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Theory of Raman scattering for a short ultrastrong laser pulse in a rarefied plasma

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Direct forward and backward Raman scattering in an underdense plasma are considered for relativistically strong laser pulses. Dispersion analysis and analytical solutions, taking account of the pulse shape, are presented. It is shown that the Raman instability has an absolute maximum of the growth rate for the laser field amplitude $(eE_0/m\omega_0 c)\simeq 1$, which corresponds approximately to the power density of 10^{18} W/cm^2 for 1 μm wavelength laser radiation. Analytical solutions are obtained for both backward- and forward-scattered light that show the quite different way that the forward and backward scattering affect the global pulse evolution. The forward scattering develops in the whole body of the pulse and affects most considerably the trailing part of the pulse. In contrast with it, the backward scattering can be expected to become saturated aleady in the leading part of the pulse and affects mainly the evolution of the leading pulse edge. The relation of the obtained results and recent three-dimensional studies of the short-pulse evolution are discussed.

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I. INTRODUCTION

Stimulated Raman scattering (SRS) [1] is of particular interest to laser-plasma interaction since it is believed to strongly affect a powerful laser pulse in plasmas. SRS suggests that an electromagnetic pump wave is scattered by ripples of electron plasma density. The latter, in their turn, are enhanced by a ponderomotive force which arises due to the beats of the pump and scattered light. There is a feedback loop and therefore an instability can occur. Not every electron plasma wave is likely to be a seed for the instability—only the waves with the wave numbers and frequencies that satisfy the matching conditions:

$$\mathbf{k}_1=\mathbf{k}_0-\mathbf{k}_e;\omega_1=\omega_0-\omega_e \ . \tag{1.1}$$

The subscripts 1,0,e correspond to scattered, pump, and electron plasma waves, respectively.

The theory of SRS has been developed and copiously studied for electromagnetic radiation of a moderate power [1–8]. Now there is quite a clear understanding of the conditions under which SRS at various angles dominates and of the parameters upon which its growth rate depends. These parameters are, namely, the pump amplitude, the ratio of the electron plasma frequency to the electromagnetic wave carrier frequency, and the angle of scattering. It is believed that the growth rate of the SRS (in a plasma with cold electrons) increases with a deviation of the scattering from the forward direction and is maximum for the backward scattering (see, for example, Refs. [1,6]). Side or backward SRS, in addition, can be strongly affected by the thermal plasma electron motion [5,6], since the Landau damping can be expected to suppress the excitation of a daughter Langmuir wave.

As to the relativistically strong electromagnetic pulses ($\gamma_E k\simeq 1$), the first attempts to understand in detail the instabilities that can affect relativistically strong electromagnetic radiation were undertaken quite a long time ago (see, for example, Ref. [9]). The recent breakthrough in the laser technology [10,11] made possible laser-plasma interaction experiments with extremely high laser field intensities in subpicosecond pulses [12,13]. Both a strong nonlinear character of laser-plasma interaction and, especially, a fairly fast scale of changes in laser intensity require at present an adequate theoretical description of these instabilities.

The recent experimental observations of backward stimulated Raman scattering (BSRS) produced by a subpicosecond laser [13] demonstrated that a considerable revision of the weakly nonlinear theory was required to explain at least some of the obtained results. Computer simulations presented in Refs. [14,15] indicate that the backward SRS can play a fairly extraordinary role in an ultrashort pulse evolution: being itself of minor importance for the pulse energy losses, BSRS could trigger off an ultrafast pulse depletion due to wake-field excitation.

This paper discusses mainly the direct forward stimulated Raman scattering (FSRS) and BSRS for relativistically strong pulses. First, we consider briefly the results that follow from the exact fully relativistic dispersion relation. Then, in contrast with the estimations based on simple models (see, for example, Ref. [5]) and numerous speculations based on the dispersion equation analysis (see, for example, Ref. [13]), we present a theory that takes account of the shape of the pulse envelope.

In a focus of our study, there are mainly analytical solutions that describe the SRS instability in rarefied plasmas where a laser carrier frequency ω_0 is much higher than the electron plasma frequency

$$\omega_0\gg\omega_{pe}\equiv(4\pi e^2 n_0/m_e)^{1/2} \ . \tag{1.2}$$

Here n_0 is an unperturbed electron density. We consider, in addition, laser pulses that are initially smooth and have a duration τ_0 fairly above the electron plasma period

$$\tau_0\gg\omega_{pe}^{-1} \ . \tag{1.3}$$

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In the opposite case, a perturbation approach to the description of the pulse evolution fails because of a strong plasma-wave generation [16–19].

II. BASIC EQUATIONS

We start from the conventional, fully relativistic one-dimensional (1D) set of equations for an electromagnetic field and cold plasma electrons, which can be readily obtained, for example, from the equations presented in Refs. [20,21]:

$$\left[\frac{\partial^2}{\partial t^2}-c^2\frac{\partial^2}{\partial x^2}+\frac{\omega_{pe}^2}{\gamma_e}\frac{n_e}{n_0}\right]\mathbf{A}_\perp=\mathbf{0}\ , \tag{2.1}$$

$$\frac{\partial}{\partial x}E_x=-4\pi e(n_e-n_0)\ , \tag{2.2}$$

$$\frac{\partial}{\partial t}p_{ex}=-eE_x-2m_ec^2\frac{\partial}{\partial x}\gamma_e\ , \tag{2.3}$$

$$\frac{\partial}{\partial t}n_e+\frac{\partial}{\partial x}(n_ep_{ex}/\gamma_e)=0\ . \tag{2.4}$$

Here $\mathbf{A}_\perp$ is a vector potential of the electromagnetic field normalized to m_ec^2/e, which in the case of a one-dimensional approximation is proportional to the transverse component of electron momentum ($\mathbf{A}_\perp=p_{e\perp}m_ec$); E_x is a longitudinal component of the electric field; $\gamma_e=(1+p_{ex}^2/m_e^2c^2+\mathbf{A}_\perp^2)^{1/2}$ is a relativistic factor; n_e is an electron density. Ions are treated as a homogeneous neutralizing background.

