Two-temperature kinetic Ising model in one dimension: Steady-state correlations in terms of energy and energy flux

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We study the nonequilibrium properties of a one-dimensional kinetic Ising model in which spins interact by nearest-neighbor ferromagnetic interactions and a spin-flip dynamics is generated by contact with heat baths that are at different temperatures on even and odd lattice sites. The average energy (ε) and the energy flux between the two sublattices (j_ε) are calculated exactly and the two-spin steady-state correlations are expressed through ε and j_ε. It is found that the correlations can be classified as ferromagnetic (for $\varepsilon<0$ and j_ε small), antiferromagnetic ($\varepsilon>0$, j_ε small), oscillating ferromagnetic ($\varepsilon<0$, j_ε large), and oscillating antiferromagnetic ($\varepsilon>0$, j_ε large). We also find a disorder line ($\varepsilon=0$, j_ε arbitrary) on which all correlations are zero. The character of spatial correlations is shown to be reflected in the time evolution of sublattice magnetizations: The dynamics is purely relaxational in the ferromagnetic and antiferromagnetic regime while it is damped oscillatory in the oscillating ferromagnetic and antiferromagnetic regions.

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I. INTRODUCTION

Temperature is not necessarily the best concept for the description of far-from-equilibrium steady states. This fact has been tacitly recognized by experimentalists who often characterize steady states in terms of currents such as fluxes of energy or momentum [1]. From a theoretical point of view, however, it is convenient to work with heat baths. First, it allows the local equilibrium approximation and thus gives a calculational tool that provides good description at least for near-equilibrium steady states. Second, the arbitrariness of possible dynamical processes is reduced by the conditions of detailed balance associated with the heat baths. Third, the interactions in the system can be controlled through the Hamiltonian in the detailed-balance condition. In view of the above, it is not surprising that there is a large body of literature [2–8] on kinetic Ising models in which the steady state is produced by dynamic processes generated by heat baths at different temperatures. Although these studies address specific questions (many of them are concerned with the possible nonequilibrium effects on phase transitions), the unifying theme seems to be an attempt to find some general features in the description of nonequilibrium steady states. Unfortunately, these attempts have not met much success so far.

A possible reason for the failure may be that the results of these models are usually analyzed in terms of temperatures. When there are several heat baths, we expect that their temperatures are not the "natural" variables and, consequently, we should try to analyze the system in terms of more basic physical quantities such as the average energy and the fluxes of various quantities, e.g., the flux of energy. In order to demonstrate the feasibility and the value of such an analysis, we consider here a one-dimensional kinetic Ising model (defined in Sec. II) in which spin flips are generated by heat baths that are at different temperatures on even and odd lattice sites. The model can be solved in the sense that the average energy (ε), the energy flux between the two sublattices (j_ε), and the two-spin correlation function (C_n) can be calculated exactly for the steady state and, furthermore, C_n can be expressed through ε and j_ε (Sec. III). The analysis in terms of ε and j_ε appears to have the following advantages as compared to the analysis in terms of the temperatures. First, the phase space that can be explored is larger compared to that when heat baths with positive temperatures are considered. Second, in this enlargened phase space, we observe a behavior that seems to be a characteristic feature of nonequilibrium steady states: Increasing the energy flux increases the complexity of correlations.

For this simple model, we can also study the relaxation to the steady state and, in Sec. IV, we calculate the dynamics of the sublattice magnetizations exactly. An interesting feature of the result is that the temporal decay of the sublattice magnetizations mirrors the spatial decay of the two-spin correlations. Namely, the character of the dynamics changes from purely relaxational to damped oscillatory when oscillations appear in the spatial correlations.

