PHYSICAL REVIEW E VOLUME 49, NUMBER 4 APRIL 1994

Bifurcation phenomena and multiple soliton-bound states in isotropic Kerr media

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(Received 4 October 1993)

An analytical and numerical study of the coupled nonlinear Schrödinger equations that govern light propagation in isotropic Kerr materials reveals the existence of a novel class of bound vector solitary waves. These solutions are shown to bifurcate from circularly polarized solitons. For certain values of the wave parameters they represent stationary bound states of several linearly polarized solitons. These solitary waves may find application to optical communications where they have the potential to increase the bandwidth of transmission lines.

PACS number(s): 42.81.Dp, 42.81.Gs, 03.40.Kf

I. INTRODUCTION

The nonlinear Schrödinger (NLS) equation governs phenomena of common occurrence in various fields of physics such as hydrodynamics, plasma physics, or nonlinear optics. In the context of optics the NLS equation describes light propagation in Kerr materials with one (spatial or temporal) transverse dimension [1,2]. This equation is integrable and possesses a fundamental soliton solution [3], here referred to as the NLS soliton, which represents a nondistorting propagating electric-field envelope with uniform phase and polarization. In optical fibers solitons are states of the field in which chromatic dispersion is exactly balanced by the refractive index nonlinearity of silica [2]. For this reason they have been suggested for application to extremely high-bit-rate transmission systems [4]. An undesirable effect of the Kerr nonlinearity, however, is to cause mutual interaction between the pulses which in turn leads to a drastic limitation of the system's bandwidth. This problem has been the subject of intensive studies, and several techniques have been proposed to reduce soliton interactions (see, e.g., Ref. [5], pp. 130–133). However, these techniques do not allow for a complete cancellation of soliton interaction and lead only to moderate increases of the fibers capacity. We show in this paper that, by considering the polarization of the field, it is possible to annihilate exactly the mutual interaction force between solitons.

When accounting for the polarization of the field in an isotropic Kerr material (e.g., a circular optical fiber) the wave equation takes the form of coupled NLS equations [6]. Although they are integrable for some peculiar nonlinearities [6,7], these equations were shown to fail the Painleve integration test [8]. The coupled NLS equations, and especially their generalization to birefringent media, were the subject of numerous works (see, e.g., Ref. [5], pp. 177–193). In particular, Tratnik and Sipe [9] and, independently, Christodoulides and Joseph [10] have shown the existence of a new type of vector solitary wave in birefringent materials. This type of solutions to the modified coupled NLS equations was later analyzed in terms of branching bifurcations from polarized NLS solitons [7,11]. We propose in this paper the study of previously unrecognized bifurcations of the coupled NLS equations for isotropic Kerr materials. We show analytically and numerically the existence of two families of vector solitary waves that branch from circularly polarized NLS solitons and tend, further from the bifurcation, to independent linearly polarized NLS soliton states. Between these two limits solutions consist of stationary trains of pulses that can be viewed as multiple soliton-bound states. Both families of vector solitary waves are characterized by the number of bound polarized solitons they contain. The resulting interactionless soliton propagation allows higher pulse repetition rates and is therefore of great interest in optical fiber transmission systems. Note that throughout this paper we use the common colloquial term of "soliton" for what should be more rigorously called a stationary solitary wave.

II. BIFURCATION ANALYSIS AND MULTIPLE SOLITON-BOUND STATES

In normalized units the circular polarization components of the field propagating in an isotropic Kerr medium with chromatic dispersion is given by the coupled NLS equations

$$i\frac{\partial U}{\partial z}+\frac{1}{2}\frac{\partial^2 U}{\partial t^2}+\left[\frac{1-B}{2}|U|^2+\frac{1+B}{2}|V|^2\right]U=0\ , \quad \text{(1a)}$$

$$i\frac{\partial V}{\partial z}+\frac{1}{2}\frac{\partial^2 V}{\partial t^2}+\left[\frac{1-B}{2}|V|^2+\frac{1+B}{2}|U|^2\right]V=0\ , \quad \text{(1b)}$$

where $U(z,t)$ and $V(z,t)$ are the envelopes of both polarization components, and $B=\chi^{(3)}_{1221}/\chi^{(3)}_{1111}$, where $\chi^{(3)}_{ijkl}$ is the nonlinear susceptibility tensor of the material [12]. Solitary-wave solutions to this equation have the form