Let us first neglect short-wavelength instabilities. This approximation is often useful to study large-scale pulse evolution and stability. Let us also consider a circularly polarized pulse for which the vector potential can be introduced in the form

$$\mathbf{A}_0=\tfrac{1}{2}A_0(x,t)\{\mathbf{e}_0\exp[i\psi_0(x,t)]+\text{c.c.}\}\ . \tag{2.5}$$

Here $\mathbf{e}_0=(\mathbf{e}_y+i\mathbf{e}_z)/\sqrt{2}$ is a unit vector of polarization, $A_0(x,t)$ is a slow varying real amplitude, and $\psi_0(x,t)$ is a phase of the pulse. The time and space derivatives of ψ_0 determine the local values of the pulse carrier frequency and wave number

$$\begin{aligned}\omega_0&=-\partial\psi_0/\partial t\gg A_0^{-1}(\partial A_0/\partial t)\ ,\\ k_0&=\partial\psi_0/\partial x\gg A_0^{-1}(\partial A_0/\partial x)\ .\end{aligned} \tag{2.6}$$

Considering the circular polarization, we avoid fast oscillations of $\mathbf{A}_0^2=A_0^2/2$ and the effects of high-frequency harmonic generation [22].

We impose limitations on the space variations of the pulse parameters

$$\omega_0^{-1}(\partial\omega_0/\partial x),\quad A_0^{-1}(\partial A_0/\partial x)\ll K_p\equiv\Omega_p/c\ . \tag{2.7}$$

Here $\Omega_p(A_0)\equiv\omega_{pe}/\gamma_0^{1/2}$ is a relativistically corrected electron plasma frequency, and $\gamma_0\equiv(1+A_0^2/2)^{1/2}$. The requirements (2.7) can be interpreted as the condition under which the plasma electron-density perturbations produced by the pulse (or, in other words, strictional nonlinearities) can be ignored.

The evolution of such a smooth pulse can be described by

$$(\partial^2/\partial t^2-c^2\partial^2/\partial x^2+\omega_{pe}^2(1+A_0^2/2)^{-1/2})\mathbf{A}_0=\mathbf{0}\ . \tag{2.8}$$

This equation leads to the usual nonlinear dispersion relation for ω_0 and k_0,

$$\omega_0^2=\Omega_p^2+k_0^2c^2\ . \tag{2.9}$$

The Raman scattering is produced by the electron-density ripples with a wavelength $\lambda_e=2\pi k_e^{-1}\le 2\pi K_p^{-1}$ that in our case is much shorter than the pulse length ($k_e^{-1}\le K_p^{-1}\ll\tau_0c$). To study SRS we consider here small-scale perturbations of $\mathbf{A}_\perp$ and n_e ($\mathbf{A}_1=\mathbf{A}_\perp-\mathbf{A}_0$, $N_1=(n_e-n_0)/n_0$) at the background of a slow large-scale pulse evolution described by Eq. (2.8). Then, linearizing Eqs. (2.1)–(2.4) in $\mathbf{A}_1$ and N_1, we obtain

$$\begin{aligned}(\partial^2/\partial t^2-c^2\partial^2/\partial x^2+\Omega_p^2)\mathbf{A}_1&\\ =-\Omega_p^2N_1\mathbf{A}_0+\Omega_p^2(\mathbf{A}_0/\gamma_0^2)(\mathbf{A}_0\mathbf{A}_1)\ ,&\end{aligned} \tag{2.10}$$

$$(\partial^2/\partial t^2+\Omega_p^2)N_1=(c^2/\gamma_0^2)\partial^2/\partial x^2(\mathbf{A}_0\mathbf{A}_1)\ , \tag{2.11}$$

where $\mathbf{A}_0$ is to be substituted as a solution of Eq. (2.8).

In the present paper we consider the instabilities that develop fast enough, so that the large-scale pulse evolution [with a characteristic time of order $(\omega_{pe}^2/\omega_0^2)\tau_0$, as it follows from Eq. (2.8)] can still be neglected during the typical time of the instability development. Consequently, we shall not focus on evolution of the pump pulse itself and treat A_0 as a given function of x and t.

III. DISPERSION EQUATION ANALYSIS

As an early approach to the problem of SRS in a relativistically strong pulse, it is instructive to study the general features of instability for the case of a homogeneous pump amplitude (A_0=const). As usual, in this case, we take all low-frequency dependencies to be proportional to $\exp(-i\omega_et+ik_ex)$, where ω_e and k_e are, respectively, a frequency and a wave number of electron-density perturbations N_1. The hybrid dispersion relation for ω_e and k_e, which can be obtained from the set of equations (2.10) and (2.11), is the same as that obtained in Ref. [9]. Using the notations

$$D_\pm\equiv\Omega_p^2+(k_e\pm k_0)^2c^2-(\omega_e\pm\omega_0)^2,\quad D_e\equiv\Omega_p^2-\omega_e^2\ , \tag{3.1}$$

we write the dispersion relation in the conventional form [1]

$$\frac{\Omega_p^2(A_0^2/\gamma_0^2)}{4}\left[\frac{k_e^2c^2}{D_e}+1\right]\left[\frac{1}{D_+}+\frac{1}{D_-}\right]=1\ . \tag{3.2}$$

Here the pump frequency ω_0 and the wave number k_0 are related by the nonlinear dispersion relation (2.9).

For a weakly nonlinear case ($A_0\ll 1$) this equation can be reduced to those discussed and well studied in a number of papers [1–4,7,8]. However, there is no consequent and complete linear analysis of SRS instability for a relativistically strong pump pulse. In this section, based on relations (3.1) and (3.2), we shall discuss the main peculiarities of the forward and backward SRS instabilities for an arbitrary pump amplitude.

