Raman-ripple lasers

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Radiation from a relativistic electron beam passing through a periodic electric field in vacuum is calculated in the self-consistent-field approximation in the framework of nonequilibrium statistical mechanics. The synergic frequencies of the emerging radiation are due to Raman scattering. The phenomena of frequency switching and introduction of additional polarization take place as electric-field wave numbers are changed. In the process of tuning, there are critical wave numbers for which radiation is ducted through the medium without modification.

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I. INTRODUCTION

In a series of papers, Raman and Nath [1] considered the problem of the passage of a monochromatic beam through a liquid medium in a cell sustaining a standing acoustic wave. The experimental arrangement is shown in Fig. 1. The monochromatic wavefront incident on the *Y-Z* surface of the cell in the negative *X* direction would experience a periodically varying refractive index and produce maximum coherence on normal incidence. If the beam is of oblique incidence, the component of the beam parallel to the *Y-Z* plane will not contribute to the diffraction pattern. On changing the frequency of the acoustic mode, the phenomenon of frequency switching takes place. The fact that the refractive index is a function of space in the case of standing waves or a function of space and time in the case of progressive waves can make Maxwell's equations nonlinear, resulting in the Raman-Nath difference differential equation for the amplitude of the diffraction orders.

Similar ideas were invoked recently by Chen and Dawson (CD) [2] in designing an ion-ripple laser. In this scheme, the liquid in the above system is replaced by a plasma, which is produced by a laser beam incident on the *Y-Z* plane of the cell containing a gas, and the plasma is produced by photoionization. On this plasma cell containing the acoustic wave a relativistic electron beam (REB) is shot in the positive *Z* direction. It was further supposed that the REB would repel all the space charges in the plasma and it would experience an undulating electric field provided by the rigidly fixed ions of the plasma.

The CD model, however, is physically unrealistic, for, as the REB enters the plasma medium, it would establish a Cerenkov cone. The beam electrons would not interact with the electrons outside the cone, while electrons within the cone (which generate Cerenkov radiation) cannot escape across the cone surface (a surface of discontinuity). Hence the Cerenkov effect is significant and cannot be ignored in a REB-plasma interaction.

In the present formulation, we investigate the problem of radiation emission in the CD scheme. In this paper we consider the motion of a REB through vacuum in which there exists a periodic electric field with a wave number l maintained externally. The more general problem of the Cerenkov effect in the passage of a REB through a medium plasma instead of vacuum sustaining an acoustic wave will be considered in a later work. We label one beam electron as a test particle (T) and the rest as field particles (i) and hence the problem is that of propagation of a relativistic test particle in an electrostatic periodic field having a relativistic electron gas. In Sec. II we formulate the problem by writing the general Hamiltonian and the Liouville equation. The operators that constitute

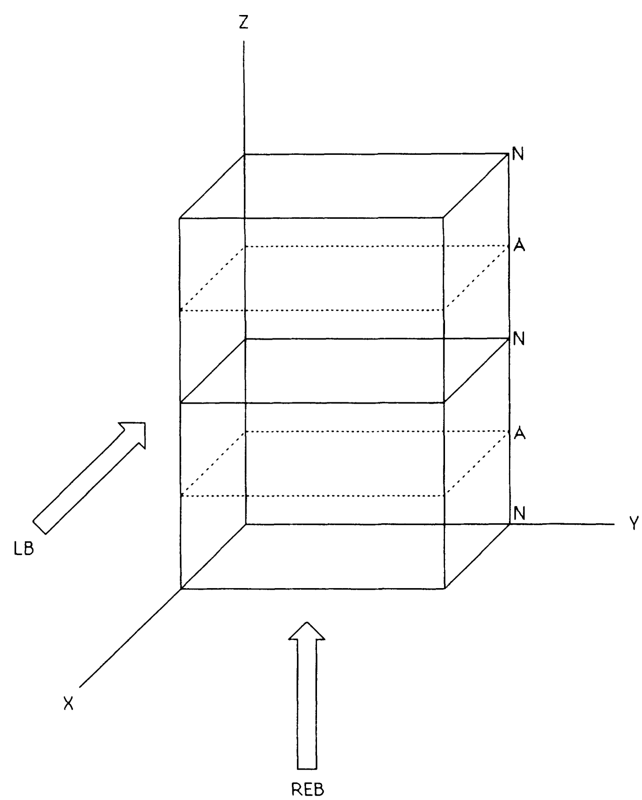

FIG. 1. Plasma cell containing the acoustic wave in hydrogen gas ionized by the laser beam (LB) in the negative *X* direction. The REB is in the positive *Z* direction. The planes of node (N) and antinode (A) are parallel to the *X-Y* plane.

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this equation have peculiar properties: the propagator consists of two noncommuting parts because of the presence of the external field. A Baker-Hausdorff expansion of these operators is effected in Appendix B. Explicit expressions for the operators containing the interaction potentials are given in Appendix A. In Sec. III, we give the general solution of the Liouville equation.

We exploit the linear nature of the Liouville equation and write the formal solution in an iterative manner. Following the diagrammatic technique [3] developed by Prigogine *et al.*, we select a subset of diagrams from the Dyson series corresponding to the interaction time scale $(m/e^2c)^{1/2}$, e and m being the electronic charge and mass, respectively, and c the concentration in thermodynamic limit ($N\rightarrow\infty$, $V\rightarrow\infty$, $N/V=c$). The results are known to be equivalent to those obtained starting from the nonlinear Vlasov integro-differential equation. This infinite set of terms can be summed up *exactly*, obtaining a response function. Since the system consists of a longitudinal mode such as the electric wave, the interaction between the transverse and longitudinal modes has to be taken systematically. We ignore scalar modes, as most of the effects are included in the longitudinal case. In Sec. IV, we have the one-particle distribution function (OPDF) having both the transverse and longitudinal components. In these evaluations, we specify an initial state constructed out of the eigenfunctions of the unperturbed part of the Hamiltonian. The arbitrariness of the initial state is reduced since we sum up the entire series consisting of the various field-particle interactions with the test particle.

We then evaluate the power radiated per unit solid angle by the system (Sec. V), by averaging the test-particle Hamiltonian with respect to the OPDF and differentiating this with respect to time and the solid angle subtended by the propagation vector. We obtain the usual resonance conditions which give the Cerenkov angle as well as the refractive index. These optical properties are detailed in Sec. VI. It may be realized that the radiated power depends on the response function of both the transverse and longitudinal modes. As the OPDF consists of two distinct modes, the transverse (λ) and longitudinal (σ) modes, we can calculate the radiated power both in the electromagnetic and acoustic modes. In the present paper we calculate only the transverse component. In the acoustic mode, the longitudinal energy would modify the acoustic frequency and we get a renormalized frequency for this mode. We shall investigate this phenomenon in a later work. The present results have a direct application to the problem of interplanetary scintillation as well as the passage of a high-energy particle through an inhomogeneous medium.

The explicit evaluation of response function is given in Appendix C. We close this paper with a discussion of the main results and indicate the future direction of work. We propose to make explicit numerical calculations of these functions later.

