Kardar-Parisi-Zhang model and anomalous elasticity of two- and three-dimensional smectic-A liquid crystals

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We reveal a deep theoretical relationship between *equilibrium* statistical physics of smectic-A liquid crystals and *nonequilibrium* statistical physics of the Kardar-Parisi-Zhang *dynamical* model [M. Kardar, G. Parisi, and Y.-C. Zhang, Phys. Rev. Lett. **56**, 889 (1986)] for interfaces growing in the presence of a flux of incoming particles. This relationship provides an *exact* approach to study Landau-Peierls phenomena and anomalous elasticity of two-dimensional smectic-A liquid crystals. Also, it yields prediction of an unusual elastic critical point in three-dimensional smectic-A liquid crystals with broken inversion symmetry (head-to-tail packing of layers). We discuss the elasticity and fluctuations of these unusual smectic-A phases.

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I. INTRODUCTION

Soft-condensed-matter systems such as liquid crystals exhibit a variety of striking effects induced by thermal fluctuations. A classical example is the well-known Landau-Peierls divergence of smectic-A displacement fluctuations which destroys true long-range translational order in these phases. For the smectic layer displacement u, Landau and Peierls found, for *three-dimensional* (3D) smectic-A liquid crystals,

$$\langle u^2 \rangle \sim \log(L) ,$$

where L is the size of the smectic sample [1].

Recently, some 60 years after Landau and Peierls, we proposed a theoretical approach that enables an essentially *exact* treatment of similar fluctuation phenomena in *two-dimensional* (2D) smectics [2]. The complexity of the 2D case, in comparison to the situation in 3D, arises from the breakdown of the linear (Hookean) elasticity in smectic-A liquid crystals. This phenomenon exists already in 3D smectic-A liquid crystals [3], but in a much weaker form. Grinstein and Pelcovits [3] demonstrated that nonlinear effects of thermal fluctuations cause, at long length scales, a nontrivial logarithmic renormalization of 3D smectic elastic constants inducing, in particular, a breakdown of the classical elasticity theory: Linear Hooke's law, (*strain*)$\sim$(*stress*), usually considered to be valid for weak external stresses, is replaced by an anomalous, nonlinear strain response to *arbitrarily* weak stresses. Such a breakdown of linear elasticity theory exists also in 2D smectics. However, it is much more severe than in 3D, since these systems are well below the critical dimension $d_c = 3$. Below d_c harmonic-fluctuation theory, used by Landau and Peierls in 3D, breaks down, at long length scales, due to anharmonic fluctuations. Thus a nonperturbative approach of Ref. [2], described here in more detail, is necessary to obtain reliable information about 2D smectics. This approach is of interest not only for smectic-A liquid crystals. Similar anharmonic phenomena exist in other liquid crystals, as well as in spin glasses, exotic magnets, membranes, and nematic polymers [4]. These phenomena are present in seemingly remote physical problems such as Rayleigh-Bénard roll instability [5], (analog of 2D smectic liquid crystals [6]) and pion condensation in neutron stars (analog of 3D smectic liquid crystals [7]).

On the other side, a similar breakdown of the classical, harmonic fluctuation theory is the main theme of another, rather different area, namely, the statistical physics of growing interfaces [8]. This field is actively being developed, in particular, due to a direct interest in the dynamical scaling properties of interfaces, e.g., of solid clusters growing in the presence of external fluxes of depositing particles [9]. Also, a strong reason for a broad interest in this field is its relationship to a number of other physical problems [8,10].

Here we describe a deep relationship between *nonequilibrium* statistical physics of growing interfaces and *equilibrium* statistical physics of smectic-A liquid crystals [2]. We relate the Kardar-Parisi-Zhang (KPZ) model [9] for the dynamics of $(d-1)$-dimensional growing interface to *nonlinear* elasticity theory of d-dimensional smectic-A liquid crystals. For 2D smectic liquid crystals [6], this relationship enables us to obtain the *exact* scaling behavior of the nonlinear elasticity theory, and to study Landau-Peierls phenomena in these systems. For a sample of a 2D smectic liquid crystal in the (x,z) plane (with layers perpendicular to the z axis), with sizes L_x and L_z, we find smectic displacement fluctuations strongly diverging as *power laws* of the system sizes,

$$\langle u^2 \rangle \sim L_x ,$$

for $L_x \ll \text{const} L_z^{2/3}$, whereas

$$\langle u^2 \rangle \sim L_z^{2/3} ,$$

for $L_x \gg L_z^{2/3}$. The divergence is thus much stronger than the logarithmic Landau-Peierls divergence of 3D smectic-A liquid crystals. A striking consequence of our results is that the classical, linear Hooke's law, $(strain) \sim (stress)$, breaks down and is replaced by the nonlinear law, $(strain) \sim (stress)^{2/3}$, for *arbitrarily weak* external stresses normal to layers, when one would normally expect the linear elasticity response to be valid. These results hold for the *stricto sensu* elastic model with no dislocations, which are actually free in a 2D smectic liquid crystal and convert it, at any finite temperature T into a nematic phase at length scales longer than the sizes of cybotactic groups ξ_x and ξ_z ($\xi_x, \xi_z \to \infty$ as $T \to 0$) [6]. Our theory enables us to obtain the exact form of the *anisotropic scaling* at the zero T transition from the 2D smectic to the nematic phase, of the form $\xi_z \sim \xi_x^{3/2}$. All these results are of interest not only for 2D smectic liquid crystals, but also for their analogs such as Rayleigh-Bénard systems [5], or stripe domain phases in thin ferromagnetic films in which smecticlike anisotropic scaling has also been observed recently [11].

An important byproduct of our theory is the elastic model for smectic-A liquid crystals with *broken spatial-inversion symmetry* [12,13], such as ferroelectric smectic-A liquid crystals with an average dipolar moment normal to the layers, resembling a lamellar phase of surfactant monolayers stacked according to "head-to-tail" rule [12]. Numerous molecular architectures capable of forming such phases were proposed in the past [13]. Their experimental 3D realization is, however, of a quite recent date [14]. Our elastic model of these phases contains a new, rotationally invariant term of the form eH, coupling the strain e and the layer curvature H. This term is forbidden in ordinary, inversion-symmetry smectic-A liquid crystals since it changes sign under spatial inversion. We find that the KPZ model in $2+1$ dimensions maps into an unusual *elastic critical point* of 3D smectic-A liquid crystals with broken inversion symmetry. At this point, displacement fluctuations strongly diverge as *power laws* of the system sizes L_x and L_z: for $L_z \gg L_x$, $\langle u^2 \rangle^{1/2} \sim L_x^{\alpha}$, $\alpha \approx 0.4$; for $L_x \gg L_z$, $\langle u^2 \rangle^{1/2} \sim L_z^{\beta}$, $\beta \approx 0.25$, in contrast to a much weaker Landau-Peierls logarithmic divergence in inversion symmetric smectic liquid crystals [1,15].

This paper is organized as follows. In Sec. II we discuss a nonlinear elastic model of smectic-A liquid crystals with broken inversion symmetry. In Sec. III we describe the relationship between smectic liquid crystals and the KPZ model. In Sec. IV we discuss 2D smectic liquid crystals. In Sec. V, the unusual elastic critical point in 3D smectic-A liquid crystals with broken inversion symmetry is discussed. Section VI outlines results of renormalization-group analysis of smectic-A liquid crystals with broken inversion symmetry. In Sec. VII, we discuss recent studies inspired by our results presented here and, in a brief form, in Ref. [2]. In particular, we discuss consequences of our results for the hydrodynamics of 2D smectic liquid crystals studied by Langer, Liu, and Toner, as well as the work of Kashuba and Pokrovsky, applying our results to stripe domain structures in thin ferromagnetic films. In Sec. VII we also discuss some unusual liquid-crystal phases that may occur in 3D smectic liquid crystals with broken inversion symmetry. In Appendix A we discuss a number of important details, such as the long-length-scale renormalization of smectic elastic constants and the derivation of the aforementioned anomalous stress-strain relationships. In Appendix B we discuss renormalization-group analysis of the elastic model of the smectic-A liquid crystals with broken inversion symmetry.

II. NONLINEAR ELASTICITY OF SMECTIC-A LIQUID CRYSTALS WITH BROKEN INVERSION SYMMETRY

We start by reviewing nonlinear elastic models of smectic-A liquid crystals. The model described in the following is, however, a more general one and applies also to smectic-A liquid crystals with broken inversion symmetry and screened long-range dipolar interactions [12–14]. Any configuration of smectic layers can be described by giving their heights $h_n(\mathbf{x})$, where $\mathbf{x}$ is $(d-1)$-dimensional "substrate" coordinate parallel to layers [Fig. 1(a)]. The nth layer fluctuates around the average height $z = nl$, where l is the equilibrium layer thickness. Thus the layer displacement $u_n(\mathbf{x}) = h_n(\mathbf{x}) - nl$ is, on average, zero. In a continuum limit, one can replace $h_n(\mathbf{x})$ by a continuous field $h(\mathbf{x},z) = z + u(\mathbf{x},z)$, which thus describes a fluctuation of a smectic layer having the average height z, Fig. 1(a).