Equation (3.2) contains as parameters only a relativistically corrected plasma frequency Ω_p and a normalized amplitude of electron quiver velocity $v_E/c \equiv A_0/\gamma_0$. It is worth mentioning that the relativistic effects here are reduced just to the relativistic electron mass growth (or to the decrease in Ω_p) caused by transverse electron oscillations in the electromagnetic field.

As follows from the dispersion relation (3.2), the forward SRS instability is always periodical $\mathrm{Re}\omega_e \simeq \Omega_p$, and has the growth rate

$$\Gamma_F \equiv \mathrm{Im}\omega_e = \mathrm{Re}\{[\Gamma_0^2 - \tfrac{1}{4}(k_e - K_p)^2 c^2]^{1/2}\} \ll \Omega_p \ , \quad (3.3)$$

where $\Gamma_0^2 \equiv \frac{1}{8}(\Omega_p^4/\omega_0^2)(A_0^2/\gamma_0^2)$. For wave numbers close to the resonant value ($|k_e - K_p| \ll \Gamma_0/c$), the growth rate Γ_F reaches its maximum $\Gamma_{Fm} = \omega_{pe}^2/4\omega_0$ for the pump amplitude $A_0 = \sqrt{2}$ and falls as A_0^{-1} with $A_0 \to \infty$.

In Fig. 1, which shows the growth rate of the FSRS instability as a function of $k_e c/\omega_{pe}$ for several values of light field amplitude, we can see a narrow resonant peak associated with the FSRS for the wave numbers which are close to the resonant value $K_p = (\omega_{pe}/c)(1 + A_0^2/2)^{-1/4}$. The dependence of a resonant wave number K_p on the pump amplitude results in the shift of this peak towards the smaller wave numbers for the greater pump amplitudes. This picture also demonstrates the rise and fall of the instability growth rate with an enlarging of the pump-field amplitude.

For the nonresonant wave numbers ($k_e < K_p$), as can also be seen in Fig. 1, an instability can develop as well. Sometimes it is referred to as the relativistic modulation instability (RMI) [7]. But since RMI has a fairly lower growth rate, it can be ignored in our further analysis.

The periodical backward SRS instability always corresponds to a weak pump [1] and needs no relativistic corrections. In the case of a large pump amplitude $[A_0 \gg (\omega_{pe}/\omega_0)^{1/2}]$, the growth rate of the instability Γ_B is comparable with $\mathrm{Re}\omega_e$. For a fixed pump amplitude the peak value of Γ_B corresponds to $k_e \simeq 2k_0 - \omega_p/c$ and equals

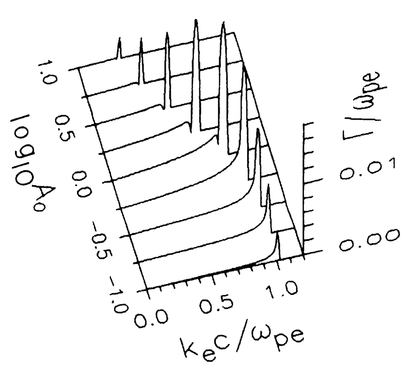

FIG. 1. Growth rate of FSRS is plotted as a function of a normalized wave number of low-frequency perturbations ($k_e c/\omega_{pe}$) for $\omega_0/\omega_{pe} = 10$. Each curve corresponds to a certain value of the decimal logarithm of the laser field amplitude $\log_{10}(A_0) = \log_{10}(eE_0/m\omega_0 c)$.

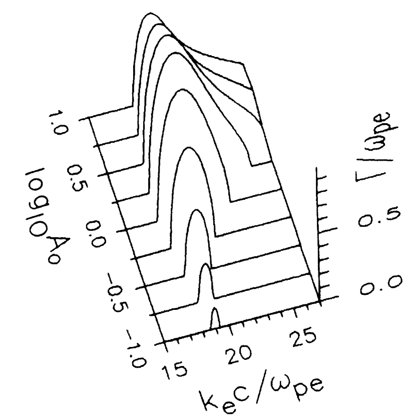

FIG. 2. Growth rate of BSRS is plotted as a function of a normalized wave number of low-frequency perturbations $k_e c/\omega_{pe}$ for the same parameters of plasma and pump field as in Fig. 1.

$$\Gamma_{Bm} = (\sqrt{3}/2)(\omega_0 \Omega_p^2/2)^{1/3}(A_0/\gamma_0)^{2/3} \ . \quad (3.4)$$

With the enlargement of the field amplitude, the growth rate reaches its absolute maximum at $A_0 = 2$ [14] and slowly decreases (as $A_0^{-1/3}$) for $A_0 \to \infty$. In Fig. 2, the value of Γ_B is plotted as a function of the wave number for several values of the pump amplitude.

The result that follows from our linear dispersion analysis is that the SRS instability has the maximum growth rate for the value of A_0 that is slightly above 1. For this value, the instability range also has a maximum width in a wave-number space. This contradicts the conclusion of the authors of the paper [9] where it was stated that growth rates of instabilities always tend to grow with increasing pump amplitude.

It is worth noting that the BSRS is a three-wave process. Concerning the direct forward SRS and RMI, we have here a four-wave process because both the down-shifted ("red") and up-shifted ("blue") scattered waves are resonant and should be taken into consideration. This deserves mention because in the early papers, when discussing electromagnetic instabilities in a plasma (see, for example, Refs. [2,4,8]), the term "Raman scattering" was associated with a three-wave process. However, the other authors [1,7] also used this term for a four-wave scattering process. Here we keep to the definition used in the book [1].

IV. FORWARD SRS FOR A PULSE OF A FINITE DURATION

For FSRS the scattered waves propagate in the same direction as the pulse itself. In a rarefied plasma ($\omega_{pe} \ll \omega_0$), their frequencies and wave numbers only differ slightly from those of the pulse. Consequently, the same is true for the phase and group velocities of the pump and scattered light. Hence, we shall look for the field of scattered waves in the form

$$A_1=\tfrac{1}{2}\{e_0 A_1(x,t)\exp[i\psi_0(x,t)]+\text{c.c.}\}\ , \tag{4.1}$$

where the phase $\psi_0(x,t)$ is the same as for the pump wave and $A_1(x,t)$ is a slow varying complex amplitude of the scattered light.