II. FORMULATION

The dynamical system considered here consists of an undulating electric field maintained externally in vacuum, in which a relativistic electron beam is pumped in. There also exists a radiation field with both transverse (λ) and longitudinal (σ) frequencies. We follow the notations of Heitler [4]. The total system is characterized by the Hamiltonian

$$\mathcal{H}=\mathcal{H}_B+\mathcal{H}_R \;, \tag{1}$$

where $\mathcal{H}_B$ is the Hamiltonian of the beam electrons and $\mathcal{H}_R$ that of the radiation field. We shall assign a label T for a particle in the beam and call it the test particle. The rest of the beam particles are denoted by the index (i). We write

$$\mathcal{H}_B=\mathcal{H}_T+\sum_i \mathcal{H}_i \;. \tag{2}$$

The explicit form for these can be written as

$$\begin{aligned}\mathcal{H}_T&=mc^2\gamma_T-e^2l\cos lz_T+e\phi_T\\&=mc^2(1+u_T^2)^{1/2}-e^2l\cos lz_T+e\phi_T\;,\\ \mathcal{H}_i&=mc^2(1+u_i^2)^{1/2}-(m\omega_{\rm pl}^2/l^2)\cos lz_i+e\phi_i\;,\\ \mathcal{H}_R&=\sum_\lambda \nu_\lambda J_\lambda+\sum_\sigma \nu_\sigma J_\sigma-\sum_{0\sigma}\nu_\sigma J_{0\sigma}\;.\end{aligned} \tag{3}$$

In the expression for $\mathcal{H}_{T,i}$, the vector $\mathbf{u}$ is defined as

$$mc\mathbf{u}=\mathbf{P}-(e/c)(\mathbf{A}_\lambda+\mathbf{A}_\sigma)\;, \tag{4}$$

where $\mathbf{P}$ is the canonical momentum and $\mathbf{A}_\lambda$ and $\mathbf{A}_\sigma$ are the interaction vector potentials which are functions of $\mathbf{q},J_\lambda,\omega_\lambda$ and $\mathbf{q},J_\sigma,\omega_\sigma$, respectively. ϕ is the interaction scalar potential which is a function of $\mathbf{q},J_{0\sigma},\omega_{0\sigma}$. The external field is represented by the harmonic potential and Poisson's equation gives the local charge density. $\omega_{\rm pl}$ is the plasma frequency $(4\pi ce^2/m)^{1/2}$ wherein c is the concentration in thermodynamic limit. The radiation Hamiltonian is in the action-angle representation (J,ω). $\mathbf{A}_\lambda$ and $\mathbf{A}_\sigma$ are characterized by the polarization vectors $\mathbf{e}_\lambda$ and $\mathbf{e}_\sigma$ such that $\mathbf{A}_\lambda$ is the transverse (or divergence free) part with $\mathbf{e}_\lambda\cdot\mathbf{K}_\lambda=0$ and $\mathbf{A}_\sigma$, the longitudinal part, with $\mathbf{e}_\sigma\times\mathbf{K}_\sigma=0$ (curl free).

We shall now define a Liouville density $\rho=\rho(\mathbf{q}_T,\boldsymbol{\beta}_T;\mathbf{q}_i;\mathbf{P}_i;J_\lambda\omega_\lambda;J_\sigma\omega_\sigma;J_{0\sigma}\omega_{0\sigma};t)$, which satisfies the linear Liouville equation

$$\frac{\partial\rho}{\partial t}+\mathcal{L}\rho=e(\delta L)\rho\;, \tag{5}$$

where $\mathcal{L}$ is the Liouville operator pertaining to the noninteracting part of the Hamiltonian and (δL) is that containing the interacting part. These operators can be written as

$$\mathcal{L}=\mathcal{L}_T+\mathcal{L}_i+\mathcal{L}_R\;, \tag{6}$$

wherein

$$\mathcal{L}_T=c\boldsymbol{\beta}\cdot\frac{\partial}{\partial\mathbf{q}_T}-(e^2l^2/mc\gamma^3)\sin(lz_T)\mathbf{k}\cdot\frac{\partial}{\partial\boldsymbol{\beta}_T}\quad\text{(test particle)}\;, \tag{7a}$$

$$\mathcal{L}_i=(\mathbf{P}_i/m\gamma_i)\cdot\frac{\partial}{\partial \mathbf{q}_i}-(m\omega_{\rm pl}^2/l)\sin(lz_i)\mathbf{k}\cdot\frac{\partial}{\partial \mathbf{P}_i}$$

$$\text{(beam particle)}\ , \qquad (7b)$$

$$\mathcal{L}_R=\bar{v}_\lambda\frac{\partial}{\partial\omega_\lambda}+\bar{v}_\sigma\frac{\partial}{\partial\omega_\sigma}-\bar{v}_\sigma\frac{\partial}{\partial\omega_{0\sigma}}\ . \qquad (7c)$$

In writing the above operators, the test-particle operators are expressed in $\boldsymbol{\beta}$ and $\mathbf{q}$ while those of beam particles are expressed in canonical variables; $\bar{v}_j^2=v_j^2+\omega_{\rm pl}^2/\gamma$ in the random-phase approximation (RPA) [5]. It may be realized that (7a) and (7b) have two parts which do not give a c number on commutation. Hence, to write the propagator $e^{-(\bar{A}+\bar{B})\tau}$ as $e^{-\bar{A}\tau}e^{+\bar{B}\tau}$ we have to make a Baker-Hausdorff expansion and this is given in Appendix B.

The interaction part (δL) in Eq. (5) can be written as

$$(\delta L)=(\mathcal{A}_T^\lambda+\mathcal{B}_T^\lambda+\mathcal{A}_T^\sigma+\mathcal{B}_T^\sigma+\mathcal{A}_T^{0\sigma}+\mathcal{B}_T^{0\sigma})+(\mathcal{A}_i^\lambda+\mathcal{B}_i^\lambda+\mathcal{A}_i^\sigma+\mathcal{B}_i^\sigma+\mathcal{A}_i^{0\sigma}+\mathcal{B}_i^{0\sigma}) \qquad (8)$$

and the explicit expressions are given in Appendix A. In writing these operators, we have used, following [4], the expressions for the interaction potentials as

$$A_j=(8c^2/V\bar{v}_j\gamma)^{1/2}\mathbf{e}_j(\sqrt{J_j}\cos\omega_j)\cos\mathbf{K}_j\cdot\mathbf{q}\ ,$$
$$\phi=(8c^2/V\bar{v}_\sigma\gamma)^{1/2}(\sqrt{J_{0\sigma}}\cos\omega_{0\sigma})\sin\mathbf{K}_\sigma\cdot\mathbf{q} \qquad (9)$$

together with the $-j$ part by replacing $\cos(\mathbf{K}_j\cdot\mathbf{q})$ by $\sin(\mathbf{K}_j\cdot\mathbf{q})$ and $\sin(\mathbf{K}_j\cdot\mathbf{q})$ by $-\cos(\mathbf{K}_j\cdot\mathbf{q})$, j being λ or σ, and $\mathbf{q}$ takes the suffix T and i, representing the test and beam particles, respectively. Further, in writing the $\mathcal{B}$ operators, we have written only a part of Poisson's bracket since the initial state is taken to be independent of ω_j. In the above operators $\boldsymbol{\beta}=\mathbf{u}_T/(1+u_T^2)^{1/2}$ and $(1+u_T^2)^{1/2}=\gamma=(1-\beta^2)^{-1/2}$. We have $\dot{\boldsymbol{\beta}}\cdot\partial/\partial\boldsymbol{\beta}=\dot{\mathbf{u}}\cdot\partial/\partial\mathbf{u}$.