The elastic energy of this stack of layers can be represented via a sum of layers' energies,

$$E = \sum_n E_n = \int \frac{dz}{l} E_{z/l} ,$$

where E_n is the sum of compressional, $E_{n,\mathrm{com}}$, and a curvature contribution, $E_{n,\mathrm{curv}}$. The compressional contribution $E_{n,\mathrm{com}}$ is generally of the form

$$E_{n,\mathrm{com}} = \frac{B}{2} \int dS e^2 ,$$

where the integral is over the area of the nth layer,

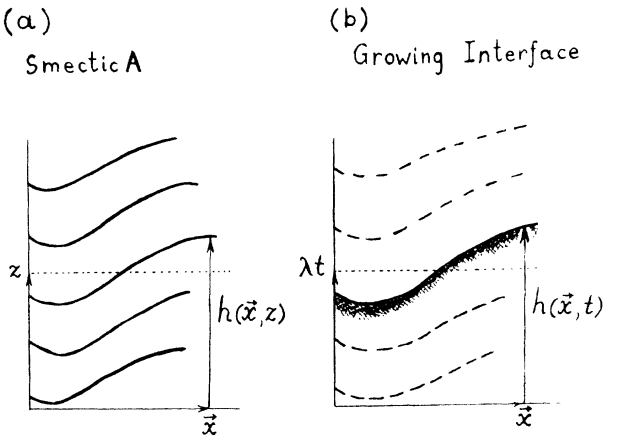

FIG. 1. (a) Smectic layer described by its height function. (b) Growing interface described by its height function. Compare snapshots of the growing interface, taken at equal time intervals [dashed lines in (b)] with the stack of smectic layers in (a).

specified by its height $h(\mathbf{x},z)_{z=nl}$, $dS=d^{d-1}\mathbf{x}(1+(\nabla_x h)^2)^{1/2}$, and

$$e=\partial_z h(1+(\nabla_x h)^2)^{-1/2}-1$$

is the Lagrangian strain [the displacement $u(\mathbf{x},z)=h(\mathbf{x},z)-z$ changes layer thickness, measured normal to the layer, by $\delta l=el$]. Smectic layers are fluid, and the layer curvature energy $E_{n,\mathrm{curv}}$ is, as for fluid membranes [16]

$$E_{n,\mathrm{curv}}=\frac{K}{2}\int dS(H^2+2H_0H)\ ,$$

with H, the layer curvature,

$$H=\nabla_x(\nabla_x h/[1+(\nabla_x h)^2]^{1/2})\ ,$$

while the constant H_0 is the so-called *spontaneous curvature*. Its presence *breaks the inversion symmetry* $(h,\mathbf{x})\rightarrow(-h,-\mathbf{x})$. A nonzero H_0 can arise only if nematogens do not have a center of inversion ("head-and-tail" molecules) *and* if most of the tails are on one while most of the heads are on the other layer side. Such a layer *alone* would tend to bend towards one, say the tail side. Now, let us stack the layers so that the head side of a layer is adjacent to the tail side of its neighbor ("head-to-tail" rule [10]). By summing layers' energies,

$$E_{\mathrm{Sm}}=\sum_n E_n=\sum_n E_{n,\mathrm{com}}+E_{n,\mathrm{curv}}\ ,$$

we obtain the *full* nonlinear, rotationally invariant smectic elastic Hamiltonian

$$\begin{aligned}E_{\mathrm{Sm}}&=\int dz\,d^{d-1}\mathbf{x}(1+(\nabla_x h)^2)^{1/2}\\&\quad\times\left[\frac{B_{\mathrm{Sm}}}{2}e^2+\frac{K_{\mathrm{Sm}}}{2}H^2+\gamma_{\mathrm{Sm}}H\right]\\&=\int dz\,d^{d-1}\mathbf{x}(1+(\nabla_x h)^2)^{1/2}\\&\quad\times\left[\frac{B_{\mathrm{Sm}}}{2}e^2+\frac{K_{\mathrm{Sm}}}{2}H^2-\gamma_{\mathrm{Sm}}eH\right]\ .\end{aligned}\tag{1}$$

Here $B_{\mathrm{Sm}}=B/l$, $K_{\mathrm{Sm}}=K/l$, and $\gamma_{\mathrm{Sm}}=KH_0/l$. The two forms of H_{Sm} in Eq. (1) are related by a partial integration, i.e., they differ by a surface term not affecting bulk fluctuations. The eH term in (1), with $\gamma_{\mathrm{Sm}}\sim H_0$, is *odd* under spatial inversion $h(\mathbf{x},z)\rightarrow -h(-\mathbf{x},-z)$ under which $e\rightarrow e$, $H\rightarrow -H$. $\gamma_{\mathrm{Sm}}=0$ in ordinary, symmetric smectic-A liquid crystals either because the heads (tails) are equally distributed between layer sides (so $H_0=0$), or, if this is not the case, asymmetric layers are arranged head-to-head, tail-to-tail, so that spontaneous curvature contributions of neighboring layers cancel.

In 2D (and *only* in 2D) the γ_{Sm} term in the smectic Hamiltonian (1) becomes a derivative contributing only to the boundary energy. Note that the spontaneous curvature of each layer contributes the energy $\sim\int dx(1+(\partial_x h)^2)^{1/2}H=\int ds(\partial\theta/\partial s)$, with s, the arc length $[ds=dx(1+(\partial_x h)^2)^{1/2}]$, θ the local layer tilt angle $\theta=\tan^{-1}(\partial_x h)$, and $H=\partial\theta/\partial s$. This is purely a boundary term. Thus, the inversion symmetry breaking term in the smectic Hamiltonian (1), does not affect bulk fluctuations in 2D smectic-A liquid crystals. From the point of view of bulk phonons, the ordinary ($\gamma_{\mathrm{Sm}}=0$) and asymmetric smectic-A liquid crystals ($\gamma_{\mathrm{Sm}}\neq 0$) are *identical* in 2D. This special property of two-dimensional smectic-A liquid crystals plays an important role in the following.

III. KPZ MODEL AND ITS RELATIONSHIP TO SMECTIC-A LIQUID CRYSTALS

Now, let us turn to a seemingly rather different class of physical problems, the dynamics of growing interfaces [8]. For a growing interface, the interesting quantity is its profile described by height function $h(\mathbf{x},t)$ of $(d-1)$ substrate coordinates $\mathbf{x}$ and one time coordinate t, Fig. 1(b). In this section we will relate the nonlinear smectic Hamiltonian (1) to the KPZ model [9] for the evolution of the profile $h(\mathbf{x},t)$ of a $(d-1)$-dimensional growing interface. The full nonlinear, rotationally invariant dynamical model for the interface of an isotropic (amorphous) cluster, growing in an isotropic flux of depositing particles, has the form

$$\frac{\partial h}{\partial t}=(1+(\nabla_x h)^2)^{1/2}(\lambda+\nu H)+[1+(\nabla_x h)^2]^{1/4}\eta(\mathbf{x},t)\ ,\tag{2a}$$

where λ is the (bare) mean velocity of the interface. In the following, we chose a time unit such that $\lambda=1$. ν, in Eq. (2a), is a surface tension, H is the interface curvature (as defined in Sec. II), and η is a white spatially uncorrelated noise, with the distribution

$$P(\eta)\sim\exp\left[-\frac{1}{4D}\int dt\,d^{d-1}\mathbf{x}\eta^2(\mathbf{x},t)\right]\ .\tag{2b}$$

The full nonlinear growth model (2) is, in the literature, usually given in a truncated form (see Appendix A).

The most interesting aspect of the KPZ model in $1+1$ and $2+1$ dimensions is that it generally (for any values of its parameters) yields *rough* interfaces with correlations of the displacements $u(\mathbf{x},t)=h(\mathbf{x},t)-t$, of the form

$$K(\mathbf{x},t)=\langle(u(\mathbf{x},t)-u(\mathbf{0},0))^2\rangle^{1/2}=|\mathbf{x}|^{\alpha}\phi(|t|/|\mathbf{x}|^{\alpha/\beta})\ ,\tag{3}$$

where $\phi(s)\rightarrow s^{\beta}$ for $s\rightarrow\infty$, while $\phi\rightarrow$const for $s\rightarrow 0$. Thus, $K(\mathbf{x},0)\sim|\mathbf{x}|^{\alpha}$, and $K(\mathbf{0},t)\sim|t|^{\beta}$. The values of the exponents α and β are *exactly* known in $1+1$ dimensions: $\alpha=\frac{1}{2}$ and $\beta=\frac{1}{3}$ [9]. In other dimensions, numerical estimates are available [8]. Nonetheless, there exists an exact relationship between α and β,

$$\beta=\frac{\alpha}{2-\alpha}\ ,$$

see Ref. [9] and Appendix B.