$$U(z,t)=u(t)\exp(i\alpha z) \quad \text{and} \quad V(z,t)=v(t)\exp(i\beta z)\ , \quad \text{(2)}$$

where the envelopes u, v, and the corresponding wave vectors α,β are real quantities. Substituting Eq. (2) into Eq. (1) leads to a set of two real ordinary differential equations:

$$\ddot{u}=2\alpha u-(1-B)u^3-(1+B)v^2u\ , \tag{3a}$$

$$\ddot{v}=2\beta v-(1-B)v^3-(1+B)u^2v\ , \tag{3b}$$

where the dots denote time derivatives. The Hamiltonian of these equations of motion is

$$H=\tfrac{1}{2}\dot{u}^2+\tfrac{1}{2}\dot{v}^2-\alpha u^2-\beta v^2+\tfrac{1}{4}(u^2+v^2)^2-\frac{B}{4}(u^2-v^2)^2\ . \tag{4}$$

The analysis of a more general Hamiltonian of this type is conducted in Ref. [7] (for the case of different nonlinear coefficients for U and V), where it is shown to be integrable for specific values of the wave- and material-dependent parameters only. In that work branching bifurcations from polarized solitons are identified when considering variations of the nonlinear coefficients. Here we do not consider the role of material-dependent parameters, and rather focus on the influence of the wave parameters (α,β). We then consider an isotropic Kerr material with a given value of B. We choose $B=\frac{1}{3}$, which corresponds to the nonlinearity of silica fibers. The parameter α can be normalized to unity in Eq. (3), leading to a single-parameter equation set

$$\ddot{u}=2u-\tfrac{2}{3}u^3-\tfrac{4}{3}v^2u\ , \tag{5a}$$

$$\ddot{v}=2\beta_v-\tfrac{2}{3}v^3-\tfrac{4}{3}u^2v\ . \tag{5b}$$

Clearly this equation admits circularly polarized (CP) NLS solitons as solutions:

$$u=\sqrt{6}\,\mathrm{sech}(\sqrt{2}t),\quad v=0\ , \tag{6a}$$

$$u=0,\quad v=\sqrt{6\beta}\,\mathrm{sech}(\sqrt{2\beta}t)\ . \tag{6b}$$

Note that these solutions exist for any real positive value of β. For the particular value of $\beta=1$ the symmetry of Eq. (5) reveals the existence of linearly polarized (LP) NLS solitons as additional solutions:

$$u=\pm v=\sqrt{2}\,\mathrm{sech}(\sqrt{2}t)\ . \tag{7}$$

We show in the following that more general solitary-wave solutions to Eq. (5) exist that, at given values of $\beta<1$, branch from the CP solitons (6a) and, as $\beta\to 1$, tend to states of decoupled LP solitons (7). Despite the nonintegrabiltiy of the Hamiltonian H with $B=\frac{1}{3}$ [7], the method of Refs. [7,11] enables the development of a partial analytical description of the bifurcation of these solutions from the CP soliton branch. Considering the solitary-wave solutions of Eq. (5) as a CP soliton (6a) plus a small perturbation, we write

$$u=\sqrt{6}\,\mathrm{sech}(\sqrt{2}t)+\varepsilon^2x\ , \tag{8a}$$

$$v=\varepsilon y\ . \tag{8b}$$

From Eq. (5) we easily verify that the linear term in ε in Eq. (8a) is zero. Substituting Eq. (8) into Eq. (5), we obtain, to the leading order, two decoupled equations in x and y. The equation in y is

$$\ddot{y}=2\beta y-8\,\mathrm{sech}^2(\sqrt{2}t)y\ . \tag{9}$$

This equation is well known in optical waveguide theory. It constitutes the modal equation of the sech^2 index profile planar waveguide [13]. The change of variable $\tau=\tanh(\sqrt{2}t)$ transforms this equation into a Legendre differential equation whose general solution is given in terms of the associated Legendre functions of the first and second kinds, $P_\nu^\mu[\tanh(\sqrt{2}t)]$ and $Q_\nu^\mu[\tanh(\sqrt{2}t)]$, where $\mu=\sqrt{\beta}$ and $\nu(\nu+1)=4$ (i.e., $\nu=1.56$ and -2.56) [14]. The solutions which decay at infinity are given by $P_\nu^{n-\nu}[\tanh(\sqrt{2}t)]$, where n is a positive integer smaller than ν. The value $\nu=1.56$ indicates that Eq. (9) admits only two solutions that decay at infinity ($n=0$ and 1). They are

$$y=\mathrm{sech}^{\nu}(\sqrt{2}t)\ \text{ at } \beta=\beta_0=2.43\ , \tag{10}$$

$$y=P_\nu^{1-\nu}[\tanh(\sqrt{2}t)]\ \text{ at } \beta=\beta_1=0.314\ . \tag{11}$$

