t$ is the threshold, and $p$ is the distribution of the lengths of the internal links with a threshold $t$ (only those links with $Q\geq t$ are included). The internal link length is the distance between two branch junctions in one channel. Figures 9(a), 9(b), and 9(c) display the structures of the channel networks with a threshold $t$ of 4, 16, and 64, respectively (only links with flows $Q\geq t$ are shown). Figure 10(a) shows the internal link length distribution for the drainage networks with thresholds $t$ of 2, 4, 8, 16, 32, 64, and 128. The internal link length distribution after the transformation is displayed in Fig. 10(b). Figure 10(b) shows that the probability $p(l,t)\delta l$ that a randomly selected link will have a length in the range $l$ to $l+\delta l$ when the threshold is $t$ can be represented by the scaling form

$$p(l,t)=t^{1/2}f(l/t^{1/2})\ . \tag{1}$$

## IV. DISCUSSION

The distribution of basin areas $N(A)$ obtained from simple statistical models [8] can be represented by the scaling form

$$N(A)=A^{-\tau}f(A/\mathcal{L}^{\gamma})\ , \tag{2}$$

where $\mathcal{L}$ is the lattice size (length scale in lattice units) and the scaling function $f(x)$ is constant for $x\ll 1$ and decays faster than any power of $x$ for $x\gg 1$. The requirement that this distribution of river basins must fill an area $\mathcal{L}^2$ leads to the exponent scaling relationship [8]

$$(2-\tau)\gamma=1\ . \tag{3}$$

In view of the power law distribution of basin areas shown in Fig. 3 it is apparent that the scaling form given in Eq. (2) should describe the distributions of basin areas in the minimum energy dissipation model as well.

If the "length" $l_b$ and "width" $w_b$ (measured perpendicular and parallel to the edges of the lattice) of the river basin's scale in the same way with the basin area,

$$l_b\sim A^{1/2}\ , \tag{4a}$$

$$w_b\sim A^{1/2} \tag{4b}$$

(see Fig. 4), then the exponent $\gamma$ in Eq. (2) will have a value of 2 so that the size distribution's exponent will have a value of $\frac{3}{2}$ [from Eq. (3)]. This is very close to the value of about 1.51 obtained from the computer simulations, and it appears that the size distribution exponent $\tau$ has a value of exactly $\frac{3}{2}$.