III. GENERAL SOLUTION OF LIOUVILLE EQUATION

We formally integrate the Liouville equation (5) and write the general solution as a Volterra equation of second kind as

$$\rho(t)=e^{-\mathcal{L}t}\rho(0)+e\int_0^t d\tau\, e^{-\mathcal{L}(t-\tau)}(\delta L)\rho(\tau)\ , \qquad (10)$$

wherein the first term on the right-hand side is a free-flow term in which the initial state $(t=0)$ is transferred to time t without any interaction, while the second term is the modification in the evolution due to interaction in the system. Equation (10) could be considered as an inhomogeneous integral equation with a kernel function as the operator, and it is non-Markovian, wherein the final state depends not only on the initial state, but on the path along which the evolution has proceeded. Equation (10) is an exact solution of Eq. (5) and hence contains all the time scales that exist in the system. The non-Markovian nature is due to this fact. We iterate Eq. (10) and write the solution as a Dyson series

$$\rho(t)=\sum_{n=0}^{\infty}e^n\int_0^t dt_1\int_0^{t_1}dt_2\cdots\int_0^{t_{n-1}}dt_n\, e^{-\mathcal{L}(t-t_1)}(\delta L)e^{-\mathcal{L}(t_1-t_2)}(\delta L)\cdots e^{-\mathcal{L}(t_{n-1}-t_n)}(\delta L)e^{-\mathcal{L}t_n}\rho(0)\ . \qquad (11)$$

In Eq. (11), since (δL) are operators independent of time, the propagators appear in a convolution form, and in a resolvant formalism Eq. (11) reduces to a geometric series. If we define the Laplace transform as in [6] we can write (11) as

$$\rho(t)=\sum_{n=0}^{\infty}e^n\oint dZ\, e^{-iZt}\mathcal{R}^0(Z)[(\delta L)\mathcal{R}^0(Z)]^n\rho(0)\ , \qquad (12)$$

where

$$\mathcal{R}^0(Z)=\int_0^\infty dt\, e^{iZt}e^{-\mathcal{L}t}=-(iZ-\mathcal{L})^{-1}\ , \qquad (13)$$

$\mathcal{L}$ being an operator and the Laplace parameter Z consisting of a small positive imaginary part to ensure convergence of the integral.

Equations (10) and (12) are exact solutions of the Liouville equation and contain the full information of the system. From this we now extract an infinite series, with the nth term having $(e^2c/m)^n$ as the coefficient corresponding to the interaction time scale and this constitutes the self-consistent-field approximation. A fluid approximation is obtained if we take terms having coefficients $(e^4c)^{-1}(m/\beta^3)^{1/2}$, which correspond to the relaxation time $(\beta^{-1}=kT)$. In these, as mentioned before, c is the concentration in the thermodynamic limit. The method of selecting the diagrams that constitute the self-consistent-field approximation has been detailed elsewhere [3]. After selecting the subset of infinite terms, we integrate over all the field variables and sum the series to obtain the OPDF.

IV. ONE-PARTICLE DISTRIBUTION FUNCTION

Any physical property of a system, given as a local function, can be obtained by averaging that particular microscopic variable with the OPDF. The OPDF can be written as

$$f_j=e^2\int_0^t dt_1\int_0^{t_1}dt_2\, e^{-\mathcal{L}_T(t-t_1)}\mathcal{A}_j^T e^{-\mathcal{L}(t_1-t_2)}\times\left[\oint dZ\, e^{-iZ(t_1-t_2)}\frac{iZ}{Z^2-\bar{v}_j^2}\times\frac{1}{1-(\mathcal{E}_\lambda+\mathcal{E}_\sigma)}\right]\mathcal{B}_{j'}^T\times e^{-\mathcal{L}t_2}\rho_T(0)\ , \qquad (14)$$

where j takes the index λ or σ to get the OPDF for the transverse or the longitudinal component. Hence in evaluating the energy loss, one can calculate the electromagnetic radiation component by taking the λ component and the acoustic mode by the σ part. In evaluating the response function $\mathcal{E}_j$, we have

$$\mathcal{E}_j^i(t_n,t_{n+2}) = \sum_i e^2 \int_{t_{n+2}}^{t_n} dt_{n+1} \int d\mathbf{P}_i d\mathbf{q}_i e^{-\mathcal{L}_i(t_n-t_{n+1})} \mathcal{B}_j^i \times e^{-\mathcal{L}_i(t_{n+1}-t_{n+2})} \mathcal{A}_{j'}^i \times e^{-\mathcal{L}t_{n+2}} \rho_i(0) \tag{15}$$

where t_n is the time at the nth instant and the summation over the particle index i gives a factor N. The matrix element connecting the test particle T at two instants at two vertices can be represented by W_{TT} and if the test particle interacts with a field particle i, then this matrix element can be represented by W_{TiiT}, where each index represents a vertex. One can show that $W_{TiiT}=W_{TT}\mathcal{E}_j$ and it is this factorization that enables one to write (14). In evaluating the response function (15), we need an initial state. We construct this as

$$\rho(0)=\sum_{n,m} \langle \psi_m^* | (1+e^{\beta\mathcal{H}_0})^{-1} | \psi_n \rangle , \tag{16}$$

where ψ_n are eigenfunctions of the unperturbed Hamiltonian of the system and β is the Boltzmann factor $(kT)^{-1}$. In the cold-plasma approximation we set the Boltzmann factor to naught. The presence of acoustic wave gives the eigenfunction in terms of Mathieu functions, and we retain the periodic part and write

$$\rho(0)=\sum_{n,i} S_{ni}^2 \sin 2nlz_i \delta(P_1)\delta(P_2)\delta(P_3-P_0) , \tag{17}$$

where now S_n satisfies the recurrence relation

$$2(a-n^2l^2)S_n+b(S_{n-1}+S_{n+1})+C(S_{n-2}+S_{n+2})=0 ,$$

with

$$a=(E^2-m^2c^4)/\hbar^2c^2+\tfrac{1}{2}(m\omega_{\rm pl}^2/\hbar c l^2)^2-(p_1^2+p_2^2)\hbar^2 ,$$
$$b=2Em\omega_{\rm pl}^2/(\hbar c l)^2, \quad C=(m\omega_{\rm pl}^2/\hbar c l^2)^2 . \tag{18}$$

After performing all the integrations appearing in (15), taking the Laplace transform, and going over to the kinetic regime by taking the asymptotic limit in time ($Z \to 0$) we get

$$\mathcal{E}_\lambda=\tilde{\omega}^2/Z^2-\bar{v}_\lambda^2 ,$$
$$\mathcal{E}_\sigma=2\tilde{\omega}^2/Z^2-\bar{v}_\sigma^2 , \tag{19}$$

where $\tilde{\omega}^2$ is the square of the renormalized plasma frequency as given in (C9). It may be seen that the renormalized plasma frequency is a complicated function of plasma frequency and wave frequency $\omega_W(=lV_z/\gamma)$ appearing in $(\omega_{\rm pl}/\omega_W)^2$ as also in the argument of Bessel function. In this sense this is a transcendental function. In the next section we evaluate the energy loss experienced by the test particle which is the same as that radiated by the system.