Assuming that the time changes of A_1 in the pulse frame of reference are fairly fast,

$$(\partial/\partial t+V_{\text{gr}}\partial/\partial x)A_1 \gg (\Omega_p^2/\omega_0^2)c\partial A_1/\partial x \tag{4.2}$$

(where $V_{\text{gr}}=c^2k_0/\omega_0$ is a group velocity of the pulse), we can neglect the linear dispersion effects. On getting a final solution, it is easy to find that this inequality holds true for the not too small pulse amplitude

$$A_0 \gg \omega_p/\omega_0\ . \tag{4.3}$$

Note that this condition on the pulse amplitude is opposite that obtained in Ref. [23], where the main focus was on the linear spreading. In other words, while the inequality (4.2) [or (4.3)] holds true, the growth of short-wavelength perturbations (with the wave number about K_p) is not suppressed or strongly affected by their linear spreading.

Neglecting the linear dispersion, we can also ignore the small deviations of group velocities of the main and scattered radiation from the speed of light in a vacuum $(c-V_{\text{gr}}\simeq c(\Omega_p^2/\omega_0^2)/2 \ll c)$. Within this approximation the carrier frequency and wave number are related just as $\omega_0=k_0c$, and the pump amplitude A_0 and phase ψ_0 depend only on $\xi=x-ct$. Further on, we use the variables ξ and t and rewrite the derivatives in the laboratory frame $\partial/\partial t$ and $\partial/\partial x$ in the form $\partial/\partial t-c\partial/\partial\xi$ and $\partial/\partial\xi$, respectively.

The results of the homogeneous case analysis of the previous section indicate that the FSRS instability can develop only from a seed, with the perturbation wave numbers close to the resonant value K_p. Consequently, we look for the electron-density perturbations and the scattered em wave amplitude in the form

$$N_1=\tfrac{1}{2}\{N(\xi,t)\exp[i\eta(\xi)]+\text{c.c.}\}\ , \tag{4.4}$$

$$A_1=A_+(\xi,t)\exp[i\eta(\xi)]+A_-(\xi,t)\exp[-i\eta(\xi)]\ . \tag{4.5}$$

Here we separate the phase of the "fast" (with $v_{\text{ph}}=c$) Langmuir wave $\eta(\xi)=\int^{\xi}K_p(\xi')d\xi'$. We denote as A_+ and A_- the amplitudes of blue and red satellites forming the amplitude of the scattered wave A_1.

Using the inequality (4.2), and assuming the envelopes N and $A_\pm$ to vary slowly in time and space [compared to the period of Langmuir oscillations $\partial/\partial t, c\partial/\partial\xi \ll \Omega_p(\xi)$], we reduce Eqs. (2.10) and (2.11) to the form

$$\partial A_+/\partial t=-\frac{i}{4}(\Omega_p^2/\omega_0)A_0N\ , \tag{4.6}$$

$$(\partial/\partial t-c\partial/\partial\xi)(\Omega_p^{1/2}N)=-\frac{i}{4}(\Omega_p^{3/2}A_0/\gamma_0^2)(A_+ + A_-^*)\ , \tag{4.7}$$

where A_+ and A_-^* are connected by a relation

$$(\omega_0+\Omega_p)\partial A_+/\partial t=-(\omega_0-\Omega_p)\partial A_-^*/\partial t\ . \tag{4.8}$$

From the set of equations (4.6)–(4.8), the equation that describes the linear stage of the FSRS instability for the quantity $Q\equiv\Omega_p^{1/2}N$ readily follows:

$$\partial/\partial t(\partial/\partial t-c\partial/\partial\xi)Q=\Gamma_0^2(\xi)Q\ , \tag{4.9}$$

where $\Gamma_0^2(\xi)\equiv\frac{1}{8}(\Omega_p^4/\omega_0^2)(A_0^2/\gamma_0^2)$. The general solution of Eq. (2.1) has the form

$$Q(\xi,t)=Q_0(\xi+ct)+\frac{1}{c}\int_{\xi}^{\xi+ct}\left[Q_0(\xi')\frac{\partial}{\partial t}I_0(Z)+\dot{Q}_0(\xi')I_0(Z)\right]d\xi'\ , \tag{4.10}$$

where the following notations are used:

$$Q_0\equiv\Omega_p^{1/2}N|_{t=0}\ ,$$

$$\dot{Q}_0\equiv(\partial/\partial t-c\partial/\partial\xi)Q|_{t=0}=-\frac{i}{4}(\Omega_p^{3/2}A_0/\gamma_0^2)(A_+ + A_-^*)|_{t=0}\ , \tag{4.11}$$

$$Z(\xi,\xi',t)=(2/c)\left[(\xi+ct-\xi')\int_{\xi}^{\xi'}\Gamma_0^2(\xi'')d\xi''\right]^{1/2}\ ,$$

and $I_0(Z)$ is the modified Bessel function.

From the form of the obtained solution it follows that the initial values of N, A_+, A_- determine completely the instability development. Either the initial pulse amplitude modulation or the electron-density perturbations met by the pulse can act as a seed for the instability. An electron-density perturbation (with $v_{\text{gr}}=0$) met by the pulse can grow, but it leaves the pulse in the end. As for the scattered em wave produced by these density perturbations, it remains traveling with the pulse and can grow.

The typical features of the obtained solution (4.10) can be demonstrated by taking as an example the simplest case of the initial perturbations of the form $N|_{t=0}=N_0\exp(i\kappa\xi)$ and $A_+|_{t=0}=A_-|_{t=0}=0$, where $\kappa=k_e-K_p$ is the suggested detuning between the electron-density perturbation with a wave number k_e and the resonant value of the wave number K_p.

