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Deterministic origin of $1/f$ noise in magnetic resonance

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A $(1/f)^{\beta}$ behavior in a high-power magnetic-resonance experiment is presented which evolved from a state of strong chaos as the driving power was increased. The experimental observation was simulated with a deterministic, dissipative, and continuous multimode model of spin-wave dynamics with no assumptions of randomness.

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I. INTRODUCTION

The phenomenon of $1/f$ noise, which refers to the scaling relation $p \propto f^{-\beta}$ of the power spectral density p with frequency f of the temporal fluctuations, has been observed in a great variety of diverse systems [1]. Here continuous strong pumping of a system of spin waves (SW's) is shown to exhibit $p \propto f^{-\beta}$ dependence of power absorption. This behavior was simulated with a model of many interacting SW modes, indicating that in this special circumstance the $1/f$ noise can be explained as a deterministic phenomenon.

In the field of nonlinear dynamics attempts have been made to explain $1/f$ scaling as a chaotic phenomenon. While dimensional analysis of $1/f$ noise from a variety of sources provided no evidence of a possibly deterministic low-dimensional dynamics underlying the observed time series [2], $1/f$ dependence has been observed even in such simple systems as for example the strongly driven chaotic pendulum [3]. The strong noise observed in Josephson-junction circuits was successfully explained as a deterministic chaotic phenomenon [4], and the deterministic motion of a particle in a two-dimensional periodic potential was found to exhibit $1/f$ scaling in a Hamiltonian system [5].

Here a $1/f^{\beta}$ behavior with $\beta=2.4$ is presented which was observed in the microwave absorption of a high-power parallel-pumping (PP) experiment in an antiferromagnet. Using a multimode continuous model of SW dynamics with dissipation, the essential properties of the experimental observations were reproduced with no assumptions of randomness suggesting that the $1/f$ behavior reported here is of deterministic origin. These results show that $1/f$ noise can develop in specific magnetic systems as the pumping power is increased beyond the values where high-dimensional chaos is observed.

In general, exchange coupled magnetic systems are ideal for the study of strong chaos due to the many degrees of freedom which may be excited simultaneously, and observations of high-dimensional chaos have been reported [6] in addition to marginal (low-dimensional) chaos [7]. The excitations are usually described as a collection of distinct SW modes in the space and time domain that interact with each other resulting, in a high-dimensional multimodes system.

II. EXPERIMENTS

Single crystals of antiferromagnetic $[NH_3CH_2]_2CuCl_4$ with a Néel temperature of 32 K were placed at the center of a 9.1-GHz cavity in the PP configuration, where the easy axis of magnetization, the static field, and the microwave field $\mathbf{h}_{\parallel}$ are mutually parallel to each other. In this configuration, SW's are directly excited by $\mathbf{h}_{\parallel}$ when the field is stronger than a threshold field $\mathbf{h}_{\text{th}}$. By increasing the power well above $\mathbf{h}_{\text{th}}$, chaotic auto-oscillations were observed in the kHz region, as reported elsewhere [8]. This high pumping power was possible without causing heating effects because of the extremely low threshold value for PP in this layered antiferromagnet ($P_{\text{th}} \approx 0.2 \mu$W at $T=1.4$ K). At low temperatures ($T \approx 1.4$ K) these auto-oscillations occurred well above the thermal noise. In some experiments a further increase in power revealed a very broad frequency spectrum. This behavior was very sensitive to the external parameters such as applied field, temperature, microwave power, and angle between the easy axis of magnetization and the applied field. Figure 1(a) shows such a time dependence of the microwave absorption $A_i(t_i)$ sampled at 50.137 kHz over a period of 100 ms for very strong pumping power $P \propto h_{\parallel}^2$, which was about 45 dB above the threshold. The signal was bandwidth limited between 100 Hz and 1 MHz. Within the limits mentioned above a large range of time scales was observed.

In Fig. 1(b) the logarithm of the power spectral density, obtained by averaging 19 power spectra from overlapping data segments of 512 points weighted by a triangular window, was plotted vs the logarithm of the frequency, revealing a nearly linear decay over 1.5 orders of magnitude. The line in Fig. 1(b) represents a fit over 1.5 decades of the low-frequency part of the power spectrum resulting in the exponent $\beta_{\text{exp}} = -\log_{10} p / \log_{10} f \approx 2.4$, and suggesting low-frequency time-scale invariance.

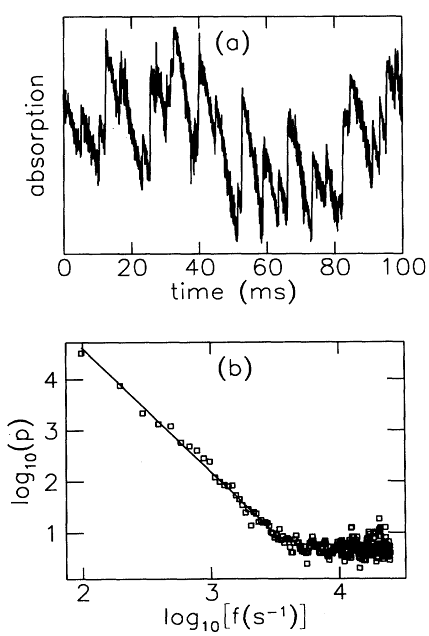

FIG. 1. (a) Time dependence of the microwave absorption measured by PP in $[NH_3CH_2]_2CuCl_4$. The very high continuous pumping power was 45 dB above the threshold for the PP process. Note the large range of time scales. (b) Logarithm of the power spectral density p vs logarithm of frequency f. The line is a linear fit over 1.5 decades of the low-frequency part of the power spectrum with an exponent $\beta_{\text{exp}} = -\log_{10} p / \log_{10} f \approx 2.4$.