II. MODEL

We consider a one-dimensional version of the so-called two-temperature kinetic Ising model [3(b)]. The state of the system $\{\sigma\}\equiv\{\ldots,\sigma_n,\sigma_{n+1},\ldots\}$, at time t is specified by stochastic Ising variables $\sigma_n(t)=\pm1$ assigned

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to lattice sites $n=1,2,\ldots,N$, where N is an even number. Periodic boundary conditions are imposed, thus $\sigma_{N+1}=\sigma_1$. The spins are in contact with heat baths with temperatures T_e and T_o on even an odd lattice sites, respectively, and the spin-flip dynamics is described by the following master equation for the probability distribution $P(\{\sigma\},t)$:

$$\frac{\partial P(\{\sigma\},t)}{\partial t}=\sum_{n=1}^{N}[w_n(\{\sigma\}_n)P(\{\sigma\}_n,t)-w_n(\{\sigma\})P(\{\sigma\},t)] , \tag{1}$$

where the state $\{\sigma\}_n$ differs from $\{\sigma\}$ by a flipping of the nth spin and the flip rates on even and odd lattice sites are given as follows:

$$w_n(\{\sigma\})=\frac{1}{2\tau}\left[1-\frac{\gamma_n}{2}\sigma_n(\sigma_{n+1}+\sigma_{n-1})\right] , \tag{2}$$

where

$$\gamma_n=\begin{cases}\gamma_e=\tanh(2J/k_BT_e), & n \text{ even}\\ \gamma_0=\tanh(2J/k_BT_o), & n \text{ odd} .\end{cases} \tag{3}$$

For $T_e=T_o=T$, Eqs. (1)–(3) define the exactly solvable Glauber model [9] which relaxes to the equilibrium state of the Ising model with the nearest-neighbor Hamiltonian $H=-J\sum_n\sigma_n\sigma_{n+1}$ at temperature T. As soon as $T_e\neq T_o$, however, there is a competition between the heat baths, each trying to drive the system towards equilibrium with the same Hamiltonian but at its own temperature. As a result, energy flows from one sublattice to the other and the stationary state that is formed is a nonequilibrium steady state. As we shall see below, this generalization of the Glauber model is solvable in the sense that the relaxation of the one-point correlations $\langle\sigma_n\rangle$ and the steady-state values of the two-spin correlations $\langle\sigma_n\sigma_k\rangle$ can be calculated exactly.

In principle, the time evolution and the steady-state value of the average of a physical quantity A could be obtained by first solving the master equation for an arbitrary initial distribution $P_0=P(\{\sigma\},0)$ and then calculating the averages through

$$\langle A\rangle_t=\sum_{P_0}\sum_{\{\sigma\}}A(\{\sigma\})P(\{\sigma\},t) . \tag{4}$$

In practice, one derives the equations governing the time evolution of the correlation functions of interest and the solvable cases are distinguished by the decoupling of these correlations from the higher-order ones. The closed set of equations are then usually solved with ease as will be seen in the example of the model discussed above. The equations for $\langle\sigma_n\rangle$ and $\langle\sigma_n\sigma_k\rangle$ can be derived by following the steps of the corresponding calculation for the Glauber model [9]. Thus we shall just present and discuss the results in Secs. III and IV.

Once $\langle\sigma_n\sigma_k\rangle$ is obtained, the average energy per spin can be expressed through the nearest-neighbor correlations as

$$\varepsilon=-J\langle\sigma_n\sigma_{n+1}\rangle . \tag{5}$$

We shall find a steady state in which $\langle\sigma_n\sigma_{n+1}\rangle$ is translationally invariant so that ε is independent of n. Since the energy needed to flip the spin at site n is $2J\sigma_n(\sigma_{n+1}+\sigma_{n-1})$, the average rate of energy transfer to the heat bath at site n is given by

$$\begin{aligned}j_\varepsilon(n)&=\langle -2J\sigma_n(\sigma_{n+1}+\sigma_{n-1})w_n\rangle\\&=\frac{J}{\tau}(\gamma_n-\langle\sigma_n\sigma_{n+1}\rangle-\langle\sigma_n\sigma_{n-1}\rangle\\&\qquad+\gamma_n\langle\sigma_{n+1}\sigma_{n-1}\rangle) .\end{aligned} \tag{6}$$

The average energy should be constant in the steady state, so the energy transfer to the heat bath at odd lattice sites must be compensated by the energy obtained at even sites. Consequently, we can define the energy current flowing from an even site to a neighboring odd site as

$$j_\varepsilon=\frac{j_\varepsilon(2n+1)-j_\varepsilon(2n)}{2} . \tag{7}$$

The two sublattices are homogeneous in the steady state found in Sec. III, so that j_ε is also independent of n.

