Statistical properties of chaotic scattering with one open channel

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(Received 2 March 1993; revised manuscript received 6 August 1993)

The correspondence between statistical properties of decaying states and fluctuations in resonance scattering is studied in a statistical model with one open channel. The model is described by an ensemble of random non-Hermitian matrices. The dependence of the correlation length on the coupling parameter both for the S matrix and the cross section is studied numerically. We show that maximal correlations in the scattering arise for a certain value of the coupling to the continuum, reflecting a specific change in the internal motion of intermediate decaying systems. Also, the Fourier transform of the two-point correlation function of the S matrix is analyzed both analytically and numerically. The self-averaging nature of this function is explicitly demonstrated.

PACS number(s): 05.45.+b, 03.80.+r, 24.60.Lz

I. INTRODUCTION

As is well known, the main problem of classical mechanics is to find the law of motion which is specific for a given law of interaction. It allows us to exactly describe the time evolution of any part of the given system. This problem, in principle, always can be solved in the case of one degree of freedom. For a long time it was believed that the problem to extend the same approach to the general case of an arbitrary number of degrees of freedom is only a technical nature. The striking result of recent developments is the discovery of a specific form of behavior which nowadays is known as dynamical chaos (see, e.g., [1]). It was found that even for a few degrees of freedom, the motion gains, under certain conditions, quite general features which are characteristic of random processes. This opens a new way for the universal description of such a motion on the basis of a statistical approach.

The possibility of chaotic motion for systems with a few degrees of freedom leads to the important problem of the existence of a similar phenomenon in microphysics, where a quantum description should be essentially used. This problem, termed "quantum chaos," nowadays attracts considerable attention (see, e.g., [2,3]). Since properties of quantum systems may be quite different from classical ones even in the deep semiclassical region [4], one of the main problems is to find proper quantities to describe the degree of chaos in quantum systems. The well-known approach in this direction is related to the study of fluctuations in energy spectra of closed physical systems. It turns out that for quantum systems which are completely chaotic in the classical limit, the fluctuations of energy spectra have a specific form and may be compared to those of eigenvalues of random matrices [5,6]. Such matrices have been used [7] to describe properties of nuclear spectra long before the very ideas of quantum chaos appeared. The efficiency of this approach has led to the fast development of random-matrix theory (RMT) [8], which nowadays has many applications in different fields of physics (see, e.g., [3,9]). One of the best examples is the description of spectra of isolated neutron resonances in complex nuclei near the threshold of emission of a nucleon. Both spectra and widths of nuclear resonances are well described within the framework of the Gaussian orthogonal ensemble (GOE) of RMT [10].

Strictly speaking, neutron resonances are not true bound states of nuclei and reveal themselves in the form of sharp peaks in neutron scattering on nuclei. Such a strong energy dependence of the cross section implies that the neutron spends a large time inside the nucleus, sharing its energy with other nucleons. As a result, an intermediate compound nuclei is formed which finally decays with the emission of a nucleon. Such a compound nucleus may be regarded as a typical example of a complex unstable (open) system. As a convenient approach to the study of such types of scattering processes, the projector operator technique may be used (see, for example, [11]). In this approach, two orthogonal sets of states are introduced. The first ("intrinsic basis," labeled by $|k\rangle$) represents the internal motion in the compound nucleus. All states of this basis are essentially bound states and the wave functions decay outside the interaction region. The commonly chosen basis in nuclear scattering theory is the basis of shell-model states as it is described in detail in Ref. [12]. Another set is used for the asymptotic motion with energy E before and after reaction (the so-called reaction "channels" labeled by $|c,E\rangle$).

In this way, the Hermitian Hamiltonian of the whole system has nonzero matrix elements within as well as between each subspace, and can therefore be represented in the following form:

$$H=\sum_{k,l=1}^{N}|k\rangle H_{kl}\langle l|+\sum_{c=1}^{M}\int dE|c,E\rangle E\langle c,E| + \sum_{c=1}^{M}\sum_{k=1}^{N}\int dE\{V_k^c(E)|k\rangle\langle c,E|+\text{H.c.}\}\ . \quad (1.1)$$

Here, N is the number of basis vectors needed to describe the internal motion by the intrinsic Hamiltonian H_{kl} with

sufficient accuracy and M is the number of open channels for a given energy range. The second part in (1.1) is assumed to be diagonal, provided that one is not interested in direct processes. Finally, the quantity V_k^c is the transition amplitude between an internal state k and a channel c. This approach is not only restricted to isolated resonances. The amplitudes V_k^c as well as the density of levels of the intermediate nucleus typically grow with growing energy. As a result, the widths of the resonances also grow, and when they strongly overlap new physical phenomena occur which can be described as well.

Using the Hamiltonian (1.1) one can get the standard representation of the S matrix for resonance scattering (see, e.g., [12]),

$$S_{cc'}(E)=\delta_{cc'}-i\sum_{kl}(V_k^c)^*\left[\frac{1}{E-\mathcal{H}}\right]_{kl}V_l^{c'}\ , \tag{1.2}$$

where the internal motion of the open system with the influence of the continuum is described by the Green's function

$$\mathcal{G}(E)=\frac{1}{E-\mathcal{H}}\ . \tag{1.3}$$

The matrix elements of the operator $\mathcal{H}$ in Eqs. (1.2) and (1.3) read

$$\mathcal{H}_{kl}=H_{kl}+\frac{1}{2\pi}\sum_c \mathrm{P}\int dE'\frac{V_k^c(E')V_l^c(E')}{E-E'} -\frac{i}{2}\sum_{c=1}^{M}(V_k^c)^*V_l^c\ , \tag{1.4}$$

with P denoting the principal-value integral. Note that the specific form of the anti-Hermitian part in Eq. (1.4) provides the unitarity of the S matrix (1.2). Due to the time-reversal invariance of the systems under consideration in this paper, the matrix elements H_{kl} as well as the amplitudes V_k^c are real. As a result, the whole matrix $\mathcal{H}$ is symmetric.