V. RADIATED POWER

In this formalism, as the test particle travels through the beam particles, in the presence of the externally maintained periodic electric field, the test particle excites the medium and establish a Cerenkov cone, generating Cerenkov radiation. If there were no external maintenance of the periodic field, the test particle would lose energy in the process of radiation. However, since there is an external supply of energy, the system is open thermodynamically, and therefore the test particle would extract energy from the acoustic wave which has infinite capacity (because of external supply) and give rise to nonlinear free electron laser (FEL) interactions. We obtain the radiated power by averaging the test-particle Hamiltonian with the OPDF and differentiating this with respect to time. Thus the power emitted per unit solid angle is

$$\frac{d^2E}{dtd\Omega}=\frac{5e^2}{2c}\frac{1}{\pi^2\gamma}\int \bar{v}_\lambda^2 d\bar{v}_\lambda \int d\boldsymbol{\beta} \int dz \int_0^t dt_2 (\mathbf{e}_\lambda\cdot\boldsymbol{\beta})(\mathbf{e}_\lambda\cdot\bar{\boldsymbol{\beta}})\cos(pz+\psi)e^{-\mathcal{L}t_2}\rho_T(0) \times \left[\oint dZ\, e^{-iZ(t-t_2)} \frac{iZ}{Z^2-\bar{v}_\lambda^2} \frac{1}{1-(\mathcal{E}_\lambda+\mathcal{E}_\sigma)} \right] , \tag{20}$$

where now

$$\mathbf{K}_{\lambda'}=\mathbf{K}_\lambda+\mathbf{k}p ,$$

which implies

$$\mathbf{e}_{\lambda'}=\mathbf{e}_\lambda-\mathbf{i}(e_{\lambda 3}/K_{\lambda 1})p ,$$
$$\bar{\boldsymbol{\beta}}=\boldsymbol{\beta}+\mathbf{k}(e^2l/mc^2\gamma^3\beta_3)\{\cos(lz-cl\beta_3\overline{t-t_2})-\cos lz\} ,$$
$$\psi=-c\boldsymbol{\beta}\cdot\mathbf{K}_{\lambda'}(t-t_2)+(K_{3\lambda'}e^2/mc^2\gamma^3\beta_3^2) \times\{\sin(lz-cl\beta_3\overline{t-t_2}) -\sin lz+cl\beta_3\overline{t-t_2}\cos lz\} . \tag{21}$$

$\bar{\boldsymbol{\beta}}$ and ψ have been obtained by applying the propagator (shift operator) given in (B5). We get a conservation condition on wave vectors as $\mathbf{K}_{\lambda'}=\mathbf{K}_\lambda+\mathbf{k}p$ with an additional wave number in the Z direction. This modifies the polarization vector $\mathbf{e}_{\lambda'}$ by introducing an additional polarization in the X direction, normal to the plane of the wave. The X and Y components of propagation remain unaltered. A straight-forward algebra would enable one to evaluate the inverse Laplace transform appearing in (20) as

$$\frac{\omega_1^2-\bar{v}_\sigma^2}{\phi}\cos\omega_1(t-t_2)-\frac{\omega_2^2-\bar{v}_\sigma^2}{\phi}\cos\omega_2(t-t_2) , \tag{22}$$

where ω_1 and ω_2 are the effective frequencies given by

$$\omega_1^2=(\bar{v}_\lambda^2+\bar{v}_\sigma^2+3\varpi^2+\phi)/2 ,$$
$$\omega_2^2=(\bar{v}_\lambda^2+\bar{v}_\sigma^2+3\varpi^2-\phi)/2 , \tag{23}$$

with

$$\phi^2=(\bar{v}_\lambda^2-\bar{v}_\sigma^2-\varpi^2)^2+8\varpi^4 . \tag{24}$$

It may be observed that ϖ^2 is always ≥ 0. Hence ω_1^2 and ω_2^2 are always >0. For those values at which $\varpi^2=0$, $\omega_1^2,\omega_2^2=\bar{v}_\lambda^2,\bar{v}_\sigma^2$. Hence for those values of the parameters for which $\varpi^2=0$ the emerging radiation will have frequencies of the original field. Again ω_1^2,ω_2^2 are both synergic functions of the parameters of the system such as density (through plasma frequency) and the Z component of the velocity of the beam particles. Having obtained these effective frequencies, we shall substitute (22) in (20) and carry out the integrations over $\boldsymbol{\beta}$, z, and t_2. We shall as before take an initial state of the test particle as given by

$$\rho_T(0)=\sum_{n=0}^{\infty} C_n^2\cos 2nlz_T\,\delta(\beta_1)\delta(\beta_2)\delta(\beta_3-\beta_0) , \tag{25}$$

wherein C_n^2 is the strength of the initial state. We shall perform all the integrations, except that of $\bar{v}_\lambda$, and take the asymptotic limit in time. We can do this also by taking the Laplace transform after combining terms in $(t-t_2)$ and t_2 and take the limit of $Z\to 0$. It is obvious that these two procedures are equivalent. We than obtain the power emitted per unit solid angle as

$$\frac{d^2E}{dtd\Omega}=\frac{e^2}{c}\int \bar{v}_\lambda d\bar{v}_\lambda(1-\mu^2)\Gamma\left[\left\{\frac{\omega_1^2-\bar{v}_\sigma^2}{2\phi}\right\}[(A\mu)^2-(\beta_0\mu\pm\omega_1/\bar{v}_\lambda)^2]^{-1/2}-\left\{\frac{\omega_2^2-\bar{v}_\sigma^2}{2\phi}\right\}[(A\mu)^2-(\beta_0\mu\pm\omega_2/\bar{v}_\lambda)^2]^{-1/2}\right.$$
$$+\frac{A(\omega_1^2-\bar{v}_\sigma^2)}{4\phi}[(A\mu)^2-(\beta_0\mu-\overline{cl\beta_0\pm\omega_1/\bar{v}_\lambda})^2]^{-1/2}$$
$$\left.-\frac{A(\omega_2^2-\bar{v}_\sigma^2)}{4\phi}[(A\mu)^2-(\beta_0\mu-\overline{cl\beta_0\pm\omega_2/\bar{v}_\lambda})^2]^{-1/2}\right] , \tag{26}$$

where now

$$A=(e^2l/mc^2)(\gamma^3\beta_0^2)^{-1} ,$$
$$\Gamma=\frac{C_n^2\beta_0^2}{\pi^2\gamma}J_0^2(A\bar{v}_\lambda\mu/cl)J_0^2(2nA)[2ncl\beta_0(A^2-1)^{-1/2}]^{-1} . \tag{27}$$

In the above, A is the ratio of the electric energy in the wave to the rest energy. In Eq. (26) we get two sets of terms, the former being implicit function of the acoustic wave number while the latter are explicit functions of l. Again the second one has the effective frequency shifted by an amount $\beta_0 cl$. The power given in (26) consists only of the most important term as the full expression would have terms from the higher harmonics.