The right-hand sides of Eqs. (5) and (7) are functions of T_e and T_o. Thus we can express these temperatures through ε and j_ε and consider the latter quantities as the independent variables. Then all physical quantities of interest can be expressed in terms of ε and j_ε.

Clearly, other quantities may be chosen as independent variables. An example is the entropy production

$$j_S=\frac{j_\epsilon(2n+1)}{T_o}+\frac{j_\varepsilon(2n)}{T_e}=\left[\frac{1}{T_o}-\frac{1}{T_e}\right]j_\varepsilon , \tag{8}$$

and other fluxlike quantities may be introduced [10]. The reason we prefer j_ε is that its definition does not explicitly contain the temperatures of the heat baths and, furthermore, the final results are significantly simpler in terms of j_ε.

III. TWO-SPIN CORRELATIONS IN THE STEADY STATE

Multiplying both sides of (1) by $\sigma_l\sigma_k$ and summing over all configurations $\{\sigma\}$, we find that the two-point correlation functions $\langle\sigma_l\sigma_k\rangle$ satisfy a closed set of differential equations ($l\neq k$):

$$\begin{aligned}2\tau\frac{\partial\langle\sigma_l\sigma_k\rangle}{\partial t}&=-4\langle\sigma_l\sigma_k\rangle+\gamma_l(\langle\sigma_{l+1}\sigma_k\rangle+\langle\sigma_{l-1}\sigma_k\rangle)\\&\quad+\gamma_k(\langle\sigma_l\sigma_{k+1}\rangle+\langle\sigma_l\sigma_{k-1}\rangle) .\end{aligned} \tag{9}$$

Since there is no process in the system which would reinforce any inhomogeneous fluctuation on a given sublattice, we expect the steady state to be invariant with respect to translations by even number of lattice sites. Then $\langle\sigma_l\sigma_{l+m}\rangle_{t\to\infty}$ should depend only on m and whether l is even or odd. Consequently, two types of correlations should be distinguished:

$$\langle\sigma_l\sigma_{l+m}\rangle_{t\to\infty}=\begin{cases}C_m^o, & l \text{ odd}\\ C_m^e, & l \text{ even} .\end{cases} \tag{10}$$

Clearly, C^o_{2n} and C^e_{2n} describe the correlations within the odd and even sublattices, respectively, while $C^o_{2n-1}=C^e_{2n-1}\equiv C_{2n-1}$ denotes the correlations between spins on different sublattices. It follows from Eq. (9) that these correlations satisfy the following set of inhomogeneous linear equations:

$$2C^o_{2n}=\gamma_o(C_{2n+1}+C_{2n-1}),\quad 1\le n<\frac{N}{2}\,,$$
$$2C^e_{2n}=\gamma_e(C_{2n+1}+C_{2n-1}),\quad 1\le n<\frac{N}{2}\,, \tag{11}$$
$$2C_{2n-1}=\frac{\gamma_o}{2}(C^e_{2n}+C^e_{2n-2})+\frac{\gamma_e}{2}(C^o_{2n}+C^o_{2n-2})\,,\quad 1\le n\le\frac{N}{2}\,.$$

Note that the inhomogeneous terms appear only in the $n=1$ and $N/2$ equations where $C^o_0=C^e_0=C^o_N=C^e_N=\langle\sigma_l^2\rangle=1$. For finite N, these equations have a unique solution. In the "thermodynamic" limit $N\to\infty$, this solution can be written in the form ($n\ge 1$)

$$C_{2n-1}=A\lambda^{n-1},\quad C^o_{2n}=A_o\lambda^n,\quad C^e_{2n}=A_e\lambda^n\,, \tag{12}$$

where

$$\lambda=\frac{1-\sqrt{1-\gamma_o\gamma_e}}{1+\sqrt{1-\gamma_o\gamma_e}}\,,$$
$$A=\frac{\gamma_o+\gamma_e}{2\gamma_o\gamma_e}(1-\sqrt{1-\gamma_o\gamma_e})\,,$$
$$A_o=\frac{1}{2}\left[1+\frac{\gamma_o}{\gamma_e}\right]\,, \tag{13}$$
$$A_e=\frac{1}{2}\left[1+\frac{\gamma_e}{\gamma_o}\right]\,.$$