As was mentioned above, the amplitudes V_k^c generally depend on the energy E. For complicated systems like atomic nuclei, which typically have a large level density, this dependence is however very smooth as compared to the sharp explicit energy dependence of the Green's function (1.3). Restricting ourselves to a limited energy region (including, nevertheless, a large number of levels), one can neglect this smooth dependence omitting simultaneously the principal-value integral in (1.4). One then can treat the operator $\mathcal{H}$ as an effective Hamiltonian which is essentially non-Hermitian.

It is useful to diagonalize the effective Hamiltonian to have the explicit resonance representation of the S matrix (1.2)

$$S_{cc'}(E)=\delta_{cc'}-i\sum_k\frac{\tilde{V}_k^c\tilde{V}_k^{c'}}{E-\mathcal{E}_k}\ . \tag{1.5}$$

The complex eigenvalues $\mathcal{E}_k=E_k-\frac{i}{2}\Gamma_k$ of $\mathcal{H}$ give the energies E_k and widths Γ_k of the resonances whereas the complex amplitudes

$$\tilde{V}_k^c=\sum_l V_l^c\psi_l^{(k)}\ , \tag{1.6}$$

with $\psi_l^{(k)}$ being the eigenstates of $\mathcal{H}$, have the meaning of decay amplitudes of unstable eigenstates.

As a result, the above approach gives a very convenient discretized version of the dynamics of the continuum. Thus one gains the possibility to use powerful matrix methods similar to those in the physics of bound states.

In the next section, the model for complex open systems used in the present paper is described. The model is based on an ensemble of random non-Hermitian Hamiltonian matrices. This is a natural generalization of the well-known random-matrix approach when describing the universal statistical properties of closed chaotic quantum systems. This model is assumed to have generic properties and may therefore be applied to different physical problems. The main properties of the model which are already known from previous analytical studies are briefly discussed in Sec. II.

In Sec. III, the correlation function for the S matrix and the cross section are investigated numerically. Special attention is paid to the dependence of their correlation lengths on the coupling to the decay channels. This allows us to reveal the underlying relation between statistical properties of the resonances and the scattering fluctuations. The latter problem is discussed in detail.

Section IV deals with the time evolution of decaying systems. The energy averaging is used to evaluate analytically the asymptotic behavior of the Fourier transform of the S matrix correlation function. We demonstrate that this asymptotic behavior is formed as a result of self-averaging.

II. DESCRIPTION OF THE STATISTICAL MODEL

Our interest is in applying the above approach to complicated quantum systems like heavy nuclei, atoms, or molecules. In this case the internal motion is supposed to be chaotic. Therefore, one makes some statistical assumptions for the effective Hamiltonian (1.4). As was introduced in [13] and now is commonly used, we take the Hermitian part H to be a member of the GOE. Correspondingly, the matrix elements H_{kl} are taken as statistically independent Gaussian random numbers with zero mean and variance:

$$\langle(H_{kl})^2\rangle=\begin{cases}1/N & \text{for } k\neq l\\ 2/N & \text{for } k=1\ .\end{cases} \tag{2.1}$$

This model has been studied analytically in a number of papers [14] where the averaged S matrix as well as two-point correlation functions for matrix elements have been calculated by using powerful methods similar to those of quantum field theory. In particular, it was shown that all statistical properties of the S matrix can be expressed in terms of the averaged S matrix. Therefore, the fluctuating part of scattering depends only on the so-called transmission coefficients which describe the probability of forming an intermediate compound system. By this, the important separation of global statistical properties and local spectral fluctuations has been explicitly demonstrated.

Another approach to the same problem has been developed in Ref. [15]. Namely, attention is mainly paid to the properties of intermediate states and their relation to that of the S matrix. In some sense, this approach is complementary to the previous one and allows us to extract information only about unstable systems. Thus, one is mainly interested in the statistics of decaying states and their complex energies. In [15], it has been shown that with the increasing coupling to the continuum a drastic change in internal motion may happen. Therefore, two dynamical regimes exist with a sharp transition between them when the coupling parameter exceeds some critical value. This phenomenon is closely related to the structure of the anti-Hermitian part of the effective Hamiltonian (1.4). The first regime (weak coupling) corresponds to the more or less homogeneous distribution of resonance energies and widths, whereas in the second one (strong coupling) the clear separation of all states into two groups appears. In the case of M open channels, M very unstable states are formed and the remaining N-M states become almost stable. It should be stressed that chaoticity of the internal motion cannot destroy the coherence, which is the origin of the formation of short-living states.