VI. OPTICAL PROPERTIES

In expression (26), μ ($=\cos\vartheta$) is the polar angle of the propagation vector in spherical polar coordinates. From (26), one can obtain various resonance conditions, viz.,

$$\mu=\pm\omega_n/\bar{v}_\lambda(\beta_0\pm A) ,$$
$$\mu=\pm(cl\beta_0\pm\omega_n)/\bar{v}_\lambda(\beta_0\pm A) , \tag{28}$$

where n can be 1 or 2. This may be compared with those obtained by Pratap and Sen [3]. One could see that the denominator is also a function of l through A, and in the second set of resonance conditions, ω_n is shifted by $\pm cl\beta_0$, forming sidebands. We shall now substitute these conditions for μ in (26) and can write it in the form suggested by Frank and Tamm as

$$\frac{d^2E}{dtd\Omega}=\frac{e^2}{c}\int\omega_n d\omega_n\left[1-\frac{1}{\beta^2N^2}\right] , \tag{29}$$

where

$$N^2=(1\pm A/\beta_0)^2\left[\frac{\bar{v}_\lambda^2}{\omega_1^2};\frac{\bar{v}_\lambda^2}{\omega_2^2}\right] \tag{30}$$

and

$$N^2=(1\pm A/\beta_0)^2\left[\frac{\bar{v}_\lambda^2}{(\omega_1\pm cl\beta_0)^2};\frac{\bar{v}_\lambda^2}{(\omega_2\pm cl\beta_0)^2}\right] . \tag{31}$$

We shall solve for $\bar{v}_\lambda^2$ in terms of ω_1 or ω_2 and write

$$\frac{\bar{v}_\lambda^2}{\omega_n^2}=1-\left[\frac{\bar{v}_\sigma^2\varpi^2}{(\bar{v}_\sigma^2+2\varpi^2)}\right]\frac{1}{\omega_n^2}-\frac{2\varpi^4}{(\bar{v}_\sigma^2+2\varpi^2)}\frac{1}{\omega_n^2-\bar{v}_\sigma^2-2\varpi^2} . \tag{32}$$

Substituting (32) in (30) we get the expressions for the refractive index N^2. We get similar expressions for the shifted frequencies case appearing in (31). Since ω_1^2 and ω_2^2 are both positive definite we could express N^2 as a function of ω_1^2 and ω_2^2. If we add expressions for ω_1^2 and ω_2^2, we can write

$$\frac{\bar{v}_\lambda^2}{\omega_1^2}=1+\frac{\omega_2^2}{\omega_1^2}-\frac{(\bar{v}_\sigma^2+3\varpi^2)}{\omega_1^2}\quad\text{or}\quad 1+\frac{\omega_1^2}{\omega_2^2}-\frac{(\bar{v}_\sigma^2+3\varpi^2)}{\omega_2^2} . \tag{33}$$

The refractive index is an explicit function of the acoustic wave number through A in (30) and (31) besides being an implicit function through the effective frequencies ω_1 and ω_2 as well as a function of $\bar{v}_\sigma^2$.

VII. CONCLUSION

We have considered the passage of a REB through vacuum in which there exists a periodic electric field. This periodic longitudinal field enables the exchange of energy between the longitudinal and transverse modes and this exchange process manifests itself in the OPDF as given in (14). The solution of the Liouville equation as given in (12) and the OPDF (14) after summing up a subset of infinite terms that contribute towards the self-consistent field are non-Markovian in nature, which implies the presence of all time scales less than the interaction time scale $(m/e^2c)^{1/2}$. It may be seen that this method gives synergic frequencies and is a direct consequence of taking Raman scattering in a systematic manner.

To obtain the kinetic regime, we make the solution Markovian by taking the asymptotic limit in time. This procedure results in a "dressed" particle approach and we get a "renormalized" plasma frequency as given by (C9), which is a highly complex function of the plasma frequency $\omega_{\rm pl}$ and the acoustic wave frequency ω_W. These also appear in the argument of the Bessel function and this synergic frequency reduces to zero for a set of critical values of the parameters for which $(2n/\gamma^3)(\omega_{\rm pl}/\omega_W)^2$ are zeros of Bessel function of zeroth order. For these critical values, the response function reduces to zero and the emerging radiation is unaffected by the presence of the longitudinal mode. The radiation will have a frequency $\bar{\nu}_\lambda$ or $\bar{\nu}_\sigma$ and the medium will behave like a vacuum.

The second significant result presented here is the pair of effective frequencies ω_1 and ω_2 given by (23). These frequencies are functions [through ϕ (24)] of $\bar{\nu}_\lambda$ and $\bar{\nu}_\sigma$, the plasma frequency $\omega_{\rm pl}$, and the acoustic wave frequency ω_W. It may be seen that ω_1^2,ω_2^2 are both >0 provided $(\bar{\nu}_\lambda^2+\bar{\nu}_\sigma^2+3\tilde{\omega}^2)>\phi$, which implies $\bar{\nu}_\lambda^2\bar{\nu}_\sigma^2+\tilde{\omega}^2(2\bar{\nu}_\lambda^2+\bar{\nu}_\sigma^2)>0$, which is always satisfied. Hence the possibility of absorption does not exist. The above condition, however, depends on the acoustic wave number l through $\tilde{\omega}^2$. Hence the longitudinal mode of the energy is always converted into the transverse mode as the acoustic mode is maintained from the outside, and since it has an infinite capacity, the system becomes thermodynamically open, and the FEL mechanism becomes operative. This results in an irreversible process.

The third feature is the Cerenkov angle given by μ $(=\cos\vartheta)$ as given by (28). The first part is that due to ω_1 and ω_2 and the dependence on l appears through these frequencies as well as through A. However, in the second expression, these frequencies are shifted by $cl\beta_0$, where c is the velocity of light. There are in all six values of μ for each ω—or we get a set of six concentric cones, one inside the other. This also manifests itself in the expression for the refractive index, as given by (30) and (31). The conditions for the refractive index to be positive definite is given by $\omega_1^2,\omega_2^2>3\tilde{\omega}^2+\bar{\nu}_\sigma^2$, which is always satisfied. Hence the system can emit only at the expense of the acoustic mode.

The above analysis can be generalized to include a random component. In this analysis, the acoustic wave number l is taken completely arbitrary. If we now generalize this in such a way that this wave number has a distribution given by a normalized Gaussian one, we can interpret this as a random distribution and one can easily extend the whole analysis. If we now integrate the various parameters such as Cerenkov angle, refractive index, the effective frequencies, etc. over l with this distribution function, we can apply this to various situations such as interplanetary scintillations as well as modulations effected in various astrophysical situations on the passage of a high-energy particle. It is, however, realized that it may be difficult to get the results in a closed form and probably one may have to expand these in series and integrate term by term, giving the series in terms of Hermite functions. This aspect of the work is in progress.

We propose to extend this analysis to a situation taking into account the medium being constituted by a quiescent plasma. In that case the Cerenkov radiation generated by the medium plasma would be more significant as compared to that produced by the beam electrons that has been considered here.