To relate the KPZ model (2) to the smectic model (1), we will perform two steps: (1) The first step is simply a renaming of variables: we will identify the *time* coordinate t of the KPZ model with the smectic *spatial* coordinate z,

$$t=z\ .$$

This is motivated by a geometric similarity between the smectic and the interfacial problem [compare Figs. 1(a) and 1(b)]. (2) Next, we map the *dynamical* problem in Eqs. (2a) and (2b) in $d-1$ spatial dimensions ($\mathbf{x}$) into an *equilibrium* statistical-mechanics problem for the field $h(\mathbf{x},z)$ in d spatial dimensions ($\mathbf{x},z$). This can be accomplished by applying the classical dynamics path-integral formalism [17], yielding the probability weight $P(h)$ of a field configuration h. The field $h(\mathbf{x},z)$ can be interpreted now as the smectic layer height function. Thus, we identify the sequence of snapshots of the KPZ interfaces taken at equal time intervals with a stack of smectic layers [compare Figs. 1(a) and 1(b)]. $P(h)$ is simply obtained from $P(\eta)$ in (2b), by changing variable $\eta \rightarrow h$. The KPZ equation (2a), with $\lambda=1$, can be rewritten as

$$\eta(h)=[1+(\nabla_x h)^2]^{1/4}(e-\nu H) ,$$

where e and H are smectic local strain and layer curvature as defined in Sec. II. Thus, $P(h)=P(\eta)J(h)$, with $J(h)=|D\eta/Dh|$. For the moment, for simplicity, let us ignore the Jacobian J (for its detailed discussion see Appendix A). Then, after inserting the above expression for $\eta(h)$ into Eq. (2b), one obtains $P(h)$ in the form of a Boltzmann factor, $P(h)\sim\exp[-H_{\rm eff}(h)]$, with the effective Hamiltonian

$$H_{\rm eff}(h)=\frac{1}{4D}\int dz\, d^{d-1}\mathbf{x}(1+(\nabla_x h)^2)^{1/2}[e-\nu H]^2 . \quad (4)$$

Note that $H_{\rm eff}(h)$ is equivalent to the smective Hamiltonian (1) for a special choice of $B_{\rm Sm}$, $K_{\rm Sm}$, and $\gamma_{\rm Sm}$, ensuring that the expression in square brackets in the second line of Eq. (1) is, as in Eq. (4), a *full square,*

$$\frac{B_{\rm Sm}}{2}e^2-\gamma_{\rm Sm}eH+\frac{K_{\rm Sm}}{2}H^2=\frac{1}{4D}[e-\nu H]^2 .$$

[Here, and in the following, we denote by $B_{\rm Sm}$, $K_{\rm Sm}$, and $\gamma_{\rm Sm}$ the reduced (i.e., divided by the temperature) elastic constants.] Thus, $B_{\rm Sm}=1/2D$, $K_{\rm Sm}=\nu^2/2D$, and $\gamma_{\rm Sm}=\nu/2D$. [So, $\nu=(K_{\rm Sm}/B_{\rm Sm})^{1/2}$.] Thus we arrive at the following.

Theorem A. For the special value of $\gamma_{\rm Sm}$,

$$\gamma_{\rm Sm}=\gamma_c=\pm(K_{\rm Sm}B_{\rm Sm})^{1/2} , \quad (5)$$

the *equilibrium* behavior of the smectic elastic model (1) is directly related to the *dynamical* behavior of the KPZ model (2) in *any d,* provided one identifies the KPZ time coordinate t with the smectic z coordinate (the direction of the translational ordering).

Recall now that, for $d=2$, the $\gamma_{\rm Sm}$ term in (1) contributes only to the boundary energy. Thus, in 2D, Eq. (4) reduces to the ordinary smectic Hamiltonian, Eq. (1), with $\gamma_{\rm Sm}=0$, $B_{\rm Sm}=1/2D$, and $K_{\rm Sm}=\nu^2/2D$. So, we arrive at the following.

Theorem B. The *equilibrium* behavior of the standard 2D smectic elastic model is *one-to-one* related to the *dynamical* behavior of the KPZ model in one spatial and one time dimension.

IV. ANOMALOUS ELASTICITY OF TWO-DIMENSIONAL SMECTIC-*A* LIQUID CRYSTALS

Theorem B of Sec. III can be combined with the *exact* results for the KPZ model in $1+1$ dimensions mentioned in Sec. III, to arrive at the following conclusions about 2D smectic liquid crystals:

(i) Correlations of smectic displacements, $u(x,z)=h(x,z)-z$, are, at long length scales, given by $\langle[u(x,z)-u(0,0)]^2\rangle^{1/2}=K(x,z)$, with

$$K(x,z)=|x|^{\alpha}\phi(|z|/|x|^{\alpha/\beta}) , \quad (6a)$$

where $\phi(s)\rightarrow s^{\beta}$ for $s\rightarrow\infty$, while $\phi(s)\rightarrow$const for $s\rightarrow 0$. Thus

$$K(x,0)\sim|x|^{\alpha} , \quad (6b)$$

and

$$K(0,z)\sim|z|^{\beta} . \quad (6c)$$

Here, $\alpha=\frac{1}{2}$, and $\beta=\frac{1}{3}$ exactly [9]. Equations (6b) and (6c) hold at length scales *longer* than certain Ginzburg length scales, ξ_{Gx} and ξ_{Gz}, respectively (see Appendix B). At shorter scales one has harmonic-theory behavior with $\alpha=\frac{1}{2}$ and $\beta=\frac{1}{4}$ [6].

(ii) Smectic elastic constants undergo a nontrivial renormalization at long length scales (small wave vectors q) of the form $K_{\rm Sm}(q)\sim|q_x|^{-1/2}$, for $q_z=0$, and $K_{\rm Sm}\sim|q_z|^{-1/3}$, for $q_x=0$, while $B_{\rm Sm}(q)\sim|q_x|^{1/2}$, for $q_z=0$, and $B_{\rm Sm}\sim|q_z|^{1/3}$, for $q_x=0$ (see Appendix A). This renormalization causes a breakdown of linear elasticity theory: the strain responds *nonlinearly* to *weak* external stresses S along the z direction,

$$\langle e\rangle\sim S^{2/3} .$$

This anomalous elastic behavior is discussed in Appendix A.

(iii) The above results hold for 2D smectics with purely phonon-type excitations. In a real 2D smectic-*A* liquid crystal, behavior at the longest length scales is, however, affected by dislocations. They are free in 2D smectics at any finite temperature T [6], and convert a 2D smectic-*A* liquid crystal, at large length scales, into a nematic liquid crystal. The smectic order and the above scaling behavior still persist inside anisotropic domains (cybotactic groups) with sizes ξ_z and ξ_x along the z and x directions. These sizes strongly diverge as $T\rightarrow 0$, i.e., one has a zero-temperature phase transition from nematic to smectic-*A* liquid crystals. Indeed, $\xi_x\xi_z=\xi_D^2=1/n_D$, where $n_D\sim\exp(-\text{const}/T)$ is the density of free dislocations. Harmonic elasticity theory, with $\xi_z\sim\xi_x^2$, then yields $\xi_x\sim\xi_D^{2/3}$ and $\xi_z\sim\xi_D^{4/3}$, as found by Toner and Nelson [6]. The above anharmonic effects produce a different scaling for large enough cybotactic groups. According to Eq. (6a), $\xi_z\sim\xi_x^{\alpha/\beta}=\xi_x^{3/2}$, and thus $\xi_x\sim\xi_D^{4/5}$, and $\xi_z\sim\xi_D^{6/5}$. Thus, our theory enables us to predict the exact form of the *anisotropic scaling* of smectic domain sizes,

$$\xi_z\sim\xi_x^{3/2} ,$$

at the zero-temperature transition from smectic A to nematic liquid crystals. We remark that at low enough T, ξ_x and ξ_z may easily exceed the system size. Then the entire sample is in the smectic state with the anomalous scaling behavior described in the items (i) and (ii) above. On the other side, if the sample is bigger than the sizes of cybotactic groups, one can still experimentally observe the anomalous behavior we predict: (a) by studying the scaling of the sizes ξ_x and ξ_z of smectic domains to confirm our form of the anisotropic scaling $\xi_z \sim \xi_x^{3/2}$; (b) by studying fluctuations at scales shorter than smectic domain sizes whose correlations are fully governed by a dislocation-free smectic elastic model disordered purely by phonons, i.e., smectic displacements $u(x,z)$. For example, one can study the q dependence of phonon hydrodynamic frequencies, as recently proposed by Langer, Liu, and Toner [see Sec. VII].

V. AN UNUSUAL CRITICAL POINT IN 3D SMECTIC-A LIQUID CRYSTALS WITH BROKEN INVERSION

We now turn to 3D smectic-A liquid crystals with broken inversion symmetry [12–14] by first discussing their behavior for the *special* value of the symmetry-breaking coupling $\gamma_{\rm Sm}=\gamma_c=\pm(K_{\rm Sm}B_{\rm Sm})^{1/2}$, Eq. (5). For this value of $\gamma_{\rm Sm}$, 3D smectics are equivalent to the KPZ model in $2+1$ dimensions, as explained in Sec. III (Theorem A). In Sec. VI we will show that this special value of $\gamma_{\rm Sm}$ actually corresponds to a novel elastic critical point in Smectic-A liquid crystals.

In this section we discuss properties of 3D smectic-A liquid crystals with $\gamma_{\rm Sm}$ exactly equal to γ_c. By using the relationship to the KPZ model (Sec. III) we find the following.

(i) The displacement correlations have the form of Eq. (3), with $\beta=\alpha/(2-\alpha)$ (in any d, see Appendix B and Ref. [9]). For $d=3$, $\alpha\approx 0.40$ and $\beta\approx 0.25$ [18].

(ii) For $\gamma_{\rm Sm}=\gamma_c$, elastic constants of 3D smectic-A liquid crystals undergo a nontrivial renormalization at small wave vectors, of the form $K_{\rm Sm}(\mathbf{q})\sim|\mathbf{q}_x|^{\alpha}$ for $q_z=0$, and $K_{\rm Sm}(\mathbf{q})\sim|q_z|^{\alpha/(2-\alpha)}$ for $\mathbf{q}_x=0$, whereas $B_{\rm Sm}(\mathbf{q})\sim[K_{\rm Sm}(\mathbf{q})]^3$ [see Eqs. (A4) and (A5) of Appendix A, with $d=3$]. So, both $K_{\rm Sm}$ and $B_{\rm Sm}$ vanish at long length scales ($\mathbf{q}\to 0$).