This effect of the formation of short-living resonances has been observed recently [16] in the numerical simulation of nuclear reactions. It is interesting to note that such kinds of phenomena were discussed long ago by Moldauer [17]. As a real example of such a broad unstable state the nuclear analog resonances are considered [18] (see also [19] where the role of this effect is described in application to the theory of giant multipole resonances). Some other examples of such a phenomenon in nuclear physics as well as in solid-state physics were mentioned in [20]. Similar effects are also observed in analytical and numerical studies of the scattering by molecules [21].

The link between the two mentioned approaches is clearly revealed by consideration of the time evolution of the processes passing through the formation of intermediate unstable systems. The correlation length of the two-point correlation function defined as the energy difference corresponding to one-half of its maximal value is of special interest. Due to the uncertainty principle, this length is proportional to the inverse value of the so-called average delay time characterizing the reaction time. In this sense, the delay time is a generalization of the lifetime of unstable systems with well-isolated levels for the case of strongly overlapping ones. The detailed analysis of the delay time and its connection to the fluctuations of the S matrix has been given in [22,23]. In particular, a relation was established [23] between the probability distribution of the delay time and the Fourier transform of the two-point correlation function. Recently, a thorough study of the time evolution of an instantaneously excited state was carried out [24] using the explicit form of the correlation function found in [14].

Statistical properties of unstable states have been systematically investigated in [15], where the randomness of amplitudes V_k^c in addition to that of H_{kl} has been assumed. The reason is that due to the complexity of the intrinsic motion, a vector $H|k\rangle$ is very complicated and can be treated as a random state. Therefore, it has random projections on any direction in the total Hilbert space. The statistical independence together with orthogonal invariance in the intrinsic subspace imply that the amplitudes V_k^c are of Gaussian character [7]. Assuming also the statistical equivalence of channels, one chooses therefore

$$\langle V_k^c \rangle = 0, \quad \left\langle V_k^c V_l^{c'} \right\rangle = \delta_{cc'}\delta_{kl} 2\gamma / N \ . \tag{2.2}$$

Here, the parameter γ determines the strength of the coupling to the continuum.

For the single-channel case, the joint distribution of complex energies of the resonances has been derived [15], showing nontrivial energy-width correlations. One of the distinctive properties of this distribution is the specific sort of repulsion between neighboring energies $\mathcal{E}$ in the complex plane. Further studies of some correlations have recently been performed numerically in [25]. But in both limits of weak and extremely strong coupling to the continuum this distribution simplifies, as far as the long-living states are concerned, to the product of the GOE joint distribution of energy levels and the Porter-Thomas distribution of their width. This means in particular that the long-living resonances cannot overlap strongly in the single-channel case [15,26]. The short-living resonances fluctuates very weakly and therefore does not influence the S matrix fluctuations. That is why one can anticipate that all fluctuations should depend only on the transmission coefficient (see below) in full agreement with the statement of Ref. [14].

In this paper, we perform a detailed numerical study for the case of one open channel, paying attention mainly to the relation between the statistical properties of resonances and fluctuations of scattering properties. In the purely elastic scattering, the latter relation reveals itself in the most clear way, since in this case the S matrix is expressed in terms of the complex eigenenergies only [27,28]. Recently, quasielastic chaotic scattering of electromagnetic microwaves in an irregular shaped cavity (which is formally analogous to the quantum scattering) was investigated experimentally in [29]; theoretical analysis of these data is given in [30] by using the random-matrix approach. The case of many decay channels was considered in [31] both analytically and numerically, where the distribution of complex eigenvalues of the non-Hermitian Hamiltonian $\mathcal{H}$ was studied in dependence on the coupling and number of channels.

III. AVERAGE S MATRIX AND CORRELATION FUNCTIONS

In what follows we are using two equivalent representations of the resonance S matrix. For the single-channel case, both of them express the S matrix only in terms of complex eigenvalues of the effective Hamiltonian (1.4). Therefore, statistical properties of the S matrix as well as the cross section are totally connected to those of complex energies of resonances. The first representation is (1.5),

$$S(E)=1-i\sum_k \frac{\left[\tilde{V}_k\right]^2}{E-\mathcal{E}_k} . \tag{3.1}$$

Another expression is given in the equivalent factorized form [27,28]

$$S(E)=\prod_k \frac{E-\mathcal{E}_k^*}{E-\mathcal{E}_k} . \tag{3.2}$$

Comparing these two expressions in the vicinity of a given resonance $\mathcal{E}_k$ one can easily find that

$$(\tilde{V}_k)^2=\Gamma_k \prod_{l\neq k} \frac{\mathcal{E}_k-\mathcal{E}_l^*}{\mathcal{E}_k-\mathcal{E}_l} . \tag{3.3}$$

Though complex, the quantities $(\tilde{V}_k)^2\equiv\tilde{\Gamma}_k e^{i\alpha_k}$ satisfy, due to the invariance of the trace, the condition

$$\sum_k (\tilde{V}_k)^2=2\,\mathrm{Im}\,\mathrm{Tr}\mathcal{H}=\sum_k \Gamma_k , \tag{3.4}$$

where the right-hand side is real. It is easy to check that $\tilde{\Gamma}_k\approx\Gamma_k$, provided that the resonances do not overlap (for $\gamma\ll 1$ or $\gamma\gg 1$).