In the initial stage (for $t\ll\tau_0$, while the change in A_0 in every fixed point x due to pulse displacement can be ignored) the instability develops just like in the homogeneous case (see Sec. III), but now the growth rate starts to depend on the local value of the pulse amplitude $A_0(\xi)$.

In the stage of a developed FSRS instability (for $t\gg\tau_0$ and $t\tau_0\Gamma_0^2\gg 1$) the resonant initial perturbation with $\kappa=0$ (which has a phase velocity equal to c) leads to the growth of electron-density perturbations that depend on time t and position ξ as

$$N\simeq N_0\exp\left[(2/c)\left[(\xi+ct)\int_{\xi}^{\infty}\Gamma_0^2(\xi')d\xi'\right]^{1/2}\right]$$
$$\propto\exp[\alpha(\xi)t^{1/2}]\ . \tag{4.12}$$

This already differs considerably from the usual exponential growth predicted by a homogeneous case analysis.

The nonresonant initial perturbations [with $\kappa>2\Gamma_0/c$,

$(\tau_0 c)^{-1}$] can also act as a seed to produce growing resonant perturbations because of the pulse amplitude inhomogeneity [i.e., the inhomogeneity in the driving term in Eq. (4.9)]. But in this case, the time required to produce considerable changes grows exponentially with an increase of the detuning κ.

The scattered em daughter wave and the beats of the pulse amplitude grow also according to the law (4.12). Behind the pulse (where $A_0 \to 0$) the solution (4.12) describes the plasma wake field produced in the course of the FSRS instability development. The amplitude of this wake field grows in time as $\exp[\{(4t/c)\int_{-\infty}^{\infty}\Gamma_0^2(\xi')d\xi'\}^{1/2}]$.

V. BACKWARD SRS FOR A PULSE OF A FINITE DURATION

BSRS is an essentially convective process with respect to the pulse, as both the enhanced electron-density perturbations and the scattered light are left behind the pulse. New portions of electron-density perturbations should enter through the leading edge of the moving pulse to provide permanent scattering. For harmonic perturbations ahead of the pulse and in the pulse frame of reference, the problem of the BSRS can be treated as a boundary [5] and, at least for time greater than τ_0, it is natural to look for a steady-state (in the pulse frame) solution to Eqs. (2.10) and (2.11). Consequently, we take N_1 and A_1 in the form

$$N_1 = \tfrac{1}{2}\{N(\xi)\exp[i\psi_e(x,t)] + \text{c.c.}\}\ , \tag{5.1}$$

$$\mathbf{A}_1 = \tfrac{1}{2}\{e_0 A(\xi)\exp[i(\psi_0(x,t) - \psi_e(x,t))] + \text{c.c.}\}\ , \tag{5.2}$$

where the amplitude of perturbation depends only on ξ, and $\psi_0(x,t)$ is a pulse field phase [see formula (2.5)].

Ahead of the pulse (where $\mathbf{A}=0$) we join a solution for N_1 with a seed plasma wave that has a wave number k_e:

$$N_e = \tfrac{1}{2}[N_0\exp(-i\omega_{pe}t + ik_e x) + \text{c.c.}]\ . \tag{5.3}$$

In contrast with the case of the previous section, a wave number of the perturbation is not too small here: $k_e \gg \omega_{pe}/c$.

Note that the problem can be easily generalized for the case of a multimode seed,

$$N_e = \tfrac{1}{2}\left[\sum_i N_0(k_{ei})\exp(-i\omega_{pe}t + ik_{ei}x) + \text{c.c.}\right]\ . \tag{5.3a'}$$

Linear approximation in the amplitude of perturbation results in independent amplification of each mode from the sum.

Let us take ψ_e in the form

$$\psi_e = (k_e - \omega_{pe}/c)x \equiv \kappa x\ . \tag{5.4}$$

Neglecting the linear dispersion effects ($\omega_0 = ck_0$) and substituting N_1 and $\mathbf{A}_1$ in the forms (5.1) and (5.2) into Eqs. (2.10) and (2.11) yields

$$(\partial^2/\partial\xi^2 + K_p^2)N = -\tfrac{1}{2}(A_0/\gamma_0^2)(\kappa - i\partial/\partial\xi)^2 A^*\ , \tag{5.5}$$

$$[\partial/\partial\xi - i(k_0 - \kappa/2)]A^* = \frac{i}{4}(K_p^2/\kappa)A_0 N\ . \tag{5.6}$$

The boundary conditions for this set are

$$A(\xi)|_{\xi=+\infty} = 0,\quad N(\xi)|_{\xi=+\infty} = N_0\exp(i\omega_{pe}\xi/c)\ . \tag{5.7}$$

The general analysis of Eqs. (5.5) and (5.6) shows that the perturbation amplitude tends to grow from the leading front into the interior of the pulse. It is convenient to write the amplitude of perturbations in the form

$$|N| = N_0\exp\int_{\xi}^{\infty} q(\xi')d\xi'\ . \tag{5.8}$$

Here we put into consideration a new quantity $q(\xi)$ that is a spatial growth rate, which depends on the local value of the pulse amplitude $A_0(\xi)$.

The most interesting case corresponds to the spatial growth that is much higher than the space variations of the pulse amplitude. In this case the simplified expressions for q can be obtained for pulse regions with certain ranges of intensity.

For the "weak" intensity pulse region $[A_0 < (\omega_{pe}/\omega_0)^{1/2}]$ we have

$$q = \mathrm{Re}\{[\tfrac{1}{8}(\omega_0\Omega_p/c^2)(A_0^2/\gamma_0^2) - \tfrac{1}{4}\Delta^2(\xi)]^{1/2}\}\ . \tag{5.9}$$

Here $\Delta(\xi)$ stands for the local detuning: $\Delta(\xi) \equiv (k_e - 2k_0(\xi) + \omega_{pe}/c)/2$.