APPENDIX A

The operators as given in (8) are

$$A_T^\lambda=-[(m\gamma^3)^{-1}(8/V\bar{\nu}_\lambda\gamma)^{1/2}]\times\frac{\partial}{\partial\boldsymbol{\beta}}\cdot\left[\frac{\bar{\nu}_\lambda}{c}\mathbf{e}_\lambda(\sqrt{J_\lambda}\sin\omega_\lambda)\cos(\mathbf{K}_\lambda\cdot\mathbf{q}_T)+\boldsymbol{\beta}\times(\mathbf{e}_\lambda\times\mathbf{K}_\lambda)(\sqrt{J_\lambda}\cos\omega_\lambda)\times\sin(\mathbf{K}_\lambda\cdot\mathbf{q}_T)\right], \tag{A1}$$

$$A_T^\sigma=-[(m\gamma^3)^{-1}(8/V\bar{\nu}_\sigma\gamma)^{1/2}]\times\frac{\partial}{\partial\boldsymbol{\beta}}\cdot\left[\frac{\bar{\nu}_\sigma}{c}\mathbf{e}_\sigma(\sqrt{J_\sigma}\sin\omega_\sigma)\cos(\mathbf{K}_\sigma\cdot\mathbf{q}_T)\right], \tag{A2}$$

$$A_T^{0\sigma}=[(m\gamma^3)^{-1}(8/V\bar{\nu}_\sigma\gamma)^{1/2}]\times\frac{\partial}{\partial\boldsymbol{\beta}}\cdot[\mathbf{K}_\sigma(\sqrt{J_{0\sigma}}\cos\omega_{0\sigma})\cos(\mathbf{K}_\sigma\cdot\mathbf{q}_T)], \tag{A3}$$

$$B_T^\lambda=(8c^2/V\bar{\nu}_\lambda\gamma)^{1/2}(\sqrt{J_\lambda}\sin\omega_\lambda)(\boldsymbol{\beta}\cdot\mathbf{e}_\lambda)\cos(\mathbf{K}_\lambda\cdot\mathbf{q}_T)\frac{\partial}{\partial J_\lambda}, \tag{A4}$$

$$B_T^\sigma=(8c^2/V\bar{\nu}_\sigma\gamma)^{1/2}(\sqrt{J_\sigma}\sin\omega_\sigma)(\boldsymbol{\beta}\cdot\mathbf{e}_\sigma)\times\cos(\mathbf{K}_\sigma\cdot\mathbf{q}_T)\frac{\partial}{\partial J_\sigma}, \tag{A5}$$

$$B_T^{0\sigma}=-(8c^2/V\bar{\nu}_\sigma\gamma)^{1/2}(\sqrt{J_{0\sigma}}\sin\omega_{0\sigma})\times\cos(\mathbf{K}_\sigma\cdot\mathbf{q}_T)\frac{\partial}{\partial J_{0\sigma}}, \tag{A6}$$

$$A_i^\lambda=[(m\gamma_i)^{-1}(8/V\bar{\nu}_\lambda\gamma)^{1/2}](\sqrt{J_\lambda}\cos\omega_\lambda)\times\left[\cos(\mathbf{K}_\lambda\cdot\mathbf{q}_i)\mathbf{e}_\lambda\cdot\frac{\partial}{\partial\mathbf{q}_i}+(\mathbf{e}_\lambda\cdot\mathbf{P}_i)\sin(\mathbf{K}_\lambda\cdot\mathbf{q}_i)\mathbf{K}_\lambda\cdot\frac{\partial}{\partial\mathbf{P}_i}\right], \tag{A7}$$

$$A_i^{\sigma}=[(m\gamma_i)^{-1}(8/V\bar{v}_\sigma\gamma)^{1/2}](\sqrt{J_\sigma}\cos\omega_\sigma)\times\left[\cos(\mathbf{K}_\sigma\cdot\mathbf{q}_i)\mathbf{e}_\sigma\cdot\frac{\partial}{\partial\mathbf{q}_i}+(\mathbf{e}_\sigma\cdot\mathbf{P}_i)\sin(\mathbf{K}_\sigma\cdot\mathbf{q}_i)\mathbf{K}_\sigma\cdot\frac{\partial}{\partial\mathbf{P}_i}\right], \quad (A8)$$

$$A_i^{0\sigma}=(8c^2/V\bar{v}_\sigma\gamma)^{1/2}(\sqrt{J_\sigma}\cos\omega_\sigma)\cos(\mathbf{K}_\sigma\cdot\mathbf{q}_i)\mathbf{K}_\sigma\cdot\frac{\partial}{\partial\mathbf{P}_i}, \quad (A9)$$

$$B_i^{\lambda}=[(m\gamma_i)^{-1}(8/V\bar{v}_\lambda\gamma)^{1/2}](\sqrt{J_\lambda}\sin\omega_\lambda)(\mathbf{e}_\lambda\cdot\mathbf{P}_i)\times\cos(\mathbf{K}_\lambda\cdot\mathbf{q}_i)\frac{\partial}{\partial J_\lambda}, \quad (A10)$$

$$B_i^{\sigma}=[(m\gamma_i)^{-1}(8/V\bar{v}_\sigma\gamma)^{1/2}](\sqrt{J_\sigma}\sin\omega_\sigma)(\mathbf{e}_\sigma\cdot\mathbf{P}_i)\times\cos(\mathbf{K}_\sigma\cdot\mathbf{q}_i)\frac{\partial}{\partial J_\sigma}, \quad (A11)$$

$$B_i^{0\sigma}=-(8c^2/V\bar{v}_\sigma\gamma)^{1/2}(\sqrt{J_{0\sigma}}\sin\omega_{0\sigma})\times\sin(\mathbf{K}_\sigma\cdot\mathbf{q}_i)\frac{\partial}{\partial J_{0\sigma}}. \quad (A12)$$

In the above, $\bar{v}_j^2=v_j^2+\omega_{\rm pl}^2/\gamma$ wherein the $\omega_{\rm pl}/\gamma$ comes from $\mathbf{A}_j^2$, which one gets by expanding $(\mathbf{P}-e\,\mathbf{A}/c)^2$ for the beam particles under the RPA [5].

APPENDIX B

The operators defined in (7a) and (7b) consist of two parts which do not give a c number on commutation. Hence we have to perform a Baker-Hausdorff expression to write $\exp(A+B)=\exp(A)\exp(\bar{\bar{B}})$. Following Fujiwara [7], we write

$$e^{-\mathcal{L}\tau}=e^{A+B}=e^{A}\exp\left[\int_0^1 d\lambda\{B-\lambda[AB]+\frac{\lambda^2}{2!}[A[AB]]-\cdots\}\right], \quad (B1)$$

where the square bracket is a commutator. We can write, from (7a), for the test particle, the operators

$$A=-c\tau\boldsymbol{\beta}\cdot\frac{\partial}{\partial\mathbf{q}},\quad B=(e^2l^2\tau/mc\gamma^3)\sin(lz)\mathbf{k}\cdot\frac{\partial}{\partial\boldsymbol{\beta}}. \quad (B2)$$

Evaluating the various commutators and substituting them in the series (A1), we can sum the series and write it as

$$(e^2l^2\tau/mc\gamma^3)\left[\sin[lz+\lambda cl\tau(\mathbf{k}\cdot\boldsymbol{\beta})]\mathbf{k}\cdot\frac{\partial}{\partial\boldsymbol{\beta}}-(\lambda cl\tau)\sin[lz+\lambda cl\tau(\mathbf{k}\cdot\boldsymbol{\beta})]\boldsymbol{\beta}\cdot\frac{\partial}{\partial\mathbf{q}}\right]. \quad (B3)$$