Using the above representations, we start our numerical study by considering the average S matrix. It turns out that for the values of γ which exceed the critical value $\gamma_c=1$ [15], the average S matrix has some characteristic energy dependence, which reminds one of the S matrix for a wide Breit-Wigner resonance. This is just the evidence of the segregation of the short-living collective state which is an intimate feature of the model under consideration, as was mentioned above. Due to this phenomenon the total average cross section [32]

$$\sigma_t(E)=2(1-\mathrm{Re}\langle S(E)\rangle) \tag{3.5}$$

can be divided into two parts in a natural way. The first one, which can be called the "shape-elastic" cross section, is given by

$$\sigma_{\mathrm{SE}}(E)=|1-\langle S(E)\rangle|^2 \tag{3.6}$$

and corresponds to the fast "direct" scattering with excitation of the short-living state. The second part,

$$\sigma_{\mathrm{abs}}(E)=1-|\langle S(E)\rangle|^2 , \tag{3.7}$$

is the cross section of absorption due to excitation of the long-living compound states. This quantity is just the transmission coefficient T.

Figure 1 represents results of numerical calculations for the cross sections. The data for the averaged S matrix are obtained by diagonalization of 2000 matrices drawn from the ensemble (1.4,2.1,2.2) with $N=100$, $M=1$, and $\gamma=2$. Due to our normalization (2.1), all resonances should be located in the energy region $[-2,2]$ in the limit of infinitely large N. Therefore, one expects that the absorption cross section is nonzero just in this region. The visible deviation in Fig. 1 is due to the finite size of the matrices. These results are in good agreement with the analytical expressions obtained in [15] for the limit $N\to\infty$.

It should be stressed that the Breit-Wigner-like wings

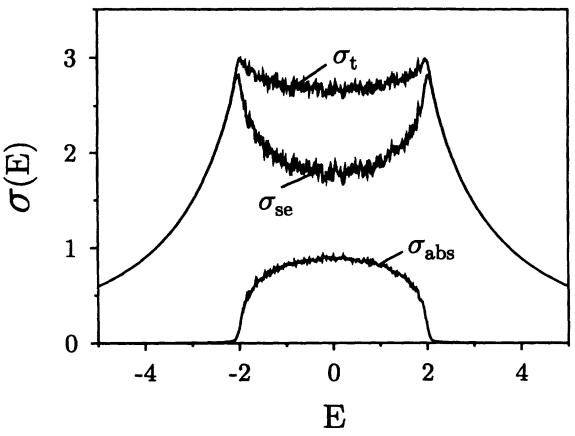

FIG. 1. Total shape-elastic and absorption cross section (3.5)–(3.7) as a function of energy for $\gamma=2$. The averaged S matrix is obtained by diagonalizing 2000 matrices with dimension $N=100$ drawn from the ensemble (1.4,2.1,2.2).

of the shape-elastic cross section are caused by the specific choice of our model for which the resonances are restricted to be within a finite-energy region. For a real physical system, resonances lying outside this region also exist. These remote resonances will influence the form of the wings and can distort or even completely destroy them. Therefore, to prove the existence of the dynamical reorganization of the unstable system discussed above, one needs to find reliable local properties of the spectrum which give clear evidence for this phenomenon. Here we would like to point out that the average value of the S matrix in the center of the considered resonance region,

$$\langle S(0)\rangle=\frac{1-\gamma}{1+\gamma} , \tag{3.8}$$

tends to -1 for $\gamma\to\infty$. This means that the corresponding scattering phase has the value $\pi/2$, which is typical for the center of a Breit-Wigner resonance.

For the same energy region, the transmission coefficient is equal to

$$T(0)=\frac{4\gamma}{(\gamma+1)^2} , \tag{3.9}$$

and both for small and large values of γ it appears to be small

$$T\approx\begin{cases}4\gamma, & \text{for } \gamma\ll 1\\ 4/\gamma & \text{for } \gamma\gg 1 .\end{cases} \tag{3.10}$$

As was stressed in [14], statistical properties of long-living resonances depend only on the transmission coefficient T itself, rather than on the coupling parameter γ. Therefore, from (3.10) it is seen that both for weak and strong coupling these properties should be the same, provided that the values of γ are inversely related to each other.

On the other hand, the transmission coefficient reaches its maximum value for $\gamma=1$, which is the critical value where the sharp transition between two dynamical regimes occurs [15]. Therefore, one can expect some manifestations of this phenomenon in the behavior of statisti-

cal characteristics of the system such as correlation functions for the S matrix or cross section. That is why these correlation functions have been numerically studied here in a wide range of the coupling parameter γ, paying special attention to the γ dependence of correlation lengths. We define the correlation functions for the S matrix and the total cross section (3.5) as

$$C_S(\varepsilon)=\langle S(E)S^*(E+\varepsilon)\rangle-|\langle S(E)\rangle|^2\,, \tag{3.11}$$

$$C_\sigma(\varepsilon)=\langle \sigma(E)\sigma(E+\varepsilon)\rangle-\langle \sigma(E)\rangle^2\,. \tag{3.12}$$

Sufficiently far from the edges of the resonance region, these functions almost do not depend on energy.

In Fig. 2, one can see the normalized absolute value of the above-defined correlation functions for three different values of γ. To obtain the data, 100 matrices drawn from the ensemble (1.4,2.1,2.2) with dimension $N=400$ have been diagonalized. To improve the statistical significance of our data, the averaging has been performed both over the ensemble and the energy. It is well known that these two kinds of averaging are equivalent because of the so-called "ergodicity" of the matrix ensembles [10]. This energy averaging was performed over 1001 values within the interval $[-5d,5d]$ in the center of resonance region $[-2,2]$ where $d=4/N$ is the mean spacing of resonances. The data clearly indicate that the correlations are decreasing with increasing ε. As one should expect, for small and large γ the two correlation functions are almost identical. For $\gamma=1$ our results for the correlation function (3.11) are in good agreement with those given in [33]. The latter have been obtained by numerical evaluation of a threefold integral which is the exact theoretical expression given in [14].