And for the "strong" intensity region $[A_0 \gg (\omega_{pe}/\omega_0)^{1/2}]$ we obtain

$$\begin{aligned} &q \simeq \frac{\sqrt{3}}{2}q_0 \gg K_p \quad \text{for} \quad |\Delta| \le q_0\ , \\ &q \simeq (q_0^3/\Delta)^{1/2} \quad \text{for} \quad q_0 < \Delta \le \tfrac{1}{4}k_0 A_0^2/\gamma_0^2\ , \\ &q = 0 \quad \text{for} \quad \Delta < -\frac{3}{2^{2/3}}q_0\ , \end{aligned} \tag{5.10}$$

where

$$q_0^3(\xi) \equiv \tfrac{1}{4}(\omega_0\Omega_p^2/c^3)(A_0^2/\gamma_0^2) = \tfrac{1}{4}K_p^3(\omega_0/\Omega_p)(A_0^2/\gamma_0^2)\ .$$

The thresholds for the BSRS instability can be defined as

$$J \equiv \int q\, d\xi \ge 1\ , \tag{5.11}$$

where J is an integral amplification factor. It follows from Eq. (5.10) that for a sufficiently long pulse ($\tau_0 \gg \omega_{pe}^{-1}$) with $A_0 \ge 1$ and $\omega_0 = \text{const}$, the threshold condition (5.11) is always satisfied.

In this section we do not suggest initially that the local value of the pulse carrier frequency and corresponding wave number [see definitions (2.6)] are constant over the whole pulse. Let us find out whether a frequency modulation can suppress the instability. Variations in the local value of the pulse carrier frequency ω_0 and corresponding wave number $k_0(\xi)$ cause the change in the local detuning $\Delta(\xi)$. Since that is the case, q turns out to be not equal to zero only in limited parts of the pulse. Supposing, for example, that, for $\partial k_0/\partial\xi = \text{const}$ in the vicinity of a single point with $\Delta=0$, we can obtain an estimation for the integral amplification factor

$$J \simeq \tfrac{1}{2}|\partial k_0/\partial\xi|^{-1}(\omega_0\Omega_p/c^2)(A_0^2/\gamma_0^2)\ . \tag{5.12}$$

Then the threshold condition $J \ge 1$ can be rewritten as

$$\omega_0^{-1}|\partial\omega_0/\partial\xi| \le \tfrac{1}{2}K_p(A_0^2/\gamma_0^2) \ . \tag{5.13}$$

As can be seen from the inequality (5.13), even a moderate gradient of ω_0 can suppress the instability for a pulse of small amplitude ($A_0 \ll 1$). As for relativistic intensities, $A_0 \ge 1$, the phase variation allowed by the conditions (2.7) (i.e., when our theory is still valid) fails to suppress BSRS.

If there are several points in a frequency modulated pulse with $\Delta=0$, the estimations of the amplification factor are more complicated and should similarly include the correlation of the phase of the amplificated Langmuir wave in different points, which, in its turn, can strongly depend on the pulse amplitude.

From Eq. (5.6) we can obtain the matching condition that determines the frequency of the scattered wave ω_1 behind the pulse (where $A_0=0$, $A_1 \propto \exp[-i\omega_1 t + ik_1 x]$) as a function of the wave number of a seed Langmuir wave ahead of the pulse:

$$\omega_1 \simeq |k_e c - \omega_{pe}|/2 \ . \tag{5.14}$$

The last relation also can be easily obtained with the help of simple considerations. In the pulse frame of reference, the Doppler-shifted frequencies of the plasma seed wave and those of the scattered em wave are equal: $\omega_1 - k_1 c = |\omega_{pe} - k_e c|$. Substituting here the wave number of the scattered light $k_1 \simeq -\omega_1/c$ yields again the relation (5.14). Note that the formula (5.14) determines the frequency but not the intensity of the scattered light. For small pulse amplitudes $[A_0 < (\omega_{pe}/\omega_0)^{1/2}]$ from the expression (5.9) it follows that only the perturbations with the wave numbers fairly close to the resonant ($k_e \simeq 2k_0 - \omega_{pe}/c$) could produce a scattered light with considerable efficiency. Consequently, according to (5.14), the backward-scattered light is down shifted: $\omega_1 \simeq \omega_0 - \omega_{pe}$. In contrast with this, for relativistically strong pulses there is also enhancement of the perturbations with $k_e > 2k_0$ [see the formula (5.10)]. As a result, the up-shifted light can exist in the scattered radiation [13].

We studied the solution of the set of equations (5.5) and (5.6) numerically for a laser pulse with a duration $\tau_0 = 10^3\omega_0^{-1}$ in a plasma with $\omega_0/\omega_{pe} = 100$. These parameters are fairly close to those of the experiment [13]. The seed waves were suggested to have a uniform spectral density $N_k^2 = \text{const}$ in k_e space for both $k_e > 0$ and $k_e < 0$. In Fig. 3, the decimal logarithm of the spectral density of the scattered radiation (normalized to N_k^2/c) is depicted as a function of a normalized scattered wave frequency (ω_1/ω_0). Every curve corresponds to a certain peak pulse intensity A_m.

At a low intensity ($A_m = 0.01$; curve 1), the initial seed plasma waves with $k_e \simeq \pm 2k_0 - \omega_p e/c$ (which correspond to the oppositely directed phase velocities) produce two peaks displaced to a value $\pm\omega_{pe}$ from the pulse frequency ω_0. There is no enhancement of the seed wave of electron-density perturbations. So it is just a backscattering by a given moving grating.