Equation (10) reveals the existence of a bifurcation at $\beta=\beta_0=2.43$ which is beyond the value $\beta=1$ corresponding to the existence of the LP NLS solitons (7). This bifurcation does not lead to multiple soliton-bound states and will not be considered here. The second solution (11) shows that, at $\beta=\beta_1=0.314$, a bifurcation exists that originates an antisymmetric solitary-wave envelope in v ($P_\nu^{1-\nu}$ is a one-node antisymmetric function).

This branch of solutions has been calculated numerically from Eq. (5) using a standard shooting method. Near the origin $[(u,v)=0]$ the motion of the point in the (u,v) plane can be approximated by neglecting the nonlinear terms in Eq. (5), resulting in decoupled equations whose solutions for $t\to-\infty$ can be written as $u=\exp(\sqrt{2}t)$ and $v=c\exp(\sqrt{2\beta}t)$, where c is an arbitrary constant. In our shooting method we used this motion in the plane (u,v) as initial conditions at large negative values of t, and we chose c as the shooting parameter. The separatrix trajectories in the plane (u,v) obtained with different values of β are given in Fig. 1 together with the corresponding solitary-wave envelopes. We verified the appearance along the u axis of a single-loop (one-node) separatrix trajectory at $\beta=\beta_1$. As β increases from β_1, the separatrix loop divides into two lobes that narrow progressively and approach the lines $u=\pm v$ as β tends to unity. For sufficiently large values of β, the solitary wave is therefore composed of two distinct pulses

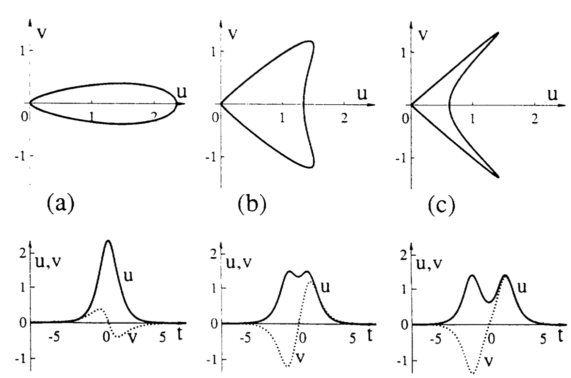

FIG. 1. One-node separatrix trajectories in the plane (u,v) and the corresponding solitary-wave envelopes. (a) $\beta=0.36$, (b) $\beta=0.78$, and (c) $\beta=0.95$.

of almost linear polarization [see Fig. 1(c)]. In the limit $\beta \rightarrow 1$ these pulses are two orthogonal LP NLS solitons separated by an infinite distance. Note that, according to Eq. (2), the linear polarization components of the field are given by $E_x = [u \exp(iz) + v \exp(i\beta z)]/\sqrt{2}$ and $E_y = [u \exp(iz) - v \exp(i\beta z)]/i\sqrt{2}$. This shows that, although the total intensity profile of the solitary wave remains constant during propagation, its electric field undergoes a uniform polarization rotation at a rate $(1-\beta)/2$. The bifurcation diagram $c(\beta)$ corresponding to these solitary waves is given in Fig. 2(a). Note that the line $c(\beta)=0$ corresponds to the branch of CP solitons (6a), and $c=1$ is the value of the shooting parameter corresponding to the LP solitons (7) that exist only at $\beta=1$.