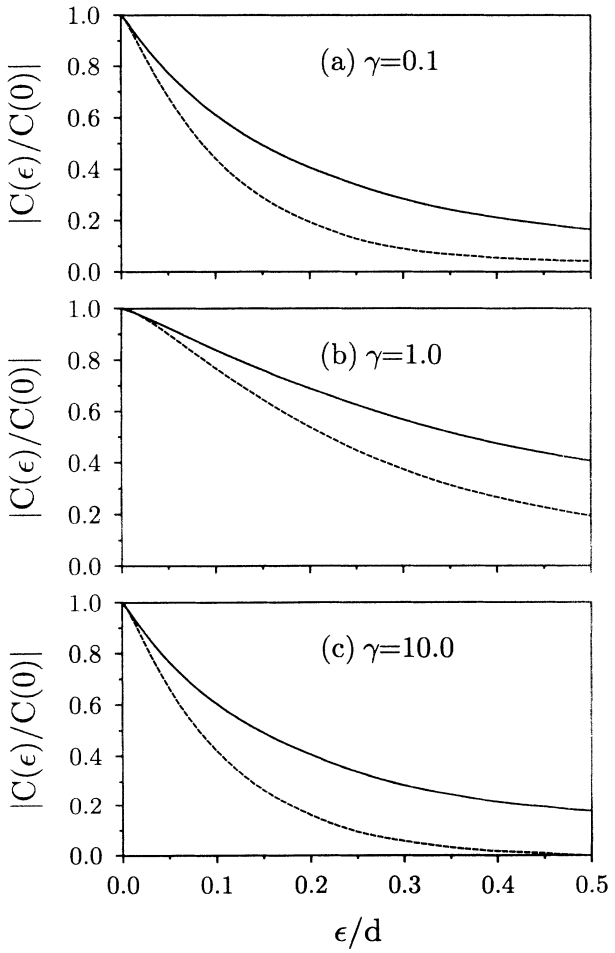

FIG. 2. Modulus of the normalized correlation functions for the S matrix (3.11) (full line) and total cross section (3.12) (dashed line) for three different values of the coupling parameter γ. The average was taken as an ensemble average over matrices of structure (1.4,2.1,2.2) with $N=400$ as well as an average over energy (for details see text).

To elucidate the γ dependence of the correlation functions it is useful to study the correlation length l. Here we define l as the value of ε for which the correlation function is equal to one-half of its maximum value. In Fig. 3 two correlation lengths are shown: one for the S matrix correlation function (l_S) and one for the cross section (l_σ). They are obtained from numerical calculations similar to that in Fig. 2, but just averaged over 81 energy values. These two lengths turn out to be proportional to each other. The γ dependence shows maxima for the critical value $\gamma_c=1$. It means that for this value of γ the maximal coherence appears in the system which results in the formation of the collective short-living state. After its segregation, when γ exceeds this critical value, this states does not influence the fluctuations in the system. This is why the correlation lengths are decreasing for γ greater than the critical value. Note that the symmetrical form of the γ dependence is due to the fact mentioned above that all statistical properties depend only on the transmission coefficient, rather than on γ itself.

It is quite clear that for nonoverlapping resonances, the correlation length actually coincides with the mean value of the width of long-living resonances. On the other hand, the latter is well known [34] to be proportional to the transmission coefficient. It means that the relation

$$T=2\pi\frac{\langle\Gamma\rangle}{d_{\rm loc}}=2\pi\frac{l_s}{d_{\rm loc}} \tag{3.13}$$

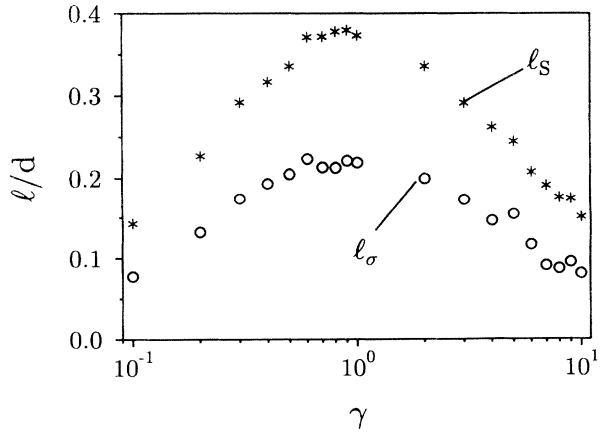

FIG. 3. Correlation length for the S matrix (l_S) and cross section (l_σ) correlation function (see text) as a function of the coupling parameter γ.

should hold in the regions of asymptotically small and large γ. Actually, the second equality is not provided automatically by our data obtained for moderate values of γ. It is because of the somewhat arbitrary definition of the correlation length for a function with non-Lorentzian form [33] that this is so. In Fig. 4 we therefore show (asterisks) the correlation length rescaled to satisfy the second equality (3.13) for $\gamma=0.1$.