With the rise of the intensity ($A_m = 0.1$; curve 2), the

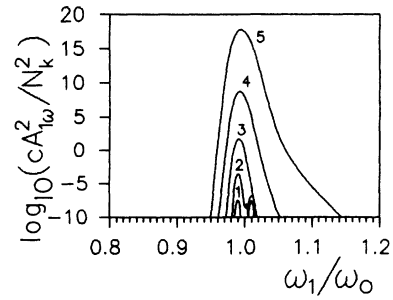

FIG. 3. Logarithm of the spectral density of backward-scattered light (normalized to N_k^2/c) is shown as a function of the normalized frequency of the scattered light (ω_1/ω_0). Pulse duration is $\tau_0 = 10^3\omega_0^{-1}$. Pulse envelope is taken in the form $A_0^2(\xi) = A_m^2[1-\cos(\pi\xi/\tau_0 c)]/2$ for $-2\tau_0 c < \xi < 0$, and $A_0^2 = 0$ for $\xi > 0$ and $\xi < -2\tau_0 c$. Uniform spectral density of seed perturbations is suggested. The ratio of carrier and electron plasma frequencies is 100. Different curves correspond to the following values of parameter A_m: $A_m = 0.01$ (curve 1), $A_m = 0.1$ (curve 2), $A_m = 0.25$ (curve 3), $A_m = 0.5$ (curve 4), and $A_m = 1$ (curve 5).

enhancement of a forward-moving seed wave appears. It gives rise to a growth and broadening of the left peak while there is a no serious enhancement in amplitude for the nonresonant seed wave with a negative phase velocity (right peak).

With the further rise of the pulse intensity ($A_m = 0.25$; curve 3), the growth and broadening of the left peak are in progress. For $A_m = 0.5$ and 1 the peak becomes much broader than even ω_{pe} (curves 4 and 5). The similar broad spectra of backscattered radiation, with both the down-shifted and up-shifted frequencies, were observed in the recent experiment with ultrastrong laser pulses [13].

Since the scattered em wave takes the energy from the pump pulse, BSRS should result in pump erosion with the rate proportional to $-\partial|A_1^2|/\partial\xi$. The scattered wave amplitude A_1 grows inside the pulse from the leading edge together with N until the validity of our solution fails at some point (further on we stand as ξ^*) due to plasma electron trapping in the field of the Langmuir wave. Even in a plasma with a fairly small electron temperature it certainly appears when the electron-density perturbations grow up to $N \simeq 1$. Note that the corresponding amplitude of the scattered wave $A_1(\xi^*)$ is still small compared to $A_0(\xi^*)$. As N cannot grow anymore for $\xi < \xi^*$, the further enhancing of A_1 is also suppressed behind ξ^*, so that the quickest erosion of the pulse may be expected to appear in the vicinity of ξ^*.

Using the local energy balance near the point ξ^* (where $N \simeq 1$), the characteristic time of the pulse profile steepening (up to a value $\partial \ln A_0/\partial\xi \simeq K_p$) can be estimated as

$$\Delta t \simeq 8\Omega_p^{-1}(\omega_0^2/\Omega_p^2) \ . \tag{5.15}$$

For $t > \Delta t$ our theory is no longer valid, as the inequalities (2.7) are broken. The jump formed on the pulse amplitude profile due to erosion begins to generate a plasma wake. More details of the 1D evolution of the relativistically strong pulse that excites a plasma wave can be found in Refs. [14,15].

The position of the point ξ^* with respect to the pulse edge is definitely determined by the pulse parameters and amplitude of the seed wave. For real plasmas, a seed perturbation for BSRS can be provided by thermal electron-density fluctuations. Since these fluctuations are essentially 3D in nature, we should make some speculations which are, in fact, already beyond the limits of the one-dimensional treatment of the present paper. However, all we need do in order to obtain the equation that approximately determines the position ξ^* is just accept (extrapolating the well-known result for a weak pump) that the spatial growth rate of backscattering is gradually decreasing with the deviation from the direct backward scattering. Under this natural assumption we obtain the equation for ξ^*:

$$2\int_{\xi^*}^{\infty} q(\xi')d\xi' \simeq \ln(N_D/(2k_0 r_D)^4(q(\xi^*)r_D)) , \quad (5.16)$$

where $r_D=(T_e/m_e\omega_{pe}^2)^{1/2}$ is the electron Debye length and $N_D=n_0 r_D^3$.

For relativistically strong ($A_0 \simeq 1$) and fairly long pulses with $\tau_0\omega_{pe} \gg 10$ (for example, for the laser and plasma parameters corresponding to the experimental conditions of Ref. [13]), it follows from (5.16) that the steepening discussed above of the pulse profile can develop on the leading edge of the pulse during a time interval determined by Eq. (5.15).

VI. CONCLUSIONS

Our analysis demonstrates that the Raman instability has a maximum growth rate for the pulse field intensity corresponding to $A \approx 1$. The same value of the pulse amplitude gives the maximum width of the instability in k space. It means that, already, available laser radiation with a peak power density of 10^{18} W/cm^2 and a wavelength of 1 μm (see, for example, Refs. [12,13]) corresponds to the maximum growth rate of the Raman instability and should be most severely affected by it.

The results of the present paper also assure us that, though both BSRS and FSRS instabilities play an important role in the laser-plasma interactions, they can be expected to affect very differently the evolution of a relativistically strong laser pulse in an underdense plasma.

The BSRS has a greater growth rate; but for a pulse of a limited duration the intensity of backscattered light stops growing in time due to the convective nature of the instability already up to the moment when the pulse has covered its own length. In fact, it means that our theory for BSRS is valid also for a sufficiently short laser pulse which does not change the shape significantly while covering its own length. Consideration of kinetic effects (see the above particle-trapping arguments) for a comparatively long pulse shows that BSRS can be expected to become saturated already in the leading part of such a pulse (see the example at the end of the previous section), where the scattered wave amplitude is still small. As a result, for ultrahigh pulse intensities, BSRS does not affect the body of the high-intensity pulse so badly as can be estimated by just basing it on the linear theory. In this case, it can be expected to mainly affect the evolution of the leading portion of the pulse radiation where it can produce an erosion of the pulse amplitude profile, as was demonstrated by PIC simulations [14,15]. Comparison between the times predicted by formula (5.15) and that of the formation of a jump on the amplitude profile in simulations [14,15] shows a good agreement of our theoretical predictions with the results of PIC simulations.

