Robustness of vortex streets

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We study the robustness of the vortex street behind a circular cylinder when there is transfer of momentum between the wake and the surrounding fluid. The model used to represent exchanges of momentum is based on Lévy walks, as implemented in lattice-gas hydrodynamics. We show that the transverse velocity, considered as a function of the maximum length of exchanges, is a good order parameter for describing how the coherence of the vortex street is affected by transport of fluid in and out of it.

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I. INTRODUCTION

The two-dimensional Boolean lattice-gas model of hydrodynamics has been applied successfully to flow around a circular cylinder into the region of Reynolds numbers above 100 [1]. The development of the cylinder wake and the velocity field in the von Kármán vortex street are reproduced with all the right qualitative features and also quantitatively [2].

Concerning the vortex street, one of the interesting issues, which is of practical importance as well [3], is the effect of free-stream turbulence on its robustness. One can say that there are two ways in which turbulence in the incoming stream can affect the vortex street behind the cylinder: one is through interference with the cylinder boundary layer and the other is through momentum transport into and out of the street from and to the surrounding fluid. The first effect is connected with what is called the drag crisis. It is the second issue we are considering in this work.

In order to implement momentum transport we use a Lévy walk algorithm, previously applied to turbulent channel flow [4]. What we call Lévy walk is only a caricature of what was proposed for fully developed turbulence in Ref. [5]. What remains of it is the following: the distances over which particle configurations, and thus momentum, are exchanged are drawn from a long-tail algebraic probability distribution [4], and also there is a waiting time associated with each jump of a given length. The implementation is simplistic in the sense that the maximum distance over which exchanges take place is limited by system size, and in the following will actually be considered a variable.

We do not claim to have a reliable model for predicting, for instance, how pressure distribution around the cylinder (the favorite experimental quantity measured) changes with free-stream turbulence at high Reynolds number. We propose to use a model—the Lévy walk model—believable from our work on turbulent channel flow [4], in order to discern which quantities one might measure in order to gain insight into the robustness of the vortex street, when there is turbulence leading to the rearrangement of the momentum transported by the vortices.

The results reported here correspond to a mean flow Reynolds number Re=100, with a characteristic velocity given by the maximum of the incoming parabolic velocity profile and a characteristic length equal to the cylinder diameter. At this Reynolds number the vortex street is well established. The two-dimensional model of lattice-gas hydrodynamics used is the Boolean model without rest particles [6], with two-particle, four-particle, and three-particle symmetric and asymmetric momentum conserving collisions. The lattice is hexagonal. The implementation on the computer involves the technique of multispin coding, which is described in Ref. [7]. Given an incoming average velocity field and kinematic viscosity, system size is determined by both the Reynolds number and the obstruction ratio, which is the ratio of cylinder diameter to channel width. The obstruction ratio we choose to work at has the value of 0.1. For a Reynolds number Re= 100, at which we will discuss our results, one then has $D = 288$ and $W = 2944$, where D and W denote, respectively, cylinder diameter and channel width, and the dimensions are given in the number of lattice sites. The length L of the channel itself is 3600. The boundary conditions transverse to the flow are no-slip. A velocity profile with value U is injected according to the method described in Ref. [1].

In Sec. II we will discuss how an order parameter emerges which describes how the vortex street reacts to momentum exchanges over larger and larger scales. This order parameter is the transverse velocity measured on the cylinder axis. Section III contains results on longitudinal velocity. Section IV starts with a remark about drag and gives a discussion of some experimental results.

One point should be made here: the exchanges of momenta we are dealing with take place only in a direction transverse to the flow. This is a situation somewhat different from the experimental one where turbulent flow, produced by a grid upstream, is characterized by a longitudinal scale, which is easily accessible in experiment. However, there is clearly also a transverse scale characterizing turbulence. At any rate, since the von Kármán street will be affected mainly by transport of momentum outside the wake into the surrounding fluid, we believe that our model captures the essential aspect of the situation we are interested in.

II. VORTEX STREET ORDER PARAMETER

The imposition of a Lévy walk onto the flow can be done in two ways: either laminar flow, i.e., the vortex street, is first established, before exchanges of momenta start taking place, or from the start momenta exchanges over many scales take place. We find that our results do not depend on which way we proceed.

What is the expectation?