The average energy ε and the energy flux between the two sublattices j_ε can now be calculated using (5)–(7), (12), and (13):

$$\frac{1}{J}\varepsilon=\frac{\gamma_o+\gamma_e}{2\gamma_o\gamma_e}(\sqrt{1-\gamma_o\gamma_e}-1)\,,$$
$$\frac{\tau}{J}j_\varepsilon=\frac{1}{2}(\gamma_o-\gamma_e)\,. \tag{14}$$

Inverting the above equations, γ_o and γ_e can be expressed through ε and j_ε and then Eqs. (12) and (13) yield the correlation function in terms of energy and flux of energy. In order to simplify the expressions, we shall set $J=1$ and $\tau=1$ (i.e., we measure ε and j_ε in units of J and J/τ, respectively) and introduce the notation

$$\eta\equiv\sqrt{1+(1+\varepsilon^2)j_\varepsilon^2}-1\,. \tag{15}$$

Then λ and the amplitudes of the correlation functions can be written as follows:

$$\lambda=\frac{2\varepsilon^2-\eta}{2+\eta}\,,$$
$$A=-\varepsilon,\quad A_o=\frac{\varepsilon j_\varepsilon}{\varepsilon j_\varepsilon+\eta},\quad A_e=\frac{\varepsilon j_\varepsilon}{\varepsilon j_\varepsilon-\eta}\,. \tag{16}$$

Having expressed the correlation functions in terms of ε and j_ε, we start the analysis by specifying the range of ε and j_ε where the model is meaningful. If the temperatures of the heat baths are restricted to be positive, then $0\le\gamma_o\le1$, $0\le\gamma_e\le1$, and Eq. (14) gives $-1\le\varepsilon\le0$, $-\frac{1}{2}\le j_\varepsilon\le\frac{1}{2}$. A more general view, however, is that the master equation (1) describes a physical process for all γ_o and γ_e for which the spin-flip rates (2) are positive. This positivity condition restricts the γ's to be in the $[-1,1]$ interval. When both γ's are negative, we can absorb the negative sign into the coupling constant $J\to-J$ and the model describes a two-temperature kinetic Ising model with antiferromagnetic couplings. When the γ's have different signs, however, such an interpretation is not possible and we have to associate the negative sign with the temperature of one of the heat baths. Such an interpretation is consistent with the usual view that negative temperature corresponds to an "inverted population of states" and is higher than the $T=+\infty$ temperature. Indeed, since $j_\varepsilon=(\gamma_o-\gamma_e)/2$, one can see that the energy flux towards the odd sublattice is increasing if we decrease γ_e to zero (corresponding to increasing the temperature of the even sublattice to $T_e=+\infty$) and then continue to decrease it to negative values.

In the following we shall take the more general view. We allow the γ's to be from the interval $-1\le\gamma_o,\gamma_e\le1$ and assume that the interactions are ferromagnetic. This means that whenever $\gamma_o<0$ or $\gamma_e<0$, the corresponding heat bath is assumed to be in an "inverted state" with negative temperature. For the given range of γ's, the values of ε and j_ε are the interval $[-1,1]$. At a fixed $\varepsilon\neq0$, however, the possible values of j_ε are more restricted as can be seen and calculated from Eqs. (13):

$$|j_\varepsilon|\le1-|\varepsilon|[1+\sqrt{1+(1-|\varepsilon|)^2}]+\varepsilon^2\,. \tag{17}$$