On integrating over λ and substituting the limits, we get (B1) as

$$e^{-\mathcal{L}_T\tau}=\exp\left[(e^2l/mc^2\gamma^3\mathbf{k}\cdot\boldsymbol{\beta})\{\cos[lz-cl(\mathbf{k}\cdot\boldsymbol{\beta})\tau]-\cos lz\}\mathbf{k}\cdot\frac{\partial}{\partial\boldsymbol{\beta}}+[e^2/mc^2\gamma^3(\mathbf{k}\cdot\boldsymbol{\beta})^2]\{\sin[lz-cl(\mathbf{k}\cdot\boldsymbol{\beta})\tau]-\sin lz+cl\tau(\mathbf{k}\cdot\boldsymbol{\beta})\cos lz\}\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{q}}-c\tau\boldsymbol{\beta}\cdot\frac{\partial}{\partial\mathbf{q}}\right]. \quad (B4)$$

A similar procedure is adopted for the field operators. As they are expressed in terms of canonical variables, we have

$$e^{-\mathcal{L}_i\tau}=\exp\left[(m^2\omega_{\rm pl}^2\gamma/l^2\mathbf{k}\cdot\mathbf{P})\{\cos(lz-lP_3\tau/m\gamma)-\cos lz\}\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{P}}+(m^2\omega_{\rm pl}^2\gamma^2/l^3P_3^2)\{\sin(lz-lP_3\tau/m\gamma)-\sin lz+(lP_3\tau/m\gamma)\cos lz\}\mathbf{k}\cdot\bar{\bar{\Psi}}\cdot\frac{\partial}{\partial\mathbf{q}}-(\mathbf{P}\tau/m\gamma)\cdot\frac{\partial}{\partial\mathbf{q}}\right], \quad (B5)$$

where $\bar{\bar{\psi}}$ is a dyadic

$$\frac{d}{d\mathbf{P}}\left[\frac{\mathbf{P}}{\gamma}\right]\text{ and }\mathbf{k}\cdot\bar{\bar{\Psi}}=\frac{d}{dP_3}(\mathbf{P}/\gamma).$$

APPENDIX C

In this appendix, we shall evaluate the response function given in (15). We shall substitute the operators and perform the photon variable integration. As $\mathbf{e}_\lambda$ and $\mathbf{e}_\sigma$ are the polarization vectors for the transverse and longitudinal modes, we shall evaluate these two separately. We add the $-\lambda$ part and after summing over the particle index i, giving a factor N, which along with V^{-1} gives the concentration c in thermodynamic limit; we get e^2c/m, which can be written as $(\omega_{\rm pl}^2/4\pi)$. We then have

$$\mathscr{E}_\lambda = +\left[2\frac{\omega_{\rm pl}^2}{m\gamma_i^3\bar{\nu}_{\lambda'}}\sin\bar{\nu}_{\lambda'}(t_{n+2}-t_{n+3})\right]\int d\mathbf{P}\int d\mathbf{q}\left\{(\mathbf{e}_\lambda\cdot\mathbf{P})\cos(\overline{\mathbf{K}_{\lambda'}-\mathbf{K}_\lambda}\cdot\mathbf{q}+\phi_\lambda)\mathbf{e}_{\lambda'}\cdot\frac{\partial}{\partial\mathbf{q}}\right.$$
$$\left.+(\mathbf{e}_\lambda\cdot\mathbf{P})(\mathbf{e}_{\lambda'}\cdot\bar{\mathbf{P}})\sin(\overline{\mathbf{K}_{\lambda'}-\mathbf{K}_\lambda}\cdot\mathbf{q}+\phi_\lambda)\mathbf{K}_{\lambda'}\cdot\frac{\partial}{\partial\mathbf{P}}\right\}e^{-\mathcal{L}t_{n+2}}\rho_i(0)\ , \qquad \text{(C1)}$$

where $\bar{\mathbf{P}}$ and ϕ_λ have been obtained by the action of the shift operators given in (B5) on $\mathbf{P}$ and $\mathbf{q}$. These are

$$\bar{\mathbf{P}} = \mathbf{P}+\mathbf{k}(m^2\omega_{\rm pl}^2\gamma_i/l^3P_3)\{\cos(lz-lP_3\tau/m)-\cos lz\}\ ,$$
$$\phi_\lambda = -\frac{\mathbf{K}_{\lambda'}\cdot\mathbf{P}\tau}{m\gamma_i}+(K_{3\lambda'}m^2\omega_{\rm pl}^2\gamma^2/l^3P_3^2)\{\sin(lz-lP_3\tau/m\gamma)-\sin lz+(lP_3\tau/m\gamma_i)\cos lz\}\ , \qquad \text{(C2)}$$

where $\tau = t_{n+1}-t_{n+2}$. Similarly

$$\mathscr{E}_\sigma = +\left[2\frac{\omega_{\rm pl}^2}{m\gamma_i^3\bar{\nu}_{\sigma'}}\sin\bar{\nu}_{\sigma'}(t_{n+2}-t_{n+3})\right]\int d\mathbf{P}\int d\mathbf{q}\left\{(\mathbf{e}_\sigma\cdot\mathbf{P})\cos(\overline{\mathbf{K}_{\sigma'}-\mathbf{K}_\sigma}\cdot\mathbf{q}+\phi_\sigma)\mathbf{e}_{\sigma'}\cdot\frac{\partial}{\partial\mathbf{q}}\right.$$
$$\left.+(\mathbf{e}_\sigma\cdot\mathbf{P})(\mathbf{e}_{\sigma'}\cdot\bar{\mathbf{P}})\sin(\overline{\mathbf{K}_{\sigma'}-\mathbf{K}_\sigma}\cdot\mathbf{q}+\phi_\sigma)\mathbf{K}_{\sigma'}\cdot\frac{\partial}{\partial\mathbf{P}}\right\}e^{-\mathcal{L}t_{n+2}}\rho_i(0)\ , \qquad \text{(C3)}$$

where $\mathbf{P}$ is the same as in (C2) while ϕ_σ is the same as in (C2) with $\mathbf{K}_{\lambda'}$ being replaced by $\mathbf{K}_{\sigma'}$.

We shall now substitute the initial state as given in (17) and perform the integration over $\mathbf{P}$ and $\mathbf{q}$. As the initial state is only a function of z, x and y integrations are trivial, giving the conservation of wave-vector components as $K_{\lambda_1'}=K_{\lambda_1}$ and $K_{\lambda_2'}=K_{\lambda_2}$. We now set $\mathbf{K}_{\lambda'}=\mathbf{K}_\lambda+\mathbf{k}p$, where p is a wave number which has been introduced because of the motion in the Z direction. This modifies the polarization vectors.