There are two important scales. One is the diameter of the cylinder, which determines also the size of the von Kármán street. The other is the maximum distance $l_{\max}$ over which momenta exchanges will take place. One expects that when the scale of these exchanges is long enough as compared to the cylinder diameter, the coherence of the vortex street will be affected by the transfer of momentum to the surrounding fluid.

What we find is in agreement with this expectation. The control parameter is $l_{\max}/D$, where D is the cylinder diameter. As to the order parameter measuring the coherence of the vortex street, we propose it to be the velocity v transverse to the flow measured on the cylinder axis downstream. That v can play the role of an order parameter is demonstrated in Fig. 1, which shows the variation of v/U as a function of reduced time. In the laminar case v is periodic with the period of the vortex street. For small distance exchanges of the size of the cylinder diameter ($l_{\max}/D = 1$) the periodicity of the street is unchanged after some transient behavior. However, when exchanges are included over distances up to at leat six times the cylinder diameter, the coherence of the street is destroyed. (The whole channel is ten diameters wide.) Results are the same at whatever positions behind the cylinder v is measured. (We do not measure velocities beyond 12 cylinder radii counting from the center of the cylinder.) In Fig. 2 we show the magnitude of v as a function of $l_{\max}/D$. The magnitude decreases continuously as the range of momenta exchanges increases and goes to zero somewhere between $l_{\max}/D = 4$ and 6. We thus interpret the magnitude of v as the order parameter which measures the spatial coherence of the vortex street. The transition between the existence and disappearance of the vortex street appears continuous. Fluctuations intrinsic to the Boolean lattice gas make a precise study of this point computationally prohibitive. A frequency analysis (Fig. 3) of the time dependent behavior of the vortex street supports the statement that transverse velocity is a good order parameter for describing the robustness of the street. Figure 3 shows the single peak corresponding to a well defined frequency for the periodic shedding of the vortex street. Though the amplitude is greatly reduced as compared to the laminar case, the frequency remains well defined at that same value as

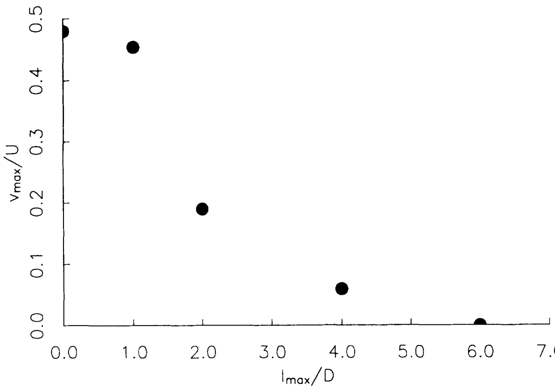

FIG. 2. Order parameter v/U as a function of control parameter $l_{\max}/D$. The value of zero for the latter corresponds to the laminar case.

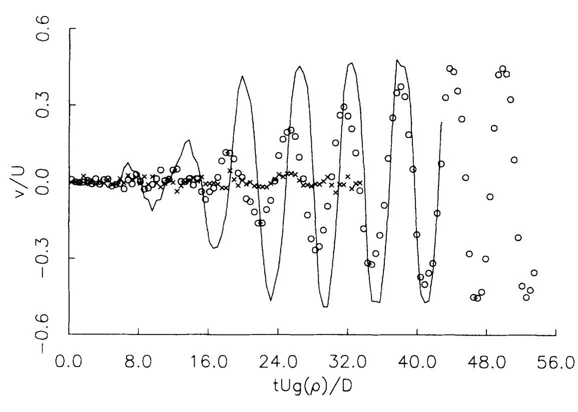

FIG. 1. Ratio v/U as a function of dimensionless time, for the laminar case (full line), the case where $l_{\max}/D$=1 (open dots), and the case where $l_{\max}/D$=6 (crosses). The tranverse velocity v is measured on the cylinder axis, at a station of four radii downstream from the cylinder center. The reduced time involves the density-dependent factor $g(\rho)$ peculiar to the Boolean lattice-gas model considered [6], equal to 0.25 in our case. U is the peak velocity of the incoming parabolic profile.