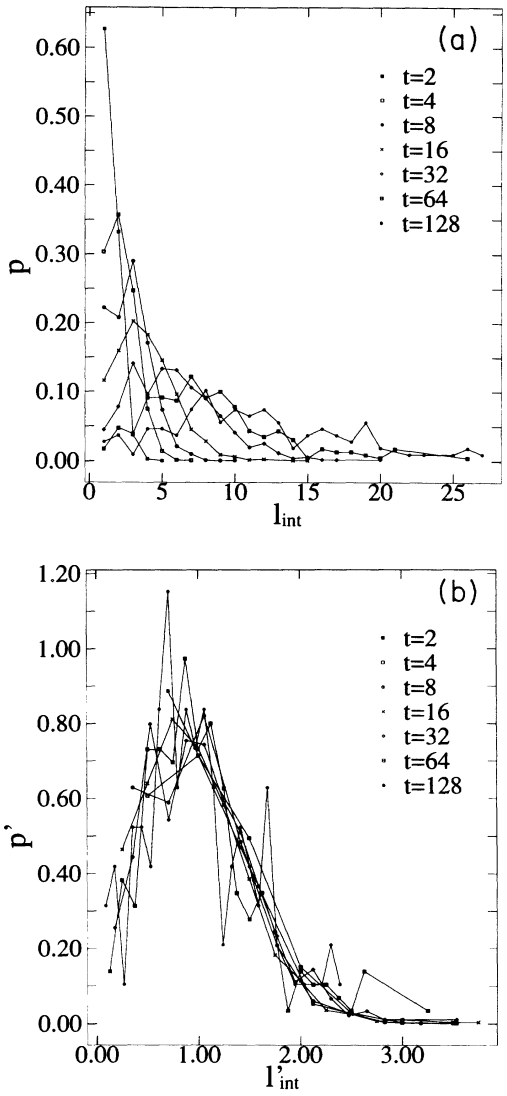


FIG. 10. Internal link length distributions for the drainage networks with different flow thresholds ($t$). (a) $p$ is the probability of finding an internal link of length $l_{\rm int}$. (b) Internal link length distribution after the transformation $l'_{\rm int}=l_{\rm int}/t^{0.5}$ and $p'=pt^{0.5}$. The thin lines that connect the points in the figures are to guide the eye.

In practice it is difficult to obtain accurate values for exponents such as $\tau$ from practical scale simulations. Consequently the very good agreement between the measured value for $\tau$ and the value of $\frac{3}{2}$ obtained from our simple scaling arguments should be regarded as fortuitous.

The simulated annealing approach has been successfully applied to a broad range of problems in areas such as statistical physics, engineering, and image analysis. However, it is not suitable for all optimization problems. The performance of this approach depends on the shape of the surface $Z(\Omega)$, where $Z$ is the quantity that is to be optimized (the energy dissipation, in our case) and $\Omega$ indicates the coordinates of the configuration space. In favorable cases the optimum configuration $\Omega_m$ is accessible from any starting configuration $\Omega_0$ via a path on

$Z(\Omega)$ that either descends continuously or requires only small barriers to be crossed. Very often the system becomes "trapped" in a deep local minimum that differs substantially from the global minimum $\Omega_m$. In our case we find that the simulated annealing procedure leads to a final ($T=0$) configuration that depends on $\Omega_0$, on a detailed level. However, all of the final configurations are statistically equivalent and appear to be equally good representations of the optimization network structure. This may reflect a degeneracy in the global minimum. More probably there are many configurations with energy dissipations close to the minimum value. We have tried a variety of initial configurations, $\Omega_0$. In particular, an initial configuration consisting of two interpenetrating "combs" (all sites with odd $x$ coordinates drain to the $y=0$ boundary and all sites with even $x$ coordinates drain to the $y=\mathcal{L}$ boundary) has been used in some of our simulations. The geometry and size distributions in this initial configuration differ drastically from those of the optimized system. However, the statistical properties that we have measured for the final configuration $\Omega_m$ are indistinguishable from those found using the Eden model starting configuration that has statistical properties that are much more similar to those of $\Omega_m$.

In many respects the drainage basins created by this purely artificial model are similar to real drainage basins in nature. The power law distributions of the optimal basin sizes, basin energy dissipations, and the energy dissipation of a link with unit length in each single drainage channel network and in the whole drainage area are similar for both real rivers and the rivers generated by this model. The boundary between each basin in the drainage area has a fractal dimension. The structure of all these drainage networks is self-similar. The connection between power law distributions, fractal structure, and self-organized criticality has been discussed by Bak, Chen, and Tang [31]. Takayasu and Inaoka [9] have shown that the channel networks developed by the erosion model display spatial self-organized criticality. A connection between optimal channel networks and self-organized criticality has also been argued recently by Rinaldo *et al.* [32]. However, it is not clear if natural channel networks and drainage basins are really optimized or if they are examples of self-organized criticality. Even if natural channel networks and drainage basins are indeed optimized, the question of how natural processes, which are far from equilibrium, lead to optimal structure also remains open. One possible answer is that in nature the Earth's topography may be regarded as the result of a process that evolves towards a state with statistically stationary properties. Such stationary states have much in common with equilibrium states. During the evolution of natural river networks, tectonic uplift and erosion processes may reach a balance. The parts of a river network that are not near optimal structures will be eroded more rapidly than those parts that are already near an optimal structure. The effects of the changes in the local structure will be propagated through the drainage network. The time scale for propagation of a perturbation may be much shorter than that for evolution of the whole drainage-basin structure.


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