In contrast with BSRS, the FSRS in an underdense plasma $[(\omega_{pe}/\omega_0) \ll 1]$ has a moderate growth rate [compare (3.3) and (3.4)], but for every pulse intensity, it can develop in the whole body of the pulse and produce a considerable modulation of the whole pulse. The rear portions of the pulse radiation should be affected more strongly by the instability development than the pulse's leading part. As soon as the pulse modulation on an electron plasma frequency has developed, there should be an ultrastrong wake-field excitation resulting in the ultrafast pulse depletion [14].

Our theory does not include some important effects that are now under discussion related to short-pulse evolution in underdense plasmas. Recently, a number of papers have appeared [21,24–30] which put an accent—discussing the pulse evolution—on the essentially three-dimensional nature of the short-pulse evolution. However, these papers demonstrate that for pulses with a fairly large transverse spot size $R \gg (\omega_0/\omega_{pe})\tau_0 c$, the transverse evolution can be slow enough to provide the validity of 1D theory during the typical time of the longitudinal large-scale evolution of a relativistically strong pulse $t_{nl} \simeq (\omega_0/\omega_{pe})^2\tau_0$ [14] (certainly, the validity of the condition $R \gg c/\omega_{pe}$, for which the 1D approach can still be used, is required). In fact, the restrictions on our theory are not so rigorous since such comparatively short-wavelength instabilities as FSRS and BSRS can develop during a time fairly less than t_{nl}. If the condition $R \gg c/\omega_{pe}$ is fulfilled, the direct forward SRS instability of a relativistically strong pulse ($A \simeq 1$) can dominate in the initial stage of the pulse evolution over the transverse effects. Since BSRS already develops during the time of order τ_0, which for $R \gg (\omega_{pe}/\omega_0)\tau_0 c$ is less than that of the transverse pulse evolution, we again can expect the 1D approximation of a given pulse to be valid for BSRS description.

Analysis of the present paper adds some important details to a possible scenario of evolution of a relativistically intense pulse in an underdense plasma and indicates a possible important role of BSRS. The modulational version of FSRS considered in this paper can be, depending on the radiation power and plasma density, either enhanced or suppressed by the transverse pulse evolution [24–26,29,30] but once triggered off, it always tends to dominate at the later stages of the instability development. Let us consider the conditions under which the BSRS acts as a triggering process.

Probably the most interesting case with which to apply our theory corresponds to the initial parameters of the 3D pulse, which provides the balance between linear diffraction and relativistic self-focusing [24,30]. This case often is referred to as the optical guiding case [17]. It is also interesting since the BSRS instability considered in the present paper was mainly ignored in the equations that were studied in the papers on 3D dynamics. The authors of the papers [21,24–26,29,30] agreed that by a proper matching of the pulse parameters, the regime of optical guiding can be provided at a distance comparable with a Rayleigh length. For this case, we can find the conditions under which not self-focusing, but rather BSRS dominates in triggering off the pulse modulation. Comparison of the Rayleigh time $t_R = R^2\omega_0/(2c^2)$ with that given by the formula (5.15) leads to the condition providing a dominant role to BSRS in triggering off the pulse modulation process:

$$(K_p R) > 4\left[\frac{\omega_0}{\omega_{pe}}\right]^{1/2} . \tag{6.1}$$

Let us now find out the condition for which the triggering action of BSRS also dominates over the growth of initial (resonant for FSRS) perturbation with amplitude. Relations (4.12) and (5.15) give a simple limitation on the level of the initial seed resonant perturbation of density

$$\ln\left[\frac{n_0}{N_0}\right] > 2(\omega_{pe}\tau_0)^{1/2}\frac{A}{\gamma_0^{5/4}} . \tag{6.2}$$

Under these limitations, neither self-focusing nor the growth of initial perturbations leads to the generation of a resonant seed Langmuir wave, but, rather, the action of BSRS. This wave, being enhanced through the action of succeeding FSRS, gives a start to pulse modulation.

Since our theory incorporates only fairly long and smooth laser pulses, it does not concern LWFA (laser wake-field accelerator) [31,16,17] and PBWA (plasma beat wave accelerator) [31] concepts. Probably the most promising laser accelerating scheme is presently the self-resonant version of the laser wake-field accelerator suggested in [24,30]. The domination of BSRS for this scheme at the early stage of the pulse evolution is undesirable, as it can lead to early pulse modulation and thus stops accumulation of pulse energy on the axis. It may result in a reduction of the accelerating field. Hence, the parameters providing the condition inverse to (6.1) should be taken to avoid BSRS action for this advance accelerator scheme. For parameters suggested in the paper [24] (where the relativistic focusing time was less than t_R), according to the estimation (6.1), BSRS can still be ignored. For the parameters of numerical calculation in Ref. [30] [where the initial relativistic self-focusing stage corresponds to a period of time already equal to $2t_R$, and thus a factor 4 should be added in the left-hand side of (6.1)], BSRS action should already be taken into account as its action can dominate over self-focusing in triggering the pulse modulation.

Deviation from the direct forward scattering can certainly result in a rise in the instability growth rate (see, for example, [6]). We have not considered side scattering here. However, the simple considerations for a moderate intensity pulse [29] (taking into account both the growth rate of scattered light and its convection out of the limited interaction region associated with the pulse) indicate that in underdense plasmas the most intense amplification can be expected for forward or near forward scattering, which can be studied within 1D theory.

In this paper, we do not include the thermal effects when considering the SRS development. For the forward SRS, the thermal motion is of minor importance since the phase velocity of the resonant daughter Langmuir wave is about the speed of light c. The thermal limiting of BSRS can be ignored while the following condition [5] is satisfied: $(2v_{\mathrm{TE}}/c)^3 < (\omega_{pe}/2\omega_0)^2(v_E/c)^2$.

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