The maximum value of j_ε is displayed on Fig. 1 by solid

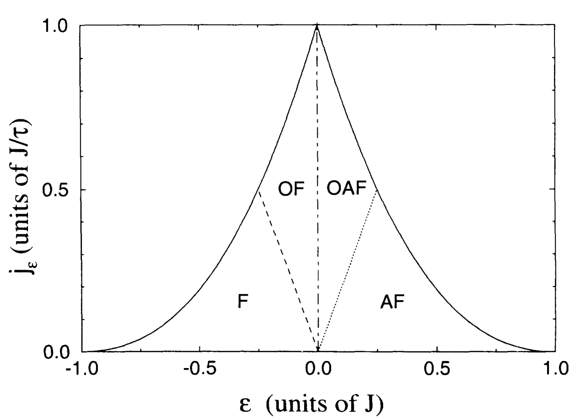

FIG. 1. "Phase diagram" in the plane of energy (ε) and energy flux (j_ε). Characteristic decay is of correlations in the "ferromagnetic" (F) and "oscillating ferromagnetic" (OF) regimes are displayed in Figs. 2 and 3. The boundary between the two regimes is shown by a dashed line. The $j_\varepsilon=0$ axis is the line of Ising equilibrium, the $\varepsilon=0$ axis is a disorder line, and the possible j_ε values are bounded by the upper solid line.

line. The curve displaying the minimum allowed values of j_ε can be obtained by reflecting this solid line through the ε axis. We shall not be concerned, however, with the $j_\varepsilon<0$ region since the $j_\varepsilon \to -j_\varepsilon$ transformation can be obtained by exchanging the temperatures of the sublattices. Then the correlations between odd and even sites do not change while the odd-odd correlations become the even-even correlations and vice versa. Indeed, examining Eqs. (12), (15), and (16), one finds

$$C_n^o(\varepsilon,-j_\varepsilon)=C_n^e(\varepsilon,j_\varepsilon)\ . \tag{18}$$

Another symmetry of the correlation functions that is apparent from (12), (15), and (16) is as follows:

$$\begin{aligned} C_n^o(-\varepsilon,-j_\varepsilon)&=(-1)^n C_n^o(\varepsilon,j_\varepsilon)\ ,\\ C_n^e(-\varepsilon,-j_\varepsilon)&=(-1)^n C_n^e(\varepsilon,j_\varepsilon)\ . \end{aligned} \tag{19}$$

For the equilibrium case ($j_\varepsilon=0$), the above relationships follow from the gauge symmetry of the Hamiltonian, i.e., from the invariance with respect to the simultaneous change of the sign of the interaction and the change of sign of spins on one of the sublattices. The corresponding gauge symmetry that yields the symmetries of the correlation functions given by (19) is the invariance of the master equation (1) with respect to the simultaneous change of the sign of the spins on one of the sublattices and the change of the signs of both γ_o and γ_e.

Combining (18) and (19) we can also find how the correlation functions are transformed under the change of the sign of ε:

$$C_n^o(-\varepsilon,j_\varepsilon)=(-1)^n C_n^e(\varepsilon,j_\varepsilon)\ . \tag{20}$$

Thus it is sufficient to investigate the correlation function in the $\varepsilon\le 0$, $j_\varepsilon \ge 0$ quadrant. The behavior in the other quadrants follow from (18)–(20).

Let us consider first the $j_\varepsilon=0$ axis. This is the line of equilibrium and, indeed, one can find from (16) that $\lambda=\varepsilon^2$, $A=-\varepsilon$, and $A_o=A_e=1$ and, consequently, the correlations are those of the equilibrium Ising model $C_n^o(\varepsilon)=C_n^e(\varepsilon)=(-\varepsilon)^n$.

The other axis ($\varepsilon=0$) is also special, since it is a disorder line where all the correlations are zero. The vanishing of the two-spin correlations is obvious from (16) since all three amplitudes of the correlation functions are zero for $\varepsilon=0$. The general statement that all correlations vanish follows from the fact that $\varepsilon=0$ corresponds to $\gamma_o=-\gamma_e$ [see Eq. (14)] and one can verify by straightforward substitution that, in this case, the steady-state solution of the Master equation (1) is the constant distribution. With all the states being equally probable, we have the $T=\infty$ Ising model where all correlations vanish.