(iii) This softening of elastic constants causes a breakdown of the Hooke's law: a weak stress S normal to layers produces a strain $\langle e\rangle\sim S^{\eta_S}$, with

$$\eta_S=2(1-\alpha)/(d-1+\alpha)\ ,$$

as discussed in Appendix A. This, with $\alpha\approx 0.4$ in $d=3$ [18], gives $\eta_S\approx 0.5$, i.e.,

$$\langle e\rangle\sim S^{1/2}\ .$$

(iv) The softening of elastic constants produces violent displacement fluctuations diverging for $\gamma_{\rm Sm}=\gamma_c$ as *power laws* of the system sizes: $\langle u^2\rangle^{1/2}\sim L_x^{\alpha}$ for $L_z\gg L_x$, and $\langle u^2\rangle^{1/2}\sim L_z^{\beta}$ for $L_x\gg L_z$. This divergence is much stronger than the well-known Landau-Peierls logarithmic divergence in ordinary smectic-A liquid crystals having $\gamma_{\rm Sm}=0$ [1,15]. For $\gamma_{\rm Sm}=\gamma_c$, strong displacement fluctuations destroy long-range translational order and produce *exponentially* decaying translational correlations. This is in marked contrast to the situations for $\gamma_{\rm Sm}=0$ with a power-law decay of translational correlations [15]. The state at $\gamma_{\rm Sm}=\gamma_c$ would appear like a nematic liquid crystal. Nonetheless, thermal undulations dephasing translational correlations do not destroy the integrity of smectic layers which only assume a rough appearance similar to that of successive snapshots of the KPZ model interfaces [8]. By Eq. (3), correlations of director fluctuations $\langle\nabla_x u(\mathbf{x},z)\nabla_x u(0,0)\rangle$ decay, for $\gamma_{\rm Sm}=\gamma_c$, as $|\mathbf{x}|^{-2(1-\alpha)}$, for $z=0$, and as $|z|^{-2(1-\alpha)/(2-\alpha)}$, for $\mathbf{x}=0$, in contrast to 3D nematics where these correlations decay as $|\mathbf{x}|^{-1}$ and $|z|^{-1}$.

VI. RENORMALIZATION-GROUP ANALYSIS

The relationship to the KPZ model provides an understanding of 3D smectic liquid crystals for the *particular* value of $\gamma_{\rm Sm}=\gamma_c=\pm(K_{\rm Sm}B_{\rm Sm})^{1/2}$, Eq. (5). For a *general* $\gamma_{\rm Sm}$, we analyzed the smectic Hamiltonian, Eq. (1), by a one-loop renormalization-group (RG) transformation detailed in Appendix B. The resulting RG flow and phase diagrams are depicted in Fig. 2, in the plane (w,v), with $w=C(d)\Lambda^{d-3}(B_{\rm Sm}/K_{\rm Sm}^3)^{1/2}$, and $v=C(d)\Lambda^{d-3}|\gamma_{\rm Sm}|/K_{\rm Sm}^2$. [Here Λ is the momentum cutoff $C(d)$, a numerical constant, see Appendix B.] So, the line $v=w$ corresponds to the line $|\gamma_{\rm Sm}|=|\gamma_c|=(K_{\rm Sm}B_{\rm Sm})^{1/2}$, whereas the line $v=0$ corresponds to ordinary smectic-A liquid crystals without broken inversion symmetry ($\gamma_{\rm Sm}=0$). For $d=3$, Fig. 2(a) [and for any $d\neq 2$, Fig. 2(b)], the RG flow pattern has a *separatrix* $v=w$ occurring for $|\gamma_{\rm Sm}|=|\gamma_c|=(K_{\rm Sm}B_{\rm Sm})^{1/2}$. For $|\gamma_{\rm Sm}|<|\gamma_c|$, flows iterate to the symmetric, ordinary smectic line with $\gamma_{\rm Sm}=0$. So, in the region $|\gamma_{\rm Sm}|<|\gamma_c|$ [the region $v<w$ in Fig. 2(a)], one has, at the longest length scales, ordinary Landau-Peierls behavior [1,15] plus logarithmic corrections of Grinstein and Pelcovits [3]. Along the separatrix $|\gamma_{\rm Sm}|=|\gamma_c|$, i.e., the line $v=w$ in Fig. 2(a), our RG actually *reduces* to that of KPZ [9], and one has the behavior discussed in Sec. V. In Fig. 2(a), we indicated, on the separatrix, the *nonperturbative KPZ* fixed point. Although this fixed point is beyond the reach of a one-loop calculation, its presence is assured by the exact arguments of Sec. III. The region $|\gamma_{\rm Sm}|>|\gamma_c|$, i.e., $v>w$ in Fig. 2(a), is a runaway region: the one-loop RG flows drive $|\gamma_{\rm Sm}|$ to ∞. So, the KPZ separatrix $|\gamma_{\rm Sm}|=|\gamma_c|=(K_{\rm Sm}B_{\rm Sm})^{1/2}$ is, in fact, a *critical line* between a Landau-Peierls phase ($|\gamma_{\rm Sm}|<|\gamma_c|$) and an "unstable" region ($|\gamma_{\rm Sm}|>|\gamma_c|$). A mean-field investigation of Eq. (1) in the region $|\gamma_{\rm Sm}|>|\gamma_c|$ indicates the onset of an undulated phase, with $\langle u(\mathbf{x},z)\rangle\neq 0$, in the form of a superposition of modulations with wave vectors normal to the z axis (see Sec. VII).

A deeper insight into the exact results of Sec. III is gained by comparing them to the results of the one-loop perturbative RG (Appendix B). For $d<3$, RG gives a stable *perturbative* anharmonic fixed point (A), belonging to the ordinary smectic line $\gamma_{\rm Sm}=0$ [the line $v=0$ in Fig. 2(b)]. This fixed point attracts RG trajectories in the re-

gion $|\gamma_{Sm}| < |\gamma_c|$, i.e., $v < w$ in Fig. 2(b). In this region, fixed point A yields displacement correlations as in Eq. (3) with $\alpha = 3(3-d)/(8-d)$, and $\beta = 3(3-d)/(7+d)$, to one-loop order. Scaling along the critical KPZ line [the separatrix $|\gamma_{Sm}| = |\gamma_c|$, i.e., the line $v = w$ in Fig. 2(b)] is regulated by a different, nonperturbative fixed point (KPZ), yielding displacement correlations again as in Eq. (3), but with *different* values of α and β, which here coincide with those of the KPZ model. For $d > 2$, the KPZ fixed point is unstable with respect to A, see Fig. 2(b). However, as $d \to 2$, this instability vanishes and a *line of critical points* containing both A and the KPZ fixed point is formed for $d = 2$, see Fig. 2(c). The appearance of this line is caused by the fact that, in 2D, the inversion symmetry-breaking γ_{Sm} term of Eq. (1) becomes a boundary term not affecting the bulk fluctuations, as discussed in Sec. II. So, all the points of the fixed line yield the *same* scaling behavior with $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$, as obtained from the above one-loop formulas for $d = 2$. In 2D, these one-loop results actually agree with the exact results for α and β of Sec. IV. Thus, both for 2D smectics and the KPZ model, the one-loop theory becomes exact in the continuum (see also Ref. [9]).

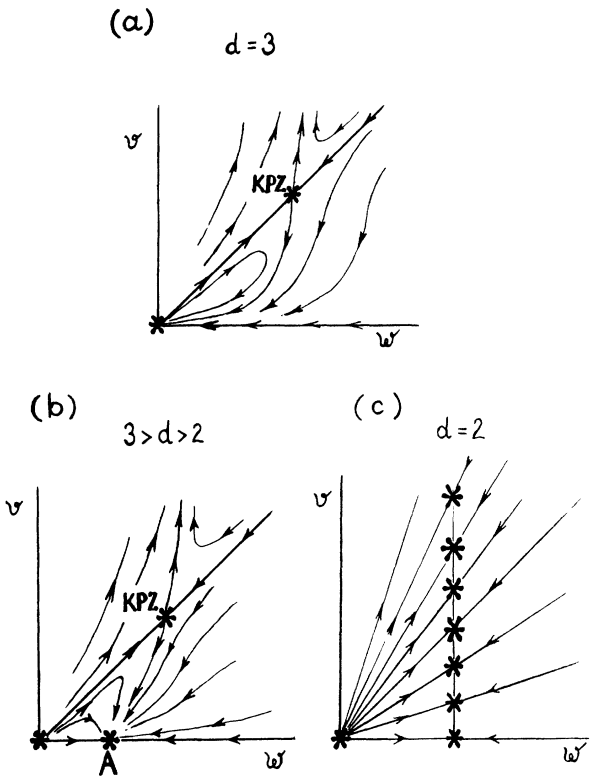

FIG. 2. Schematic renormalization-group flow diagrams for smectic-A liquid crystals with broken inversion symmetry in the plane (w,v). Here $w = C(d)\Lambda^{d-3}(B_{Sm}/K_{Sm}^3)^{1/2}$, and $v = C(d)\Lambda^{d-3}|\gamma_{Sm}|/K_{Sm}^2$. The ordinary smectics-$A$ liquid crystals (without broken inversion symmetry, i.e., $\gamma_{Sm} = 0$) correspond to the horizontal axis $v = 0$. The KPZ separatrix, $|\gamma_{Sm}| = |\gamma_c| = (B_{Sm}K_{Sm})^{1/2}$ is the diagonal line, $v = w$. We indicate, on this separatrix, the nonperturbative KPZ fixed point. Its presence is assured by the exact arguments of Sec. III. (a) RG flows in $d = 3$. (b) RG flows for $3 > d > 2$. Note the appearance of a stable fixed point A with $\gamma_{Sm}^* = 0$. (c) RG flows for $d = 2$. Note the presence of a line of fixed points.