One can see from Fig. 4 that the mean width exceeds the transmission coefficient for all values of γ given in this figure. Note that for $\gamma>\gamma_c=1$ the broad short-living resonance is not taken into account when averaging. We conclude that the overlapping of long-living resonances is not at all extremely small. It is clearly seen that the γ dependence of the correlation length follows that of the transmission coefficient rather than that of the mean width. This is in agreement with the connection between the time delay and the S-matrix fluctuations established in [22,23,35].

It is interesting to compare (3.9) with the well-known formula

$$T=1-\exp\left[-2\pi\frac{\langle\Gamma\rangle}{d_{\rm loc}}\right], \tag{3.14}$$

derived in [36,37] for the case of strongly overlapping resonances. It connects the transmission coefficient to an arbitrary value of the mean width $\langle\Gamma\rangle$. The result is also presented in Fig. 4 (diamonds) and is in good agreement with the theoretical curve. Nevertheless, this formula is not exactly correct. Using (3.9) we find for the mean width of long-living resonances the expression

$$\frac{\langle\Gamma\rangle}{d_{\rm loc}}=\frac{1}{\pi}\ln\left|\frac{1+\gamma}{1-\gamma}\right|, \tag{3.15}$$

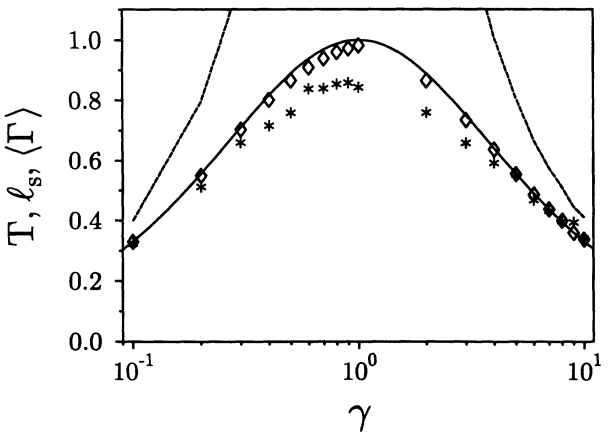

FIG. 4. Dependence of some characteristics of the model on the coupling parameter γ. The full curve refers to the transmission coefficient (3.9). The dashed curve represents the mean width in units of the local mean spacing ($d_{\rm loc}=\pi/N$) obtained by diagonalizing 100 matrices for each value of γ with $N=400$ drawn from the ensemble (4.1,2.1,2.2). The diamonds correspond to Eq. (3.14). The asterisks give the S matrix correlation length rescaled to be equal to the transmission coefficient for $\gamma=0.1$.

which is singular at the point of the critical value of γ. Therefore, the relation (3.14) fails in the domain of transition between two different types of internal motion. This domain is very narrow because of the logarithm on the right-hand side of (3.15).

In Fig. 5 we demonstrate how sharp the transition between these two different regions appears. Plotted are the values of the three largest resonance widths as a function of the coupling parameter γ. To get smooth curves we averaged over 20 matrices. One can clearly see how the width corresponding to the wide unstable resonance increases rapidly (note the logarithmic scale) passing the critical value γ_c, while the remaining widths decrease together. In Fig. 5(a) data are shown which are taken from diagonalization of matrices with dimension $N=400$, while the data in Fig. 5(b) represent matrices with $N=1000$.

IV. FOURIER TRANSFORM OF THE TWO-POINT CORRELATION FUNCTION

As was already mentioned in the Introduction, the decay law of an unstable state is closely connected to the

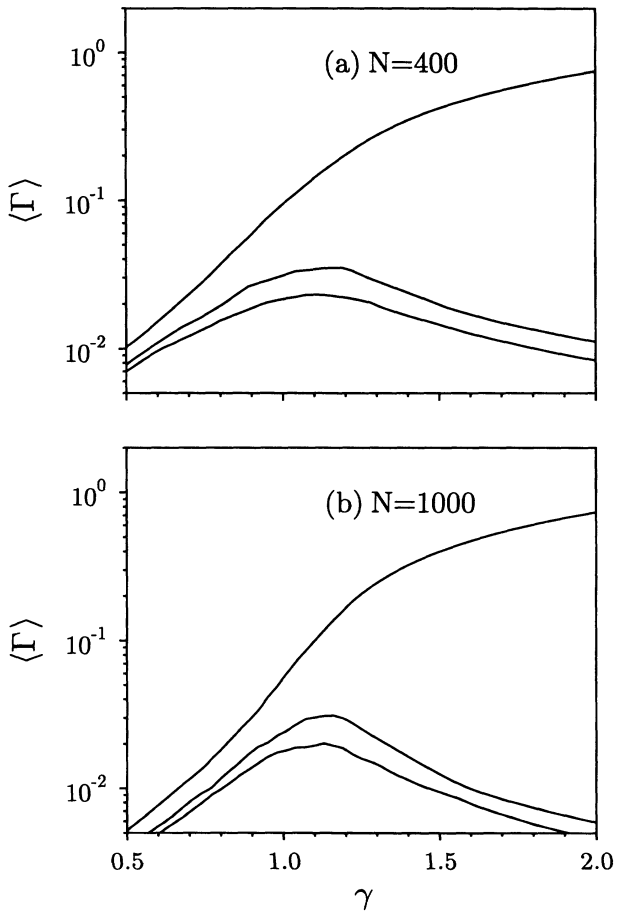

FIG. 5. The values of the three largest resonance widths as function of the coupling parameter γ are shown. The data are obtained by diagonalizing 20 matrices drawn from the ensemble (4.1,2.1,2.2) for different values of the coupling and two dimensions of the matrices.