A second type of solitary-wave solution was identified that bifurcates from the CP soliton at $\beta=0$. These solitary waves are symmetric functions of t and possess two nodes in v. They originate from a more complex bifurcation which cannot be studied by means of a linear perturbation analysis because the linear equation (9) does not admit two-node solutions which decay at infinity. The symmetry in the profiles of these solitary waves corresponds to separatrix trajectories that are folded upon themselves at a turning point (u_0, v_0) where the velocities $\dot{u}, \dot{v}$ are zero. Introducing the polar coordinates $u_0 = r_0 \cos\theta_0$ and $v_0 = r_0 \sin\theta_0$, the Hamiltonian (4) gives the following relation between r_0 and θ_0:

$$r_0^2 = 4[1-(1-\beta)\sin^2\theta_0]/(1-\tfrac{1}{3}\cos^2\theta_0) . \tag{12}$$

For the calculation of the symmetric solitary waves it is convenient to use θ_0 as the shooting parameter. Figure 2(b) shows the corresponding bifurcation diagram $\theta_0(\beta)$. Note that the line $\theta_0(\beta)=0$ corresponds to the CP soliton (6a) and the value $\theta_0=45°$ at $\beta=1$ is the angle corresponding to the LP solitons (7). Figure 3 shows the evolution of the two-node separatrix and the corresponding solitary wave for increasing values of β. As β tends to 1 the solitary wave transforms into a bound state of three LP NLS solitons (the distance between them is infinite at $\beta=1$). Note that a similar type of composite soliton state has recently been described in the study of nonlinear fiber couplers [15].

As illustrated in Fig. 2 by the dotted curves for both types of separatrices, other branches were found that bifurcate from CP soliton states. These additional branches correspond to higher-order (i.e., with larger

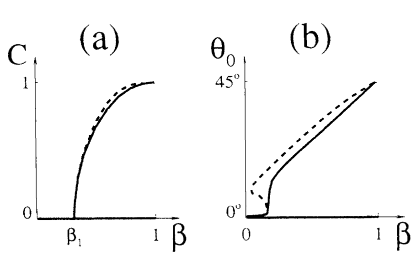

FIG. 2. Bifurcation diagrams of the (a) one-node and (b) two-node solitary-wave solutions (solid lines). The dotted lines represent higher-order solitary-wave families: the three- and six-node solutions in (a) and (b), respectively.

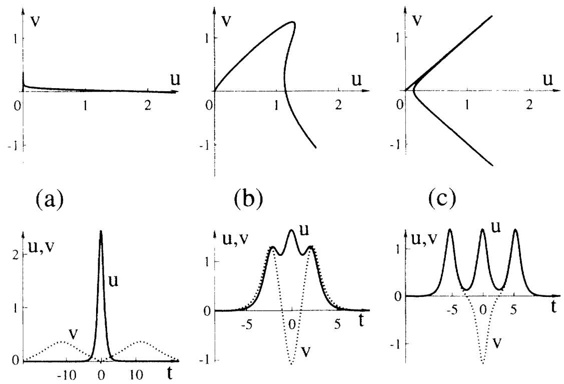

FIG. 3. Two-node separatrix trajectories in the plane (u,v) and the corresponding solitary-wave envelopes. (a) $\beta=0.02$, (b) $\beta=0.7$, and (c) $\beta=0.996$.

numbers of nodes) separatrix trajectories that merge at the bifurcation point with the lower-order separatrices studied above. The dotted curve of Fig. 2(a) corresponds to the three-node solution represented in Fig. 4. Near the bifurcation it represents bound states of two CP solitons (at $\beta=\beta_1$ the distance between them is infinite). As shown in Fig. 4(b), as β tends to unity the solitary wave becomes a bound state of four equidistant LP solitons. In both limiting regions, the total energy carried by this solitary wave is therefore twice the energy of the one-node solitary wave. This is illustrated by the energy bifurcation diagram $Q(\beta)$ plotted in Fig. 5, where the energy Q is defined by

$$Q = \int_{-\infty}^{+\infty} (u^2+v^2) dt . \tag{13}$$

We also found higher-order separatrices with even numbers of nodes in v. The dotted line of Fig. 2(b) is the branch of six-node solitary waves. For sufficiently large β they represent bound states of seven LP solitons. In fact, we found no apparent limitation in this soliton multiplication process, and were limited only by the growing numerical accuracy that is required to calculate the correspondingly more complex trajectories. As illustrated in

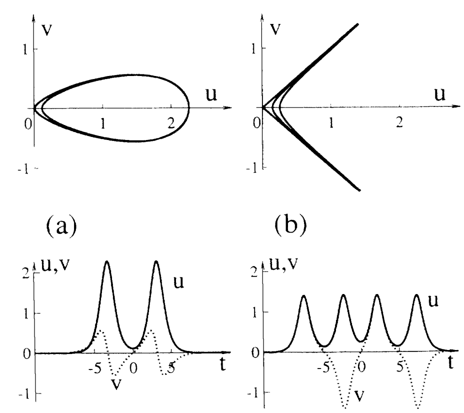

FIG. 4. Illustration of the existence of higher-order solitary waves: the three-node solitary wave corresponding to the dotted line of Fig. 2(a). (a) $\beta=0.4$. (b) $\beta=0.99$.