Let us consider now the "ferromagnetic" region denoted by F in Fig. 1. Although there is no real ferromagnetic order in this region, we use this name to describe the following observation: The correlations are positive at all distances (see Fig. 2). The only difference from the equilibrium Ising correlations is the splitting of the C_{2n} correlations into C_{2n}^o and C_{2n}^e. The inequality $C_{2n}^e \le C_{2n}^o$ observed in Fig. 2 is consequence of the energy flux flowing towards the odd sublattice [$j_\varepsilon=(\gamma_o-\gamma_e)/2>0$]. The

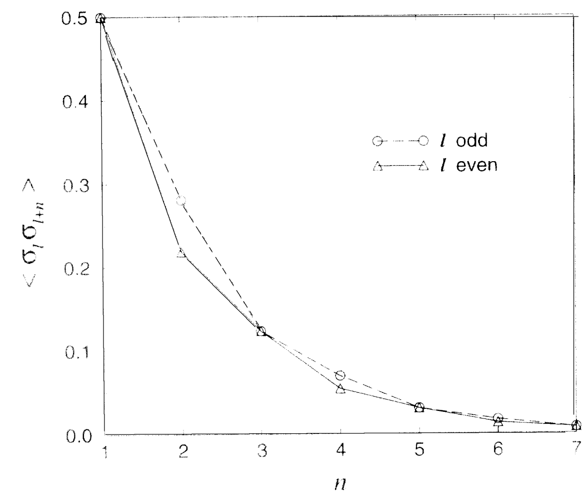

FIG. 2. General form of two-spin correlations in the ferromagnetic regime, i.e., in the region denoted by F in Fig. 1. For this particular example we get $\varepsilon=-0.5$ and $j_\varepsilon=0.1$. The correlation function in the AF region at $-\varepsilon$ and j_ε is obtained by exchanging the squares and the circles and by changing the sign of the correlations at odd n.

odd sublattice is thus at a lower temperature than the even sublattice and so the odd-odd correlations are stronger than the even-even ones.

The region of ferromagnetic correlations is bounded from the large flux side by the line:

$$j_\varepsilon=-2\varepsilon\ , \tag{21}$$

which is shown as a dashed line in Fig. 1. One can find from (16) that $\lambda=0$ on this line and all the correlations vanish except the nearest-neighbor correlation and the second-nearest-neighbor odd-odd one. This result can be partly understood in terms of the temperatures of the heat baths. First, $\lambda=0$ means that $\gamma_o\gamma_e=0$ [see Eq. (13)]. Further, since $j_\varepsilon>0$ means $T_e>T_o$, we must have $\gamma_e=0$, i.e., $T_e=\infty$. Since the flipping of spins is entirely random on the $T=\infty$ sublattice, there can be no correlations built between the even sites.

Above line (21), we have $\lambda<0$. According to Eq. (13), this means that $\gamma_o\gamma_e<0$, i.e., one of the heat baths is at positive while the other is at negative temperature. In this region, both C_n^o and C_n^e decay through oscillations of period four lattice spacings as shown in Fig. 3. It is remarkable that the plot seems to be rather chaotic, although there is a simple underlying structure to it. We call these correlations "oscillating ferromagnetic" (and denote the region above the line $j_\varepsilon=-2\varepsilon$ by OF) because, apart from the oscillations, we also see that the dominating nearest-neighbor and second-nearest-neighbor correlations are ferromagnetic. Note that the corresponding region for $\varepsilon>0$ is denoted by OAF (oscillating antiferromagnetic) since there the nearest-neighbor and the dominating second-nearest-neighbor correlations and antiferromagnetic [see Eq. (20)].

Looking at the "phase diagram" and the correlation functions in Figs. 1–3, we can observe the following two

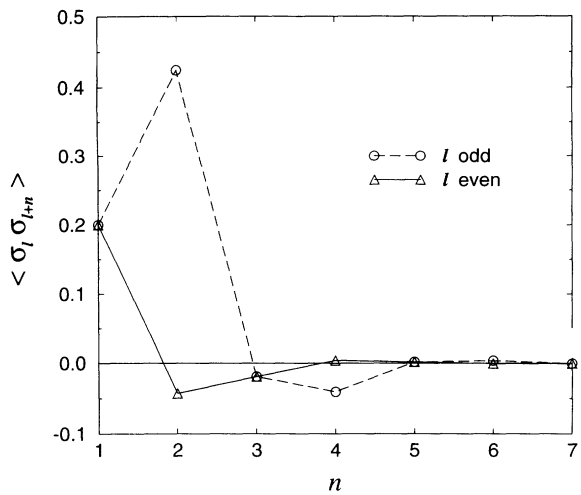

FIG. 3. General form of two-spin correlations in the oscillating ferromagnetic regime, i.e., in the region denoted by OF in Fig. 1. In this actual example we set $\varepsilon=-0.2$ and $j_\varepsilon=0.5$. The correlation function in the OAF region at $-\varepsilon$ and j_ε is obtained by exchanging the squares and the circles and by changing the sign of the correlations at odd n.