VII. DISCUSSION AND RELATED STUDIES

Our prediction of the breakdown of the linear elastic behavior in 2D smectic liquid crystals (Sec. IV and Ref. [2]) attracted the attention of other researchers. Thus, Langer, Liu, and Toner discussed the consequences of this breakdown on the hydrodynamics of 2D smectic liquid crystals formed in films adsorbed on fluid surfaces (as in a Langmuir film) [19]. By using the renormalized (momentum-dependent) smectic elastic constants (Sec. IV), they calculated the hydrodynamic frequencies and obtained the dynamic structure factor. They find the dispersion relation $\omega(\mathbf{q})$ of "phonons" (i.e., periodically modulated displacements) of the smectic structure, of the form

$$\omega(\mathbf{q}) \sim (\pm\sqrt{3} - i)q^{14/9}[\cos(\theta)]^{4/3}[\sin(\theta)]^{14/9},$$

where $q = |\mathbf{q}|$, and $\pi/2 - \theta$ is the angle between $\mathbf{q}$ (the direction of propagation) and the normal to smectic layers (z axis in Sec. IV). This form of $\omega(\mathbf{q})$ is (in the absence of dislocations) valid in the long-wavelength limit $q \to 0$, whereas for wavelengths shorter than a Ginzburg length scale ξ_G (more precisely, ξ_{Gz} defined in Appendix B) another form of $\omega(\mathbf{q})$, obtained from linear, Hookean elasticity, becomes valid [19]. For an experimental observation (via a dynamic light scattering) of the linear elasticity breakdown, one needs: (i) a small enough ξ_G, so that a light-scattering probe can be applied at all; (ii) the dislocation length scale ξ_D, Sec. IV, has to be much larger than ξ_G. The latter condition is satisfied, at least, at low enough temperatures since ξ_D/ξ_G diverges as $T \to \infty$ (see Appendix B). However, ξ_G also typically diverges in this limit, and may exceed the optical range of wavelengths.

These considerations can be used to select the best experimental system for observing, via dynamic light scattering, the breakdown of the Hookean elasticity theory. Langer, Liu, and Toner thus suggest *dense* systems of so-called "hairy rod polymers" (rigid rods with flexible side chains). These polymers form stable films on water with the rods lying in the plane, and may develop a smectic ordering with the direction of translational order perpendicular to the rods. In a dense hairy-rod system, where the hairs of neighboring polymers strongly overlap, the ξ_G can be made small, some 30 smectic layers (as estimated in Ref. [19]) which should be readily accessible experimentally by light scattering.

On the other hand, for a *dilute* hairy-rod system, having only a weak steric repulsion between polymers, the Ginzburg length ξ_G is typically large, beyond the optical range, and the light scattering cannot be used to reveal the Hookean elasticity breakdown. For these systems one can still use *uniform* (zero-momentum) external probes, such as weak uniform (in-plane) external stresses normal to smectic layers, to which, in the non-Hookean regime, the strain responds nonlinearly, $(strain) \sim (stress)^{2/3}$, Sec. IV. Instead of external stresses, it is, perhaps, better to use uniform external electric fields or flow alignment which, through the coupling to the molecular anisotropy,

may produce effects similar to that of external stresses.

Anisotropic, smecticlike scaling has been observed also in thin ferromagnetic films exhibiting stripe domain structures [11]. Recently, Kashuba and Pokrovsky [20] used our results to discuss translational correlations in these 2D smectic analogs. For the correlation function of the order parameter $M(x,z) \sim \mathrm{Re}\exp[iq_0 u(x,z)]$, Eq. (6) implies the form

$$\langle M(x,z)M(0,0)\rangle \sim \exp\left[-\frac{q_0^2}{2}(K(x,z))^2\right] .$$

$K(x,z)$ here is as in Eq. (6) with $\alpha=\frac{1}{2}$ and $\beta=\frac{1}{3}$. These correlations can be checked experimentally by polarized electron microscopy or by a study of the electron microscopy photographs. It should be stressed, however, that the actual situation in these physical systems is more complex than in *stricto sensu* smectic-A liquid crystals. First of all, spatial anisotropy is present and cuts off true smectic behavior at long length scales [20]. Its effect is similar to that of an external uniform stress applied to a smectic liquid crystal. Secondly, domain walls might be strongly pinned by quenched defects producing a glassy state of the domain liquid (see the work of Golubović and Kulić in Ref. [4]).

We turn now to 3D smectic-A liquid crystals with broken inversion symmetry [12–14] such as ferroelectric smectic-A liquid crystals with an average dipolar moment normal to layers. These phases have been proposed in the literature a long time ago by Pikin and Indenbom and by Blinc and Zekš [12], and various experimental realizations have been suggested in the past [13]. However, their first experimental realization, by Tournilhac, Blinov, Simon, and Yablonsky, is quite recent [14]. An interesting prediction of our work is the existence of an unusual elastic critical point in these phases, Sec. V. It has scaling behavior of the KPZ model in $2+1$ dimensions, with the KPZ time coordinate identified with the smectic z coordinate (normal to layers). We believe that this critical point is at the border between the phase with ordinary smectic behavior (asymptotically flat layers with Landau-Peierls behavior) and another phase in which layers develop a modulation ("rippled" phase). The existence of such a phase for large enough γ_{Sm} can be inferred from a mean-field analysis. For $|\gamma_{\mathrm{Sm}}|<|\gamma_c|=(K_{\mathrm{Sm}}B_{\mathrm{Sm}})^{1/2}$, the energy (1) is minimized by the usual smectic ground state with flat, equidistant layers. Indeed, for $|\gamma_{\mathrm{Sm}}|<|\gamma_c|=(K_{\mathrm{Sm}}B_{\mathrm{Sm}})^{1/2}$ the quadratic form in e and H entering (1) is *PD*, so (1) reaches its absolute minimum, zero, only for $e=0$ and $H=0$. On the other side, for $|\gamma_{\mathrm{Sm}}|>|\gamma_c|$, i.e., in the runaway region of Fig. 2(a), this ordinary smectic state is not necessarily the ground state. However, amazingly enough, the ordinary smectic state is, for *any* γ_{Sm}, a *locally* stable state. This comes from the fact that, upon expanding $h(\mathbf{x},z)=z+u(\mathbf{x},z)$, the γ_{Sm} of (1) contributes only a term *cubic* in u (see Appendix A). Thus, the harmonic (quadratic) approximation to (1) is actually *insensitive* to the value of γ_{Sm} and has the ordinary Landau-Peierls form depending only on K_{Sm} and B_{Sm}. This ensures local stability of the ordinary, flat smectic state for any value of γ_{Sm}. Nonetheless, for $|\gamma_{\mathrm{Sm}}|>|\gamma_c|$ nothing ensures that this ordinary smectic state is the actual ground state. To study this problem, we examined energies of possible modulated ground states of the form

$$\langle h(\mathbf{x},z)\rangle=(1+\epsilon)z+\mathrm{Re}\sum_{j=1}^{N} A_j \exp(i\mathbf{q}_j\mathbf{x}) , \tag{7}$$

representing a "rippled" phase of undulated layers. Undulation here is a superposition of N plane waves with wave vectors $\mathbf{q}_j$ perpendicular to the z axis. Parameters ϵ, $\mathbf{q}_j$, and A_j are to be obtained from the mean-field condition that the state (7) minimizes the energy (1). Note that a simple one-dimensional undulation ($N=1$) always has an energy bigger than that of the ordinary, flat smectic state which is zero. [For such undulation the contribution of the γ_{Sm} term of (1) vanishes, for the same reason this contribution vanishes in 2D smectic-A liquid crystals, see Sec. II. The remaining K_{Sm} and B_{Sm} contributions are always positive.] Thus we considered a more complex case with three modulations ($N=3$) with wave vectors $\mathbf{q}_j$, of the same magnitude, forming 120° angles with respect to each other (triangular undulation) [21]. We found that, with $|\gamma_{\mathrm{Sm}}|$ increasing above $|\gamma_c|$, this state eventually energetically wins over the ordinary flat smectic state. This happens through a phase transition occurring at $|\gamma_{\mathrm{Sm}}|=c|\gamma_c|$, with $c=\sqrt{3}>1$. So, within the mean-field theory, this undulated state would occupy only a portion of the runaway region, $|\gamma_{\mathrm{Sm}}|>|\gamma_c|=(K_{\mathrm{Sm}}B_{\mathrm{Sm}})^{1/2}$ in Fig. 2(a). The runaway character of RG flows in this region most likely indicates that thermal fluctuations (neglected in mean-field theory) extend the region of the undulated state to the entire runaway region.