Fourier transform of the two-point correlation function of the S matrix. Here, we are interested in its behavior for asymptotically large times. This asymptotic behavior corresponds to small values of ε and was analyzed theoretically in [24] using the analytical results of Ref. [14]. Taking the single-channel case as an example, we show below that the same asymptotic behavior can be obtained in a much simpler way by the direct energy averaging procedure. To make the averaging over energy it is very convenient to use the Lorentzian weight function

$$P_E(E')=\frac{I}{2\pi}\frac{1}{(E'-E)^2+\frac{1}{4}I^2} . \tag{4.1}$$

The interval I of the averaging should contain a sufficiently large number of resonances. This amount of resonances is nevertheless much smaller than the total amount of them, so that $d \ll I \ll 4$. The form (4.1) for the weight function allows one to perform exactly the energy averaging, provided that one uses the resonance representation (3.1) for the S matrix. In the next step, without any trouble one can get the Fourier transform, which for times $t \gg 1/I$ reads

$$\begin{aligned} F(E;t)&=\frac{1}{4\pi}\int_{-\infty}^{\infty} d\varepsilon\, C_S(E;\varepsilon)e^{i\varepsilon t} \\ &\approx i\sum_k \tilde{V}_k^2 e^{-i\mathcal{E}_k t}\left|\frac{1}{E-\mathcal{E}_k+\frac{i}{2}I}-\frac{1}{E-\mathcal{E}_k-\frac{i}{2}I}\right| \\ &\quad\times\sum_{k'}\left|\tilde{V}_{k'}^*\right|^2 e^{i\mathcal{E}_{k'}^* t} , \end{aligned} \tag{4.2}$$

where $C_S(E;\varepsilon)$ is the S matrix correlation function defined in (3.11) but with an energy average rather than an ensemble one. The only term omitted in (4.2) is proportional to $\exp(-It/2)$ and, therefore, is small.

Since the interval I contains many resonances, one can neglect Γ_k as compared to I in the denominators in (4.2). This is not possible for the wide resonance that appears when γ exceeds $\gamma_c=1$. However, the corresponding term is exponentially small for the considered time and can be omitted. Therefore, for arbitrary values of γ one gets

$$F(E;t)=\sum_k \tilde{V}_k^2\frac{I}{(E-E_k)^2+\frac{1}{4}I^2}e^{-i\mathcal{E}_k t}\left|\sum_k \tilde{V}_k^2 e^{-i\mathcal{E}_k t}\right|^* , \tag{4.3}$$

where the sum is taken over all resonances but the wide one. Averaging over the whole energy region, one finds the energy-independent expression

$$F(t)=\frac{1}{Nd}\int_{-\infty}^{\infty} dE\, F(E;t)=\frac{2\pi}{Nd}\left|\sum_k \tilde{V}_k^2 e^{-i\mathcal{E}_k t}\right|^2 . \tag{4.4}$$

One can see that the interval I dropped out of the expression for $F(t)$. The asymptotic behavior of the Fourier transform is given by some sort of self-averaging. For large times, the cross terms in the squared modulus in (4.4) cancel each other out due to the randomness of the phases $E_k^t-\alpha_k$, and one obtains

$$F(t)=\frac{2\pi}{Nd}\sum_k \tilde{\Gamma}_k^2 e^{-\Gamma_k t} . \tag{4.5}$$

We would like to stress that even small fluctuations of GOE levels are enough to provide this cancellation at asymptotically large times. As was pointed out above, for small and large values of γ the quantities $\tilde{\Gamma}_k$ coincide with the resonance widths Γ_k. Since for these cases they are distributed in accordance with the Porter-Thomas law

$$\rho(\Gamma)=\frac{1}{\sqrt{2\pi\langle\Gamma\rangle\Gamma}}\exp\left|-\frac{\Gamma}{2\langle\Gamma\rangle}\right| , \tag{4.6}$$

one can approximately write

$$\begin{aligned} F(t)&=\int_0^{\infty} d\Gamma\,\rho(\Gamma)\Gamma^2 e^{-\Gamma t} \\ &=\frac{6\pi}{d}\frac{\langle\Gamma\rangle^2}{(1+2\langle\Gamma\rangle t)^{5/2}} . \end{aligned} \tag{4.7}$$

This result is in accordance with that obtained in [24]. In particular, it gives the $t^{-5/2}$ dependence for asymptotically large times, in agreement with the general asymptotic formula given in [33]. On the other hand, for values of γ near γ_c the quantities $\tilde{\Gamma}_k$ differ from the widths, and their distribution is, in essence, unknown. Therefore, for this case one cannot explicitly estimate the sum in (4.5).

For numerical calculations of the Fourier transform, the expression (4.4) is used where the quantities $\tilde{V}_k^2$ are computed according to (3.3). Figure 6 presents numerical data for the normalized Fourier transform (fluctuating line) for one matrix taken from the ensemble (1.4,2.1,2.2) with $N=400$ and $\gamma=0.1$. The dependence (4.7) is shown by the smooth line. As one can see, Fig. 6 clearly reflects the self-averaging nature of the calculated quantity.