features that appear to be rather general characteristics of driven systems. (i) When the system is ordered ($|\varepsilon|\approx 1$) it cannot be driven. Indeed, the maximum available energy flux is $j_\varepsilon^{\max}\sim(1-|\varepsilon|)^2$ and the character of correlations does not change as a result of driving. (ii) The range of available flux values increases as the order decreases in the system ($|\varepsilon|\to 0$) and the correlations become more complex at large fluxes.

Note that the above conclusions appear naturally in the $\varepsilon, j_\varepsilon$ representation. It would have been much more difficult to arrive at similar conclusions using the temperatures of the heat baths. Indeed, had the model been analyzed in terms of positive temperatures of heat baths, the most interesting regions of the phase diagram, namely the OF and OAF regions, would have been left out of considerations.

IV. DYNAMICS OF SUBLATTICE MAGNETIZATIONS

The equation for the time evolution of the average magnetization at site n is obtained from (1) by multiplying both sides by σ_n and summing over all configurations $\{\sigma\}$:

$$\tau\frac{\partial\langle\sigma_n\rangle}{\partial t}=-\langle\sigma_n\rangle+\frac{\gamma_n}{2}(\langle\sigma_{n+1}\rangle+\langle\sigma_{n-1}\rangle)\ . \quad (22)$$

Thus, if an initial state had homogeneous but distinct sublattice magnetizations

$$\langle\sigma_{2n+1}\rangle_{t=0}=m_o(0),\ \ \langle\sigma_{2n}\rangle_{t=0}=m_e(0)\ , \quad (23)$$

then each remain homogeneous at all times. Furthermore, $m_o(t)$ and $m_e(t)$ satisfy the following equations:

$$\tau\dot{m}_o=-m_o+\gamma_o m_e,\ \ \tau\dot{m}_e=\gamma_e m_o-m_e\ . \quad (24)$$

The solution to these equations is a sum of two exponentials

$$m_\alpha(t)=a_+^{(\alpha)}e^{-\Gamma_+ t}+a_-^{(\alpha)}e^{-\Gamma_- t}\ \ (\alpha=o,e)\ , \quad (25)$$

with the rates of relaxation given by

$$\Gamma_\pm=\frac{1}{\tau}(1\pm\sqrt{\gamma_o\gamma_e})\ . \quad (26)$$

The interesting feature of results (25) and (26) is that the purely relaxational behavior in the F and AF regions ($\gamma_o\gamma_e>0$) changes over to damped oscillatory relaxation in the OF and OAF regions ($\gamma_o\gamma_e<0$). As it was shown in Sec. III, a similar changeover takes place in the spatial dependence of the two-point correlation functions. Thus we see here an explicit example of what is otherwise well known, namely, the coupling between the dynamics and the statics in a nonequilibrium steady state is much stronger than in an equilibrium system where the Hamiltonian in the detailed balance condition determines all the static correlations. Another remarkable feature of the dynamics is that $\mathrm{Re}\Gamma_\pm$ is a constant ($1/\tau$) in the OF and OAF regions. Thus the exponential envelope of the damped oscillatory relaxation is fixed in the whole oscillatory region. While this is interesting, we believe it is not a general feature; it is just a consequence of the simplicity of the model.

V. FINAL REMARKS

We should emphasize that the descriptions of the two-temperature model in terms of ε and j_ε or in terms of T_e and T_o are equivalent, provided we allow for negative temperatures. Indeed, we switched between the two descriptions frequently finding one or the other to be more convenient in explaining various features of the results. Thus one might argue that Secs. II–IV contain no more than an exact calculation of both the two-spin correlations in the steady state and the relaxation of sublattice magnetizations towards the steady state. We believe, however, that there is a more general point to our calculation. Namely, it shows that it is possible and perhaps useful to analyze nonequilibrium steady states in terms of local physical quantities such as energy and energy flux instead of using external parameters such as the temperature of heat baths.

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