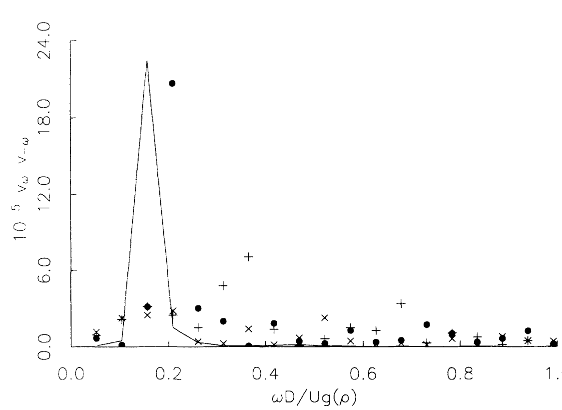

FIG. 3. Velocity power spectrum as a function of frequency. Frequency is normalized; see the caption of Fig. 1. The laminar curve is a full line reduced by a factor of 256 for comparison with the other curves. The full circles are for $l_{\max}/D = 4$. The crosses are for $l_{\max}/D = 6$ at two different stations behind the cylinder $x/D = 3.5$ and 4, showing the noisiness of the signal.

long as $l_{\max}/D$ is smaller than 4. When exchanges up to $l_{\max}/D = 6$ are included, the peak disappears. The data, taken at two different stations behind the cylinder, show (within the statistical precision of the simulation) a broad noisy background. One should note that as the range of exchanges increases, the peak remains narrow before fading into the noisy background rather than broadening progressively.

III. LONGITUDINAL VELOCITY AND MOMENTUM TRANSPORT

Figures 4 and 5 highlight some interesting behavior of the longitudinal velocity u measured on the cylinder axis at different stations. The point of interest concerns a comparison between two modes of momentum transport. The first one, operative fully in laminar flow and less and less as Lévy walks take place (cf. Fig. 2), concerns entrainment of the fluid by the large scale structure of the vortex street itself, which transports flow over distances of the order of the cylinder radius. The second one is the Lévy walk, which can transport momenta over distances large compared to cylinder radius, between the wake and the flow outside of it. The Lévy exchanges may be thought of as a stochastic model for large scale turbulent eddies. The picture is different at two stations, one close to the cylinder at $x/D = 1$, the other farther, namely, at $x/D = 4$ (distances along the axis are measured from the cylinder center). Close to the cylinder, where the swirling movement of the laminar vortex street has a small amplitude, the longitudinal velocity u for large exchanges ($l_{\max}/D = 6$) is slightly higher than the one for laminar flow (cf. Fig. 4). This is consistent with results on drag mentioned in the following section. Farther away from the cylinder where the entrainment of the vortex street works fully in the laminar case, u has the same value (approximately) for the laminar case as when $l_{\max}/D = 6$, indicating that the mixing due to Lévy walks and that due to the vortex street achieve similar results (cf. Fig. 5). For $l_{\max}/D = 2$ the mixing is clearly

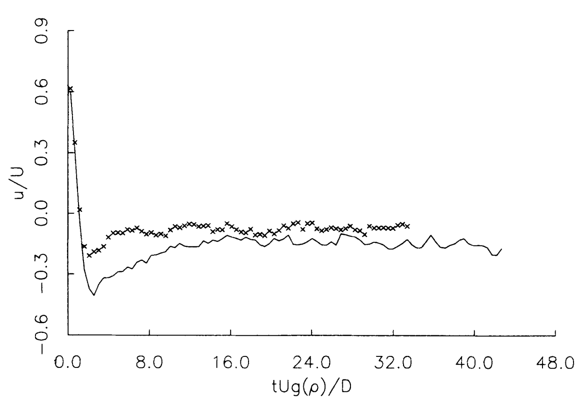

FIG. 4. Longitudinal velocity normalized by the peak velocity as a function of reduced time (see the caption of Fig. 1.). The full line corresponds to the laminar case, the crosses to $l_{\max}/D = 6$. These data are taken at the station $x/D = 1$ behind the cylinder.