A detailed theoretical understanding of the behavior in the runaway region is beyond the scope of this paper. It is complex even at the mean-field level. We have not explored a variety of other possible ground states. We remark, however, that for a very large layer spontaneous curvature H_0, i.e., very large γ_{Sm} (see Sec. II), phases with *topology* different from the smectic, i.e., lamellar one, are favored. For example, the curved, globular ("micellar") phase of membranes is then certainly more favored than the lamellar one. Then, the important question is, would the system, with increasing H_0, undergo a change of topology before getting to our novel smectic elastic point at $|\gamma_{\mathrm{Sm}}|=|\gamma_c|=(K_{\mathrm{Sm}}B_{\mathrm{Sm}})^{1/2}$? Let us consider, for example, a lamellar phase stabilized by Helfrich's entropic repulsion yielding $B_{\mathrm{Sm}}\approx k_B T^2/Kl^3$ [22]. This, with $K_{\mathrm{Sm}}=K/l$ and $\gamma_{\mathrm{Sm}}=KH_0/l$ (see Sec. II), implies that our KPZ-type critical point occurs for $|H_0|=H_c$, with

$$H_c \approx \frac{k_B T}{Kl} . \tag{8}$$

Thus, by tuning, for example, the spontaneous curvature H_0 (e.g., by changing the salt concentration of the solvent) one can, perhaps, approach our elastic critical point. Is this going to happen before the spontaneous

curvature triggers a transition from lamellar phase to a phase with different topology, say a highly curved globular phase? We examined this question within a simple "phenomenological" approach to phase equilibria in fluid membrane systems [23]. It turns out that the transition from the lamellar to globular phase occurs for the spontaneous curvature $|H_0|=H^*$, with

$$H^* \approx \left[\frac{k_B T}{K}\right]^{1/6} \frac{1}{l} , \tag{9}$$

for zero saddle splay bending rigidity [23]. Note that H^* is, for rigid membranes with $K \gg k_B T$ much larger than H_c in Eq. (8). Thus, in practice, our novel elastic critical point at $|H_0|=H_c$ may lie well inside the region $|H_0|<H^*$ in which the membrane system still has the topology of a lamellar phase and can be described by the smectic elastic model (1). So, rigid layers are good candidates to search for our novel elastic critical point and examine the nature of "rippled", undulated phases proposed here.

Finally, we stress that our model (1) for smectic-A liquid crystals with broken inversion symmetry ("ferroelectric" smectic-A liquid crystals) presumes that long-range dipolar forces are screened, as is the case if these phases are formed in an electrolytic solvent. Unscreened dipolar interactions have also been considered recently [24]. As in a more familiar case of the Heisenberg model, these long-range interactions would cut off long-distance fluctuations and stabilize long-range translational order both in the Landau-Peierls phase and at our novel KPZ-type elastic critical point. Their effects are similar to those of the external stress (see Appendix A).

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APPENDIX A

In this appendix we discuss the truncated forms of the KPZ model and the smectic elastic model that usually appear in the literature. Then, we discuss the renormalization of smectic elastic constants at long length scales (Secs. IV and V). We discuss here also the anomalous stress-strain relationship, i.e., the breakdown of Hooke's law induced by the renormalization of smectic elastic constants. Finally, we discuss the functional Jacobian involved in relating the KPZ model to smectic liquid crystals (Sec. III).

The rotationally invariant KPZ model Eq. (2) is usually represented in the truncated form

$$\partial_z u = \nu \nabla_{\mathbf{x}}^2 u + \frac{\lambda}{2}(\nabla_x u)^2 + \eta(\mathbf{x},z) . \tag{A1}$$

Only the terms up to the second order in $u(\mathbf{x},z)=h(\mathbf{x},z)-z$ are kept since higher-order terms are irrelevant [9]. The rotationally invariant smectic model Eq. (1) can be (with the same justification) truncated up to the terms of the fourth order in u yielding

$$E_{\rm Sm} = \int dz\, d^{d-1}\mathbf{x} \left[\frac{B_{\rm Sm}}{2}[\partial_z u - \tfrac{1}{2}(\nabla_{\mathbf{x}} u)^2]^2 - \gamma_{\rm Sm}[\partial_z u - \tfrac{1}{2}(\nabla_{\mathbf{x}} u)^2]\nabla_{\mathbf{x}}^2 u + \frac{K_{\rm Sm}}{2}[\nabla_{\mathbf{x}}^2 u]^2 \right] . \tag{A2}$$

Note that the *harmonic* contribution of the $\gamma_{\rm Sm}$ term *vanishes,* i.e., contributes only to the boundary energy, as can be verified by a partial integration. Thus the inversion symmetry breaking $\gamma_{\rm Sm}$ term in Eq. (1) contributes, to the *lowest* order in u, the cubic term $(\nabla_{\mathbf{x}} u)^2 \nabla_{\mathbf{x}}^2 u$. In other words, the quadratic, harmonic approximation to (A2) is *insensitive* to the value of $\gamma_{\rm Sm}$.

Let us now discuss the renormalization of the smectic elastic constants $B_{\rm Sm}$ and $K_{\rm Sm}$ (Secs. IV and V). As noted above, the $\gamma_{\rm Sm}$ term contributes only an anharmonic contribution, and the harmonic propagator is of the standard smectic form

$$G_0 = \langle |u(q)|^2 \rangle_0 = \frac{1}{K_{\rm Sm} q_x^4 + B_{\rm Sm} q_z^2} , \tag{A3}$$

even in the presence of the inversion symmetry breaking term. The *renormalized* propagator G_R can be thus obtained by replacing $B_{\rm Sm} \to B_{\rm Sm}(q)$, and $K_{\rm Sm} \to K_{\rm Sm}(q)$,

$$G_R(q) = \langle |u(q)|^2 \rangle = \frac{1}{K_{\rm Sm}(q) q_x^4 + B_{\rm Sm}(q) q_z^2} .$$

The form of the q-dependent smectic elastic constants is extracted by requiring G_R to yield the scaling of the displacement correlations as given by Eq. (3). This requirement, and the scaling relation $\beta=\alpha/(2-\alpha)$ (see Appendix B and Ref. [9]), then yield, for $|q| \to 0$,

$$\begin{aligned} K_{\rm Sm} &\sim q_x^{d-3+\alpha} , \\ B_{\rm Sm} &\sim q_x^{d-3+3\alpha} \end{aligned} \tag{A4}$$

for $q_z \ll \text{const} q_x^{1/(2-\alpha)}$, whereas

$$\begin{aligned} K_{\rm Sm} &\sim q_z^{(d-3+\alpha)/(1-\alpha)} , \\ B_{\rm Sm} &\sim q_z^{(d-3+3\alpha)/(2-\alpha)} \end{aligned} \tag{A5}$$

for $q_z \gg \text{const} q_x^{1/(2-\alpha)}$.

Let us now discuss the breakdown of Hooke's law induced by the above long-distance renormalization of smectic elastic constants. A *compressional* stress S applied to a smectic-A liquid crystal along the z direction contributes to the smectic elastic Hamiltonian the term $S\partial u/\partial z$. Within harmonic Hookean elasticity theory, a weak external compressional stress S produces the strain

$$e = -S/B_{\rm Sm} , \tag{A6}$$

where $B_{\rm Sm}$ is the bare smectic compressibility constant. Another effect of the stress is a suppression of long-distance fluctuations, which arises due to stress-induced breaking of the rotational invariance. Indeed, in the presence of S, the harmonic propagator assumes the form

$$G_0(q)=\frac{1}{Sq_x^2+K_{\rm Sm}q_x^4+B_{\rm Sm}q_z^2} . \quad \text{(A7)}$$

At long length scales ($q\to 0$), the stress term Sq_x^2 dominates over the soft, $K_{\rm Sm}q_x^4$ term and kills the strong fluctuations of the isotropic model. The renormalized, anharmonic counterpart of (A7) has the form

$$G_R(q)=\frac{1}{Sq_z^2+K'_{\rm Sm}(q)q_x^4+B'_{\rm Sm}(q)q_z^2} . \quad \text{(A8)}$$

The bare smectic elastic constants are replaced by renormalized constants $K'_{\rm Sm}(q)$ and $B'_{\rm Sm}(q)$, the form of which is discussed in the following. On the other hand, the stress term Sq_x^2 is not renormalized by anharmonic effects: the smectic-A liquid crystal with externally applied stress S is analogous to the Heisenberg ferromagnet in the presence of an external magnetic field that breaks rotational symmetry. Then, by applying standard symmetry arguments analogous to those used in proving the Goldstone theorem for the Heisenberg ferromagnet, one finds that the symmetry-breaking contribution Sq_x^2 to the full propagator is not renormalized (like the magnetic-field contribution to the transverse correlation function of the Heisenberg ferromagnet). What is the form of $K'_{\rm Sm}(q)$ and $B'_{\rm Sm}(q)$ in (A8)? To answer this, note that for $q_x \gg q^*(S)$, with $q^*(S)$ defined by

$$S=K'_{\rm Sm}(q^*(S))(q^*(S))^2 , \quad \text{(A9)}$$

the $K_{\rm Sm}$ term in (A8) dominates over the stress term. Physically, this means that, for $q_x \gg q^*(S)$, fluctuations are not affected by the presence of the stress S, i.e., they behave as in a system with $S=0$. Thus, the q-dependent smectic elastic constants in the full propagator (A8) are, for $q_x > q^*(S)$, given by Eq. (A4), which applies to the $S=0$ case. This fact and Eq. (A9) imply that, for a weak stress,

$$S=K_{\rm Sm}(q^*)(q^*)^2\sim(q^*)^{d-1+\alpha} .$$

Thus

$$q^*(S)\sim S^{1/(d-1+\alpha)} . \quad \text{(A10)}$$

On the other side, at the longest length scales, i.e., smallest momenta $q_x \ll q^*(S)$, the stress term Sq_x^2 in (A8) *completely* suppresses the violent long-length-scale fluctuations causing a nontrivial renormalization of the elastic constants of the isotropic model. Thus $B'_{\rm Sm}(q)$ and $K'_{\rm Sm}(q)$ in (A8) do not have any (essential) q dependence for $q_x < q^*(S)$, i.e., by (A4)

$$B_{\rm Sm}(q_x)\approx B_{\rm Sm}(q_x=q^*(S))\sim(q^*(S))^{d-3+3\alpha} . \quad \text{(A11)}$$

This and Eq. (A10) then imply

$$B_{\rm Sm}(q=0)\sim S^{(d-3+3\alpha)/(d-1+\alpha)} . \quad \text{(A12)}$$

So, the presence of external stress S stabilizes, at long length scales, the effective smectic compressibility constant to a finite value, given for a weak S by (A12). The strain e induced by S can be then calculated by the renormalized version of Hooke's law (A6), i.e., by

$$e=-\frac{S}{B_{\rm Sm}(q=0)} .$$

This and Eq. (A12) imply

$$e\sim S^{\eta_S} ,$$

with

$$\eta_S=\frac{2(1-\alpha)}{d-1+\alpha} . \quad \text{(A13)}$$

Equation (A13) with $\alpha=\frac{1}{2}$ in $d=2$, gives $\eta_S=\frac{2}{3}$, as mentioned in Sec. IV. In $d=3$, $\alpha\approx 0.4$, so by (A13), $\eta_S\approx 0.5$, as mentioned in Sec. V in the discussion of the KPZ-type elastic critical point in smectic-A liquid crystals with broken inversion symmetry.