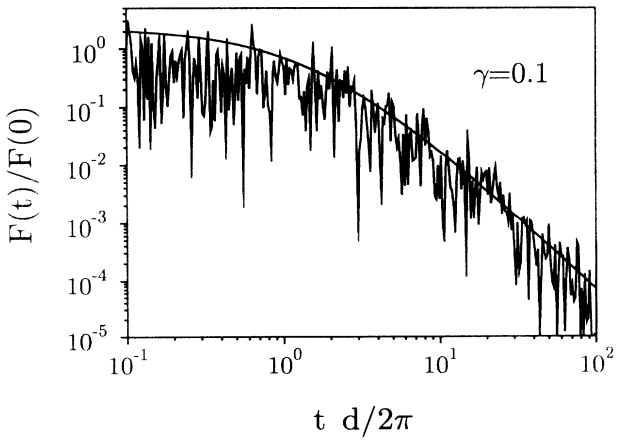

FIG. 6. Normalized Fourier transform of the S matrix correlation function. The fluctuating curve is obtained numerically from the expression (4.4) using (3.3) by diagonalizing one matrix of the ensemble (1.4,2.1,2.2) with $N=400$ and $\gamma=0.1$. The smooth curve is given by (4.6) and normalized as explained in the text.

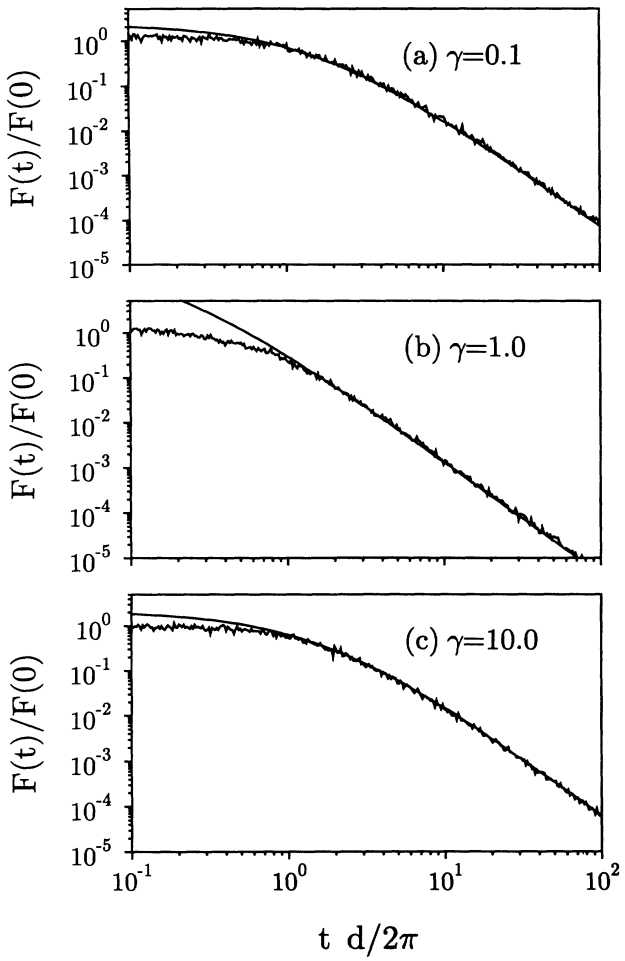

FIG. 7. Same as Fig. 1 but for three different values of the coupling parameter γ and averaged over 100 matrices of the ensemble (1.4,2.1,2.2) with $N=400$.

Large fluctuations in Fig. 6 are removed by averaging over an ensemble of matrices (see Fig. 7). This averaging was performed over 100 matrices.

In Figs. 7(a) and 7(c), the very good correspondence between the numerical data and analytical formula (4.7) is seen in the entire region of its validity. As should be expected, these two curves practically coincide. Figure 7(b), where the case $\gamma=1$ is given, demonstrates, in agreement with the above discussion, a clear deviation of the numerical data from (4.7). Nevertheless, our data demonstrate that the asymptotic behavior at extremely large times has the same form as in (4.7) [24]. Due to the fact that the expression (4.7) is not valid for small times, this function was normalized in the asymptotic region $td/2\pi \gg 1$.

V. CONCLUSIONS

In this paper, we have thoroughly studied the fluctuations of elastic resonance scattering, as they reflect the statistical properties of the underlying intermediate resonance states. The scattering model under consideration is based on the random-matrix approach. The data obtained are in good agreement with previous analytical results and demonstrate clearly the existence of a critical value of the coupling to the continuum corresponding to a certain reorganization of the intrinsic motion of the intermediate complicated decaying system.

ACKNOWLEDGMENTS

We would like to thank Fritz Haake and Hans-Jürgen Sommers for enlightening discussions. We are also grateful to C. M. Lewenkopf and V. G. Zelevinsky for a careful reading of the manuscript and many useful comments and suggestions. Two of us (F.I. and V.S.) acknowledge financial support by the Russian Fund for Fundamental Investigations, Contract No. 93-02-6888, as well as by the Sonderforschungsbereich "Unordnung und große Fluktuationen" (SFB 237) der Deutschen Forschungsgemeinschaft during their stay in Essen, where the main part of this work has been done. D.S. would like to thank the SFB 237 and the Budker Institute of Nuclear Physics for financial support and hospitality during his stay in Novosibirsk. The numerical part of this work has been done at the Höchstleistungsrechenzentrum des Forschungszentrums Jülich and the Rechenzentrum der Rheinisch-Westfälischen Technischen Hochschule Aachen.

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