The modified transverse polarization is $\mathbf{e}_{\lambda'}=\mathbf{e}_\lambda-i(e_{\lambda_3}/K_{\lambda_1})p$, which makes $\mathbf{K}_{\lambda'}\cdot\mathbf{e}_{\lambda'}=\mathbf{K}_\lambda\cdot\mathbf{e}_\lambda=0$. Thus a new polarization in the radiation component appears because of the motion of the beam in the Z direction. With regard to the longitudinal component, since $\mathbf{e}_\sigma\times\mathbf{K}_\sigma=0$, $\mathbf{e}_\sigma$ and $\mathbf{K}_\sigma$ are parallel to each other and hence $\mathbf{e}_\sigma=\mathbf{K}_\sigma/|\mathbf{K}_\sigma|$ or the polarization vector is the unit vector in the direction of $\mathbf{K}_\sigma$. With regard to the new wave vector $\mathbf{K}_{\sigma'}$, we have $\mathbf{e}_{\sigma'}=\mathbf{e}_\sigma+\mathbf{k}p/|\mathbf{K}_\sigma|=\mathbf{K}_{\sigma'}/|\mathbf{K}_\sigma|$, which will set $\mathbf{e}_{\sigma'}\times\mathbf{K}_{\sigma'}=0$. After a partial integration with respect to $\mathbf{P}$ and $\mathbf{q}$ and having done the x and y integrations trivially, we have

$$\mathscr{E}_\lambda = -2\frac{\omega_{\rm pl}^2}{m\gamma_i^3\bar{\nu}_{\lambda'}}\sin\bar{\nu}_\lambda(t_{n+2}-t_{n+3})$$
$$\times\int dz\int d\mathbf{P}\left[\sin(pz+\phi_\lambda)\left\{e_{\lambda3}^2p\Omega-P_1(e_{\lambda3}^2p/K_{\lambda'})+(\mathbf{e}_\lambda\cdot\mathbf{P})\left[\mathbf{K}_{\lambda'}\cdot\frac{\partial\Omega}{\partial\mathbf{P}}\right]-(\mathbf{e}_\lambda\cdot\mathbf{P})\left[\mathbf{e}_{\lambda'}\cdot\frac{\partial\phi_\lambda}{\partial\mathbf{q}}\right]\right.\right.$$
$$\left.-\frac{2}{\gamma_i}\left[\mathbf{K}_{\lambda'}\cdot\frac{\partial\gamma}{\partial\mathbf{P}}\right](\mathbf{e}_\lambda\cdot\mathbf{P})(\mathbf{e}_{\lambda'}\cdot\bar{\mathbf{P}})\right\}$$
$$\left.+\cos(pz+\phi_i)\left\{-\frac{2}{\gamma_i}\left[\mathbf{e}_{\lambda'}\cdot\frac{\partial\gamma}{\partial\mathbf{q}}\right](\mathbf{e}_\lambda\cdot\mathbf{P})+(\mathbf{e}_\lambda\cdot\mathbf{P})(\mathbf{e}_{\lambda'}\cdot\bar{\mathbf{P}})\left[\mathbf{K}_{\lambda'}\cdot\frac{\partial\phi}{\partial\mathbf{P}}\right]\right\}\right]e^{-\mathcal{L}t_{n+2}}\rho_i(0) \qquad \text{(C4)}$$

and

$$\mathcal{E}_\sigma=-2\frac{\omega_{\rm pl}^2}{m\gamma_i^3\bar{\nu}_{\sigma'}}\sin\bar{\nu}_\sigma(t_{n+2}-t_{n+3})$$

$$\times\int dz\int d\mathbf{P}\left[\sin(pz+\phi_\sigma)\left\{\Omega\left[1+\frac{K_{3\sigma}^2}{K_\sigma^2}\right]p+\Omega K_{3\sigma}\left[1+\frac{p^2}{K_\sigma^2}\right]+(\mathbf{K}_\sigma\cdot\mathbf{P})\left[1-\frac{p^2}{K_\sigma^2}\right]\right.\right.$$

$$+pP_3\left[1+\frac{K_{3\sigma}p}{K_\sigma^2}\right]-\frac{(\mathbf{K}_\sigma\cdot\mathbf{P})\left[\mathbf{K}_{\sigma'}\cdot\frac{\partial\phi}{\partial\mathbf{q}}\right]}{K_\sigma^2}-\frac{2}{\gamma}\left[\mathbf{K}_{\sigma'}\cdot\frac{\partial\gamma}{\partial\mathbf{q}}\right]\frac{(\mathbf{k}_\sigma\cdot\mathbf{P})(\mathbf{K}_{\sigma'}\cdot\bar{\mathbf{P}})}{K_\sigma^2}$$

$$\left.+\frac{\mathbf{K}_\sigma\cdot\mathbf{P}}{K_\sigma^2}\mathbf{K}_{\sigma'}\cdot\frac{\partial}{\partial\mathbf{P}}(\mathbf{K}_{\sigma'}\cdot\bar{\mathbf{P}})\right\}$$

$$\left.+\cos(pz+\phi_\sigma)\left\{-\frac{2}{\gamma}\frac{(\mathbf{K}_\sigma\cdot\mathbf{P})\left[\mathbf{K}_{\sigma'}\cdot\frac{\partial\gamma}{\partial\mathbf{q}}\right]}{K_\sigma^2}+\frac{(\mathbf{K}_\sigma\cdot\mathbf{P})(\mathbf{K}_{\sigma'}\cdot\bar{\mathbf{P}})}{K_\sigma^2}\frac{\partial\phi}{\partial\mathbf{P}}\cdot\mathbf{K}_{\sigma'}\right\}\right]e^{-\mathcal{L}t_{n+2}}\rho_i(0)\ , \tag{C5}$$

where

$$\Omega=(m^2\omega_{\rm pl}^2\gamma_i/l^2P_3)[\cos(lz-lP_3\tau/m\gamma)-\cos lz]\ . \tag{C6}$$

We shall sum over the polarization vector in the transverse component using the relation

$$(\mathbf{e}_\lambda\cdot\mathbf{a})(\mathbf{e}_\lambda\cdot\mathbf{b})=\frac{(\mathbf{K}_\lambda\times\mathbf{a})\cdot(\mathbf{K}_\lambda\times\mathbf{b})}{K_\lambda^2}$$

where **a** and **b** are two arbitrary vectors. In the present case

$$e_{\lambda3}^2=\frac{(\mathbf{K}_\lambda\times\mathbf{k})^2}{K_\lambda^2}=(1-\mu^2)\ ,$$

where μ is the cosine of the polar angle made by $\mathbf{K}_\lambda$. We shall retain only the isotropic part in (C4) and (C5) by averaging over the angles of the propagation vector. We get $(1-\mu^2)$ in (C4) and $(1+\mu^2)$ in (C5), which when averaged would give factors such as $(8\pi/3)$ and $(16\pi/3)$, respectively. We then have (C4) and (C5) after carrying out all the integrations and going over to the Laplace transform and taking the asymptotic limit in time as

$$\mathcal{E}_\lambda=\frac{\tilde{\omega}^2}{Z^2-\bar{\nu}_{\lambda'}^2}\ , \tag{C7}$$

$$\mathcal{E}_\sigma=\frac{2\tilde{\omega}^2}{Z^2-\bar{\nu}_{\sigma'}^2}\ , \tag{C8}$$

where $\tilde{\omega}^2$ is the renormalized plasma frequency given by

$$\tilde{\omega}^2=\omega_{\rm pl}^2\left[\frac{4\pi}{3\gamma_0^3}\left[\frac{\omega_{\rm pl}}{\omega_W}\right]^2\frac{2n+1}{2n}S_n^2J_0^2\left[2n\frac{\omega_{\rm pl}^2}{\omega_W^2}\right]\right]\left[\left[\frac{\omega_{\rm pl}}{\omega_W}\right]^4-1\right]^{-1/2}\ . \tag{C9}$$

In the above ω_W is the frequency $lP_0/m\gamma=lV_z/\gamma$, where V_z is the Z component of the velocity of the beam particles. In evaluating (C7) and (C8) we have only retained the dominant part and not the higher harmonics in the expansion of $\cos(2nlz+\phi)$. These terms have come from the term $e^{-\mathcal{L}t_{n+2}}\rho_i(0)$. We also get $p=(2n+1)l$.

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