Finally, let us discuss the Jacobian factor $J(h)=|D\eta/Dh|$ involved in relating the KPZ model to smectic-A liquid crystals via, as discussed in Sec. III,

$$P(h)=J(h)\exp(-H_{\rm eff}) , \quad \text{(A14)}$$

with $H_{\rm eff}$ given by Eq. (4). Actually, by applying the classical dynamics path-integral formalism [17], $P(h)$ can be calculated also as

$$P(h)\sim\left\langle\left[\prod_{z,\mathbf{x}}\delta(\partial_z h(\mathbf{x},z)-v(\mathbf{x},z;h,\eta))\right]\exp\left[-\tfrac{1}{2}\int dz\int\int d^{d-1}\mathbf{x}_1 d^{d-1}\mathbf{x}_2\delta^{d-1}(\mathbf{x}_1-\mathbf{x}_2)\frac{\delta v(\mathbf{x}_1,z)}{\delta h(\mathbf{x}_2,z)}\right]\right\rangle_\eta , \quad \text{(A15)}$$

for a general first order in the time z random evolution equation of the form

$$\partial_z h=v(\mathbf{x},z;h,\eta) . \quad \text{(A16)}$$

Here v is velocity at $\mathbf{x}$, being a functional of the field h and the noise η configuration at the time z. Equation (A15) can be transformed into the form in (A14) [and, thus, $J(h)$ calculated], by using the Fourier transform representation of the δ functions in the product entering (A15), and then taking the average over the noise η. Let us first consider the truncated KPZ and smectic models, Eqs. (A1) and (A2). Both models preserve so-called "infinitesimal rotational invariance," known as the "Galilean" invariance in the KPZ context [9]. Moreover, both the truncated models contain all relevant interactions, i.e., those yielding, at *long length scales,* divergencies responsible for the anomalous fluctuation behaviors both of the KPZ model and of the smectic model. By applying (A15) to the truncated KPZ model (A1), we obtain as the effective Hamiltonian $H_{\rm eff}$ the truncated smectic model (A2), while the Jacobian factor $J(h)$ turns out to be an h-independent constant.

To discuss the form of $J(h)$ for the case of the full nonlinear KPZ model (which has the full rotational invariance), Eq. (1), let us first clarify an important issue of the

equilibrium statistical mechanics of smectic-A liquid crystals. The partition function corresponding to the full, nonlinear rotationally invariant smectic elastic model should be properly written as

$$Z=\int Dh\, M(h)\exp(-H_{\rm Sm})\ . \tag{A17}$$

$H_{\rm Sm}$ here is as in Eq. (1) and preserves the full rotational invariance [in contrast to the truncated smectic model (A2), which preserves "infinitesimal rotational," i.e., "Galilean" invariance]. Note that (A17) contains, in addition to the usual Boltzmann factor, an additional factor $M(h)$ in its measure. Namely, in the Lagrangian elasticity picture used in this paper, with layer heights $h(\mathbf{x},z)$ being the field variables (see Sec. II), use of the naive integration measure $\int Dh$ would actually break the *full* rotational invariance. It would produce, e.g., some dependence of the calculated free energy on the direction of smectic translational ordering through terms that actually *diverge* in the continuum limit ($\Lambda\to\infty$, with Λ the upper momentum cutoff). The trouble of the naive measure comes, physically, from *short-scale* fluctuations: the ordinary short-distance cutoff $|\mathbf{q}_x|<\Lambda$, $-\infty<q_z<\infty$, which is natural if $h(\mathbf{x},z)$ is the field variable, manifestly breaks rotational invariance. For this reason, the actual measure used in (A17) is $\int Dh\, M(h)$, with

$$M(h)\sim\exp\left[-\frac{1}{2}C(\Lambda)\left(\frac{K_{\rm Sm}}{B_{\rm Sm}}\right)^{1/2}\times\int dz\, d^{d-1}\mathbf{x}\frac{(\nabla_{\mathbf{x}}h)^2}{1+(\nabla_{\mathbf{x}}h)^2}\right], \tag{A18}$$

where Λ is the upper momentum cutoff for $\mathbf{q}_x$ (z coordinate is presumed to be continuous), and

$$C(\Lambda)=\frac{1}{d-1}\int\frac{d^{d-1}\mathbf{q}_x}{(2\pi)^{d-1}}(\mathbf{q}_x)^2\ . \tag{A19}$$

With this measure the full rotational invariance of the smectic partition function (A17) is restored since the presence of $M(h)$ actually removes ("substracts") contributions of short-scale fluctuations breaking the full rotational invariance arising if the naive measure is used.

Now, let us return to the calculation of the Jacobian $J(h)$ for the full nonlinear KPZ model (2). By calculating $P(h)$ by means of (A15) we obtain $J(h)$ in the form of $M(h)$, as in Eq. (A18) with the factor $(K_{\rm Sm}/B_{\rm Sm})^{1/2}$ replaced by v, the "surface tension" constant entering the KPZ model, Eq. (2). [As before, we set $\lambda=1$ in (2).] The condition $v=(K_{\rm Sm}/B_{\rm Sm})^{1/2}$ we already met in Sec. III while discussing the equivalence of the KPZ model to smectic-A liquid crystals with a special choice of elastic constants (see the lines preceding Theorem A, Sec. III). Thus, the Jacobian $J(h)$ is nothing else but the smectic functional $M(h)$ in (A18), for smectic-A liquid crystals at the elastic critical point characterized by the KPZ scaling behavior.

APPENDIX B

Here we outline renormalization-group (RG) equations for smectic-A liquid crystals with broken inversion symmetry. These equations lead to the conclusions summarized in Sec. VI. To derive them one can start from the truncated smectic model (A1) (see Appendix A) which contains all relevant nonlinearities. The RG equations were obtained by eliminating fluctuations in the momentum shell $\Lambda e^{-l}<|\mathbf{q}_x|<\Lambda$ (with Λ the momentum cutoff), $-\infty<q_z<\infty$, and by performing the rescaling

$$\mathbf{x}=e^{l}\mathbf{x}',\quad z=e^{l\alpha/\beta}z',\quad u=e^{l\alpha}u'\ . \tag{B1}$$

This *anisotropic* rescaling ensures scaling of displacement correlations as in Eq. (3). The exponents α and β in (B1) can be related by a symmetry argument: the form of truncated nonlinear strain $e=\partial_z u-\frac{1}{2}(\nabla_{\mathbf{x}}u)^2$ is dictated by the infinitesimal rotational invariance (see Appendix A). The rescaling (B1) must preserve this form of e. Then, as, by (B1), $\partial_z u=e^{l(\alpha-\alpha/\beta)}\partial_{z'}u'$, and $(\nabla_{\mathbf{x}}u)^2=e^{l(2\alpha-2)}(\nabla_{\mathbf{x}'}u')^2$, one must impose $\alpha-\alpha/\beta=2\alpha-2$, i.e.,

$$\beta=\frac{\alpha}{2-\alpha}\ . \tag{B2}$$

With this, we find the following RG equations, to one-loop order,

$$\frac{dB_{\rm Sm}}{dl}=B_{\rm Sm}[d-3+3\alpha-w]\ , \tag{B3}$$

$$\frac{dK_{\rm Sm}}{dl}=K_{\rm Sm}\left[d-3+\alpha+\frac{5-d}{d+1}w-\frac{4d(d-2)}{d^2-1}gw\right], \tag{B4}$$

$$\frac{d\gamma_{\rm Sm}}{dl}=\gamma_{\rm Sm}\left[d-3+2\alpha-\frac{2(d-2)}{d-1}w\right]. \tag{B5}$$

Smectic elastic constants used above are reduced ones (i.e., divided by temperature). Here, w and g are dimensionless parameters defined by

$$w=C(d)\Lambda^{d-3}\frac{B_{\rm Sm}^{1/2}}{K_{\rm Sm}^{3/2}}\ , \tag{B6}$$

$$g=\frac{\gamma_{\rm Sm}^2}{B_{\rm Sm}K_{\rm Sm}}\ , \tag{B7}$$

with $C(d)=S_{d-1}/8(2\pi)^{d-1}$, where S_d denotes the area of a unit d-dimensional sphere. By (B3) to (B7) one finds the following two RG equations for w and g:

$$\frac{dw}{dl}=w\left[3-d-\frac{8-d}{d+1}w+\frac{6d(d-2)}{d^2-1}gw\right], \tag{B8}$$

$$\frac{dg}{dl}=\frac{4d(d-2)}{d^2-1}gw\left[g-\frac{d+3}{2d}\right]. \tag{B9}$$

RG flows implied by (B8) and (B9) are sketched in Fig. 3 in $d=3$ [Fig. (a)], $3>d>2$ [Fig. (b)], and $d=2$ [Fig. (c)]. RG flows depicted in the (w,v) plane (see Fig. 2, Sec. VI) were obtained from those in the (w,g) plane (Fig. 3) by noting that $v=w\sqrt{g}$.