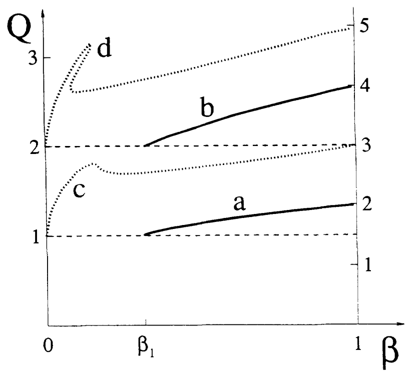

FIG. 5. Energy bifurcation diagram of the two families of solitary waves. Curves a,b, and c are the branches of one-, two-, and three-node solutions of Figs. 1, 3, and 4. Curve d is the branch of four-node solutions. The horizontal dashed lines are the branches of CP soliton states. For clarity the axes of the left- and right-hand sides are scaled in units of CP and LP soliton energies, respectively.

Fig. 2(b) the higher-order branches exhibit a more complex behavior related to nonunicity of the solutions in a certain range of the wave parameter β. Figure 5 clearly illustrates the soliton multiplication process. The branches that originate at β_1 correspond to the transformation of states of n-independent CP solitons into states of $2n$-independent LP solitons. Similarly the branches of symmetrical solutions go from states of n CP solitons to states of $2n+1$ LP solitons. A similar soliton multiplication process in the bifurcation from polarized solitons has already been observed [16] with the more general Hamiltonian system of Refs. [7,11].

In summary, we have found new families of vector solitary waves in isotropic dispersive Kerr materials. These solitary waves branch from circularly polarized soliton states and, as they go further from the bifurcation, become bound states of linearly polarized solitons. Apart from a uniform polarization rotation the field envelope of the solitary waves is stationary. This result demonstrates the possibility of exact annihilation of the interaction forces between polarized solitons in an isotropic Kerr medium. Such a result can have important implications in soliton-based optical fiber transmission systems.

III. APPLICATION TO SOLITON TRANSMISSION SYSTEMS

Due to its practical importance, the problem of soliton interaction in optical fibers has attracted much attention in recent years. Several techniques for the reduction of soliton interaction forces have been suggested such as amplitude [17] or phase [18] shifting, the use of third-order dispersion [19], active temporal modulation schemes [20,21], as well as passive filtering methods [22,23]. With respect to these techniques the result of our analysis seems to provide a simple and promising alternative way to increase the bandwidth of soliton-based fiber transmission systems. In order to show the practicality of the multiple soliton-bound states studied above, we must address three important issues, namely the stability of these complex solutions, the problem of their generation, and the validity of the theoretical model [Eqs. (1)]. In the following we present a preliminary study of these practical aspects of the problem.

Let us first analyze in more detail the structure of the multiple bound-soliton states. In a practical communication system, the soliton pulses must propagate sufficiently far apart so as to be detected individually at the output of the fiber link. We must therefore consider the multiple bound-soliton solutions for values of β close to unity, such as those represented in Figs. 1(c), 3(c), and 4(c). The separatrices of these solutions are close to the separatrices of the LP NLS solitons. At $z=0$ the envelopes of the linear polarization components are given by $E_x=(u+v)/\sqrt{2}$ and $E_y=(u-v)/i\sqrt{2}$. As we have seen above, for $z\neq 0$ these envelopes are conserved in a reference frame rotating at a rate $(1-\beta)/2$ (see Ref. [24] for a more detailed analysis of this feature). Since u and v are real functions, the envelopes in E_x and E_y are $\pi/2$ out of phase. Therefore we see from Figs. 1(c), 3(c), and 4(c) that, provided the pulses are sufficiently far apart, the multiple soliton-bound states can be well approximated by trains of equidistant LP NLS solitons in which adjacent solitons are orthogonally polarized and $\pi/2$ out of phase. The difference in profile between the multiple soliton-bound states and the sech pulses is a perturbation too small to be accurately generated in practice. The problem of the generation of the multiple soliton-bound states can therefore be limited to the generation of such trains of pulses. Naturally, since the multiple soliton-bound states are not generated with their exact shape, one must expect the appearance of a slight interaction between the solitons.