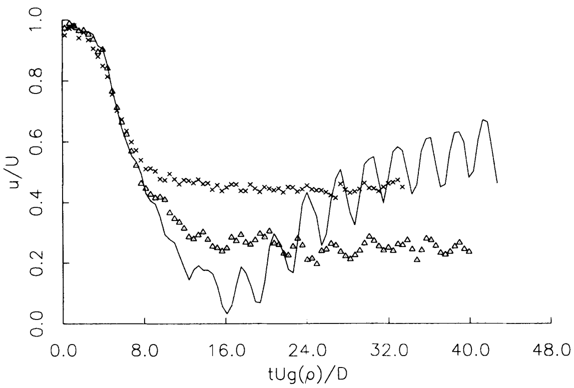

FIG. 5. Longitudinal velocity normalized by the peak velocity as a function of reduced time (see the caption of Fig. 1). These data are taken at the station $x/D = 4$ behind the cylinder. The full line corresponds to the laminar case, triangles to $l_{\max}/D = 2$, and crosses to $l_{\max}/D = 6$.

less than for $l_{\max}/D = 6$, and therefore the corresponding longitudinal velocity is lower, since neither the Lévy walk with its exchanges over relatively small distances nor the weakened vortices are able to transfer much momentun into and out of the the free-stream flow.

IV. DISCUSSION

The main result is that transverse velocity measured on the cylinder axis is a good order parameter for studying the robustness of the von Kármán street to transport of momentum into and out of the wake due to turbulence. The crucial parameter is $l_{\max}/D$, the ratio of the maximum length of exchanges and cylinder diameter. Experimentally it is also found [8] that the relevant parameter for studying the influence of turbulence on flow past a cylinder is the quantity L_x/D, where L_x is the integral scale of turbulence in the direction of the flow. The other parameter is the intensity of turbulence measured by the ratio of fluctuating to average velocity. This value is fixed in our case to about 0.08, which is in the range considered in experiments. Unfortunately the issue of the robustness of the von Kármán street is not the main concern of the experimental literature we have surveyed. Thus the result most directly relevant we have been able to find is the statement that for a circular cylinder placed in a turbulent mixing layer (away from the region of intense shear) the vortex street is destroyed because of turbulence [9]. Here the relevant parameter is the ratio of cylinder diameter to the width of the shear region, and when this parameter becomes sufficiently small, loss of vortex shedding is observed [9]. Since the scale of turbulence in the mixing layer must be of order of the size of the shear layer, this parameter is the inverse of our $l_{\max}/D$ and our analysis is therefore compatible with the experimental result on the loss of vortex shedding of Ref. 9. Also, measurements [8] of base pressure on square and rectangular plates in turbulent flow give results considerably below that of laminar flow. It is thus suggested [8] that it is indeed the extra entrainment of fluid out of the wake, due to turbulence, which explains these results.

We have presented results on longitudinal velocity behavior, when $l_{\max}/D$ varies, which we believe are of potential experimental interest (cf. Figs. 4 and 5).

Two additional remarks need be made.

(i) As pointed out in Sec. III, closely behind the cylinder, the longitudinal velocity is slightly higher when there is turbulence than in the laminar case. Since macroscopic velocities are averages over microscopic ones, the meaning of this is that on average fewer lattice particles will impinge on the cylinder at the downstream side for the turbulent as compared to the laminar case. Since drag in lattice gases is calculated by summing up momenta transferred to the cylinder by the lattice gas [1], the total drag being a combination of upstream particles pushing the cylinder one way and downstream particles pushing it the opposite way, the drag will consequently be higher in the turbulent than in the laminar case. We have measured the increase to be of the order of 5%. This increase is consistent with an expression of turbulent drag as a function of laminar drag which appears in Ref. [8], where the increase is expressed as a function of turbulent intensity.

(ii) We stress that our model is meant to reveal what are reasonable choices for the control parameter and order parameter for the study of vortex street robustness. Whether the street actually falls apart when exchanges of momenta take place over sufficiently large distances compared to cylinder diameter, as happens in our model, has to be decided in the end by experiment. There is an interesting connection here, however, because Lévy walks are not just characterized by the possibility of large exchanges. With each exchange distance a characteristic waiting time [5] is associated, and the form of the relationship between time and distance is crucial for describing enhanced diffusion in fully developed turbulence [5]. In this work the chosen relationship is *ad hoc,* the same which was used previously [4] for turbulent channel flow which appears insensitive to its precise form. Nevertheless here our results are in principle sensitive to the precise connection there is between the length and time scale of momenta exchanges, and our estimate of when the vortex street falls apart could be changed. Any experimental information on the robustness of the vortex street would therefore give insight into this matter.

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