The most important features of these RG flows are the following.

(1) By (B9), the line

$$g = g^*(d) = \frac{d+3}{2d} , \tag{B10}$$

is invariant under RG flow. This line is a horizontal separatrix in the (w,g) plane (see Fig. 3), corresponding to the diagonal separatrix in the (w,v) (see Fig. 2). For $g = g^*(d)$, our Eq. (B8) for w reduces to the central RG equation of Kardar, Parisi, and Zhang [Eq. (5b) of Ref. [9]] for a single dimensionless parameter $\bar{\lambda} = \lambda(D/\nu^3)^{1/2}$ present in their model. (To see this, note that our $w = [S_{d-1}/4(2\pi)^{d-1}]\bar{\lambda}^2$, due to the relationship between the KPZ and smectic models, Sec. III, implying $B_{\rm Sm} = 1/2D$, and $K_{\rm Sm} = \nu^2/2D$. Note that d entering the KPZ equation equals our $d-1$. Also, recall that here we set $\lambda = 1$, see Sec. III.) For $d = 3$, by (B10), this KPZ separatrix occurs for

$$g = g^*(d=3) = 1 ,$$

or, by (B7), for

$$|\gamma_{\rm Sm}| = |\gamma_c| = \sqrt{K_{\rm Sm} B_{\rm Sm}} .$$

Thus, our RG analysis agrees with the nonperturbative results of Sec. III, which indicate that, for the above critical value of $\gamma_{\rm Sm}$, 3D smectics exhibit an anisotropic scaling behavior directly related to that of the KPZ model in 2+1 dimensions. This behavior is believed to be regulated by a nonperturbative KPZ fixed point indicated in Figs. 2 and 3 (though it is beyond the reach of the one-loop RG which only indicates that the parameter w is marginally relevant along the KPZ separatrix).

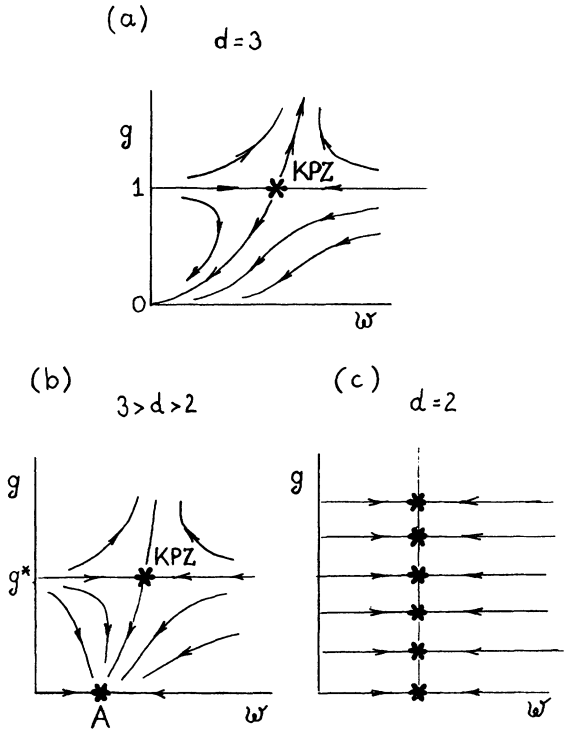

FIG. 3. Schematic renormalization-group flow diagrams for smectic-A liquid crystals with broken inversion symmetry in the plane (w,g). Here w has the same meaning as in Fig. 2, while $g = \gamma_{\rm Sm}^2/K_{\rm Sm}B_{\rm Sm}$. Figures (a) $d=3$, (b) $3>d>2$, and (c) $d=2$, are directly related to Figs. (2a), (2b), and (2c), via $v = w\sqrt{g}$. A indicates a perturbative anharmonic fixed point with $\gamma_* = 0$, while KPZ indicates the nonperturbative fixed point, the presence of which is assured by the arguments of Sec. III.

(2) The line $g = 0$ is, by (B9), invariant under RG flows, as it should since it represents ordinary smectic-A liquid crystals invariant under inversion, i.e., with $\gamma_{\rm Sm} = 0$. Along this line, for $d = 3$, our RG equation (B8) reduces to the RG equation of Grinstein and Pelcovits [3] yielding logarithmic corrections to the effective elastic constants that arise due to a marginally irrelevant parameter w. On the other hand, for $d < 3$, Eqs. (B8) and (B9) yield a stable perturbative anharmonic fixed point (A) with $g^* = 0$ and

$$w^* = \frac{(3-d)(d+1)}{8-d} .$$

For this fixed point, by (B3) (with $dB_{\rm Sm}/dl = 0$) one obtains

$$\alpha = \frac{3(3-d)}{8-d} , \tag{B11}$$

and, by (B2),

$$\beta = \frac{3(3-d)}{7+d} . \tag{B12}$$

These one-loop formulas are strictly correct only to leading order in $\epsilon = 3-d$, i.e., in the form

$$\alpha = \frac{3\epsilon}{5} + O(\epsilon^2), \quad \beta = \frac{3\epsilon}{10} + O(\epsilon^2) .$$

(3) For $d = 2$, by (B9),

$$\frac{dg}{dl} = 0 ,$$

whereas the RG equation for w, Eq. (B8), as well as Eqs. (B3) to (B5) become independent of $g \sim \gamma_{\rm Sm}^2$. Thus, in 2D, the RG flows become insensitive to the value of $\gamma_{\rm Sm}$. This is in agreement with the fact that the $\gamma_{\rm Sm}$ term of the smectic energy becomes a boundary term not affecting bulk fluctuations (see Sec. II). In effect, in 2D, one obtains in Figs. 3(c) and 2(c) a line of fixed points all yielding the same scaling with, by (B11) and (B12), $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$ to one-loop order. These values of the exponents turn out to be exact. This is related to the fact that, in 1+1 dimensions, the one-loop theory of the KPZ model becomes exact at long length scales [9].

We remark that Eqs. (B8) and (B9) are strictly valid only to first order in $\epsilon = 3-d$. To this order, all one-loop integrals should be calculated in $d = 3$ exactly, and the proper form of the RG equations is

$$\frac{dw}{dl} = w[\epsilon - \tfrac{5}{4}w + \tfrac{9}{4}wg] ,$$

$$\frac{dg}{dl} = \tfrac{3}{2}wg[g-1] .$$

Thus $g^* = 1$, i.e., the KPZ separatrix is, to this order in ϵ, given by $|\gamma_{\rm Sm}| = \sqrt{K_{\rm Sm}B_{\rm Sm}}$, in agreement with Eq. (5) of Sec. III. We remark this because Eq. (B10) might suggest

that this agreement holds only in $d=3$.

Finally, we discuss Ginzburg length scales for the anomalous fluctuation behavior described in this paper. There are two of them, ξ_{Gx} parallel to layers, and ξ_{Gz} normal to smectic layers. Only at scales longer than these can one observe the anomalous behavior, whereas at shorter scales one has nonanomalous behavior well described by harmonic fluctuation theory. Ginzburg length scales can be estimated by calculating one-loop anharmonic corrections to bare elastic constants, as already done in deriving RG equations (B3) to (B5). A substantial change of bare elastic constants, due to long-length-scale fluctuations, occurs if the system's size along the x direction exceeds the length ξ_{Gx} for which the dimensionless w in (B6), with $\Lambda \approx \xi_{Gx}^{-1}$, becomes of order 1. So, $\xi_{Gx}^{3-d} B_{\rm Sm}^{1/2}/K_{\rm Sm}^{3/2} \approx 1$, for $d<3$. For $d=2$, this gives

$$\xi_{Gx} = \text{const} \frac{K_{\rm Sm}^{3/2}}{B_{\rm Sm}^{1/2}} , \tag{B13}$$

for the Ginzburg length scale parallel to layers. The Ginzburg length scale normal to smectic layers, ξ_{Gz}, is related to ξ_{Gx} via the harmonic theory scaling $[z \sim (B_{\rm Sm}/K_{\rm Sm})^{1/2} x^2]$ yielding

$$\xi_{Gz} = \left[\frac{B_{\rm Sm}}{K_{\rm Sm}} \right]^{1/2} \xi_{Gx}^2 . \tag{B14}$$

In order to observe the anomalous fluctuation behavior, one needs a sample with sizes satisfying $L_x > \xi_{Gx}$ and $L_z > L_{Gz}$. We recall that smectic elastic constants in (B13) and (B14) are reduced (divided by the temperature T). If ordinary (nonreduced) elastic constants have finite $T \to 0$ limits, then, by (B13) and (B14),

$$\xi_{Gx} \sim \frac{1}{T}, \quad \xi_{Gz} \sim \frac{1}{T^2} , \tag{B15}$$

at low temperatures. In practice, however, smectic elastic constants are frequently entropic in origin, and, at low T, change as power laws of T. This would change only the power laws in Eq. (B15). On the other side, the dislocation length scales ξ_x and ξ_z behave as $\sim \exp(\text{const}/T)$ at low T (see Sec. IV). Thus, at least at low T one has a broad range of length scales where the smectic anomalous elastic behavior of Sec. IV can be observed.

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