From a practical point of view, the problem of the stability of the bound-soliton states is reduced to that of the stability of the relative position of the polarized solitons in the train. Since any perturbation introducing a phase asymmetry leads to a change in the soliton velocity [25], one must expect the multiple soliton-bound states to be unstable [although highly overlapping waves such as those of Fig. 1(a) may be stable due to self-trapping]. However, such an instability is inherent to the use of solitons as information carriers, and is common to all soliton-based optical transmission systems. The positive aspect of our analysis, with respect to previous studies, is that the existence of the multiple bound-soliton solutions of the coupled NLS equations suggests the possibility of a significant reduction in mutual interaction.

We have observed this numerically by integrating Eqs. (1) with trains of orthogonally polarized and $\pi/2$ phase-shifted sech pulses as initial conditions. Figure 6(a) shows the propagation of two bound pulses which constitute an approximation of the soliton-bound state of Fig. 1(c) by NLS solitons. The separation between solitons is $T=2.3$ their full width of half maximum (FWHM). We see that the perturbation due to the approximation is only responsible for small oscillations of the solitons around fixed positions. This behavior occurs over a distance longer than the collision length of the two in-phase

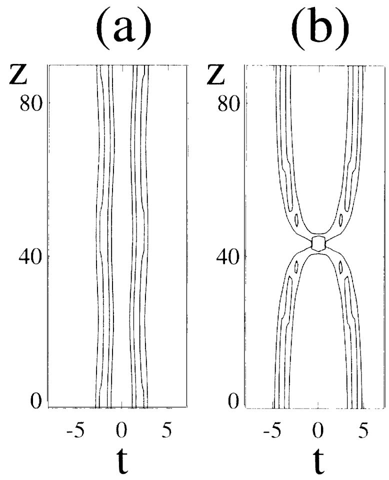

FIG. 6. Contour plot showing (a) the evolution of the total intensity profile of the two-soliton-bound state obtained by approximation with orthogonally polarized and $\pi/2$ out-of-phase LP NLS solitons of the exact solution shown in Fig. 1(c). The separation between the solitons is $T=2.3$ their FWHM. (b) The evolution of two in-phase, likewise polarized LP NLS solitons separated by $2T=4.6$ FWHM.

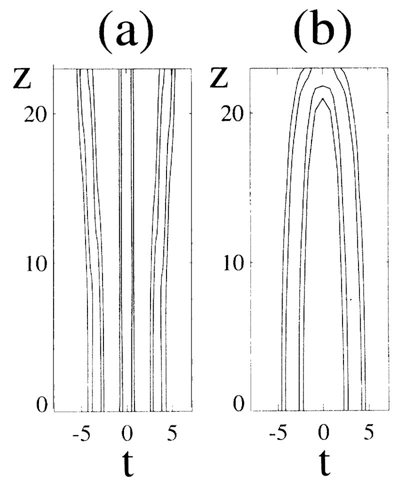

FIG. 7. Contour plot showing (a) the evolution of the total intensity profile of the approximated three-soliton-bound state corresponding to $\beta=0.95$. The separation between the pulses is 2 FWHM. (b) The evolution of two in-phase likewise polarized LP NLS solitons in the absence of the central orthogonally polarized one. We observe a drift instability of the soliton-bound state with a characteristic length of the order of the collision length of the in-phase solitons. This result shows that the use of multiple soliton-bound states leads to an increase, by a factor of 2, of the bandwidth of the fiber links.

and like polarized solitons separated by $2T=4.6$ FWHM shown in Fig. 6(b).

To appreciate the importance of the role of polarization, this result must be compared with the study of N-soliton interaction in the scalar approximation [26]. The authors of that work analyze the interaction between solitons of unequal amplitude. It is shown that with specific arrangements between the solitons, stable propagation can be obtained as long as the distance between them is larger than five times their FWHM.

In an actual soliton-based communication system, the information would be coded by removing solitons from the generated train of equidistant orthogonally polarized and $\pi/2$ out-of-phase sech pulses separated by the bit period T. The train would then unavoidably contain adjacent solitons with identical polarization and phase. Naturally, these solitons are liable to interact and will be responsible for bandwidth limitation. The most unfavorable situation corresponds to two solitons separated by twice the bit period, $2T$. This is illustrated in Fig. 7, which shows the evolution of the three-soliton-bound state (corresponding to $\beta=0.95$) approximated by sech pulses together with the evolution of the two in-phase and like polarized solitons in the absence of the central orthogonally polarized pulse. We see in Fig. 7(a) that perturbation resulting from the approximation by sech pulses is large enough to break the bound state. The breakup is due to a repulsive force between the pulses. These forces result in a detrimental temporal jitter of the pulses. If we choose a pulse translation of one-half pulse FWHM as the criterion of maximum acceptable jitter, the maximum propagation distance is $L=23$ (see Ref. [27] for jitter and detection considerations). This distance corresponds approximately to the collision length of the in-phase likewise polarized solitons separated by twice the bit period. This result shows that the use of multiple soliton-bound states potentially leads to an increase of a factor of 2 of the transmission bit rate.

In fact, use of orthogonally polarized solitons in long-distance transmission links has recently been studied theoretically and experimentally [27,28]. In particular, it was observed, but not properly explained, that in the case of orthogonally polarized adjacent solitons pulse-to-pulse interaction is much weaker than in the case of solitons with identical polarization (scalar case). An increase of a factor of 2 in the transmission bit rate has been reported in the polarization multiplexing technique based on this feature [27]. We show in the following that this result can be explained through the existence of the multiple bound-soliton solutions of the coupled NLS equations (1).

In the theoretical analysis above we considered pulse propagation in ideal isotropic media with Kerr nonlinearity of electronic origin ($B=\frac{1}{3}$). Naturally this analysis cannot be applied directly to practical transmission systems. A single-mode fiber always exhibits varying residual birefringence, and light propagation should in principle be described by a generalized form of the coupled NLS equations including varying birefringence parameters [27]. However, on the basis of a simple reasoning the authors of Ref. [27] showed that an averaging procedure over all polarization states of the nonlinear terms and the group-velocity dispersion can be applied that leads to the simple Manakov model [29]. In other words, they showed that Eqs. (1) describe pulse propagation in fibers in real practical conditions provided that $B=0$. In fact, we found no qualitative difference in the multiple bound-soliton solutions for variation of B between zero and $\frac{1}{2}$.

For the case $B=0$ the multiple soliton-bound states originate from the same bifurcations as for $B=\frac{1}{3}$ (but the bifurcation of the even soliton-bound states occurs now at $\beta=\beta_1=0$), and their envelopes are well approximated by sech pulses. An important difference for the $B=0$ case is that the relative phase between adjacent polarized solitons can be chosen arbitrarily. This can easily be explained by the fact that, when $B=0$, Eqs. (1) are invariant with respect to the change of variables $U \rightarrow E_x=(U+V)/\sqrt{2}$ and $V \rightarrow E_y=(U-V)/i\sqrt{2}$; that is, they are identical for circular and linear polarization components. Moreover, they are invariant with respect to a change of relative phase between the polarization components. Using E_x and E_y instead of U and V, the soliton-bound states will represent trains of solitons in which adjacent solitons are in phase and orthogonally polarized. This is precisely the situation considered in the polarization multiplexing technique of Ref. [27]. We can therefore interpret the observations reported in that work in terms of the existence of the multiple soliton-bound states as solutions of the coupled NLS equations. Note finally that a preliminary study of the propagation of soliton-bound states in fiber transmission systems governed by the averaged coupled NLS equations (i.e., with $B=0$) is performed in Ref. [30] on the basis of a variational approach.

IV. CONCLUSION

In conclusion, we have carried out analytically and numerically a bifurcation analysis of the circularly polarized NLS soliton solutions of the coupled nonlinear Schrödinger equations describing light propagation in isotropic Kerr media. The bifurcation branches correspond to previously unrecognized one-parameter families of vector solitary-wave solutions. These solutions are interesting in exhibiting, in a certain parameter range, N-soliton-bound states. The existence of such soliton-bound states suggests the possibility of reducing soliton interactions in optical-fiber transmission systems. The use of trains of orthogonally polarized solitons which approximate the multiple soliton-bound state potentially leads to an increase of a factor of 2 of the transmission bit rate. This result corroborates recent theoretical and experimental studies of soliton polarization multiplexing in long-haul transmission links, and provides an understanding of the success of such schemes.

ACKNOWLEDGMENTS

The authors are indebted to N. N. Akhmediev for many helpful discussions. This work is supported by the Australian Photonics Cooperative Research Center (APCRC).

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