III. NUMERICAL SIMULATIONS

In order to confirm the possibility of a deterministic origin of the $(1/f)^{\beta}$ behavior observed in the experiment, a numerical simulation of a strongly excited many-mode SW system was performed.

The model used was the stroboscopic model (SM), which is a classically derived continuous model of SW's [9]. In contrast to the established S theory [10], which uses the approximations of small precession angles, the SM remains valid and finite for very strong excitations of the spin system and is thus well suited to simulate the experimental situation described above. The starting point of the SM was a classical equation of motion for the magnetization

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} , \qquad (1)$$

where $\mathbf{M}$ is the macroscopic magnetization, γ the gyromagnetic ratio, and $\mathbf{H}_{\text{eff}} = \mathbf{h}(t) + \mathbf{h}_A + \mathbf{h}_{\text{int}}$ is the effective field, which included a pump field $\mathbf{h}(\mathbf{t})$, an effective uniaxial anisotropy field in the x direction $h_{A,x} = -d_{Ax} S_x$, and in the z direction $h_{A,z} = -d_{Az} S_z$, respectively, and the interaction field h_{int}, $x = -2 A_{x,k} S_{x,j}^2$, which was restricted to quadratic terms in two modes k and j. Magnetic excitations which for the PP case consist of standing SW's were represented by a fictive classical spin $\mathbf{S}$ of constant magnitude. The transformation to slow motions was implemented by analytically integrating the precession of the fictive spins using the approximation that the static field is much larger than all other fields. In the case of PP the integration ranges over two periods of the pump frequency. For the PP case the following set of differential equations for the polar angle ϕ_k and the azimuthal angle θ_k of the spin S_k, strobed every second pump period, was obtained in normalized dimensionless units:

$$\begin{aligned} \frac{d\theta_k}{dt} &= h_{\parallel} a_k \sin(\theta_k) \sin[2(\phi_k)] - r_k \sin\theta_k \\ &\quad + \sin(\theta_k) \sum_j \sin^2(\theta_j) B_{kj} \sin[2(\Delta\phi_{kj})] , \\ \frac{d\phi_k}{dt} &= h_{\parallel} a_k \cos(\theta_k) \cos[2(\theta_k)] + \Delta\omega_k + d_k(1 - \cos\theta_k) \\ &\quad + \cos(\theta_k) \sum_j \sin^2(\theta_j) B_{kj} \cos[2(\Delta\phi_{kj})] , \end{aligned} \qquad (2)$$

where $a_k = d_{Ax,k}/2\omega_p$ is the coupling of the pump field $h_{\parallel}$ to mode k, $d = d_{Ax,k}/2 - d_{Az,k}$ is the self-detuning, $\Delta\omega_k = \omega_p/2 - \omega_k$ is the detuning of the SW frequency $\omega_k = h_{\parallel} + d_{Ax,k}/2 - d_{Az,k}$ from half the pump frequency ω_p, $h_{\parallel}$ is the PP field amplitude, and $\Delta\phi_{kj} = \phi_k - \phi_j$. The interaction between the two modes is described by the coupling matrix B_{jk}. Finally r_k is the damping of mode k which was introduced phenomenologically.

When the equations of the SM [Eqs. (2)] are expanded with respect to small polar angles θ_k and only the first terms in the expansion are retained ($\sin\theta_k \approx \theta_k$, $\cos\theta_k \approx 1$), they are equivalent to the results of the S theory as shown in [9]. In this case the dynamical variables of the S theory, which are the magnon number n_k and phase ψ_k, are related to the dynamical variables of the SM by $n_k = \theta_k^2$ and $\psi_k = 2\phi_k$.

In the PP experiment a single microwave photon directly excites a SW pair with equal and opposite wave vector, resulting in the creation of a standing spin-wave mode. In general a broad manifold of SW's exists [11]. Therefore, there is a degenerate band of SW's which can be directly excited by the pump term $h_\parallel$ in Eq. (2). In addition, the SW modes are directly coupled to other modes by the coupling term B_{kj} in Eq. (2). Thus it is in general not possible to determine from the experiment how many modes are excited.

In order to test how many modes are required to simulate the experimental data, a numerical integration of Eq. (2) was performed by considering an increasing number N of modes driven by a strong pumping field ($h_\parallel = 105$). The coupling of the pump field to the mode k decreased exponentially $[a_k = (1/2)^{N-1}]$. The damping $r_k = 0.1$ and self-detuning $d_k = 0.5$ were equal for all modes and the detuning decreased in a linear way from $\Delta\omega_k = 0$ for mode 1 to $\Delta\omega_k = -3.6$ for mode 10. The neighboring modes were coupled by $B_{kj} = -7.5$ for $j = k \pm 1$ and next neighboring modes by $B_{kj} = 0.5$ for $j = k \pm 2$. The microwave absorption observed in the experiment is proportional to the sum of power fed to each mode and is obtained from the model [Eq. (2)] as

$$A_{\text{th}} = h_\parallel \sum_{i=1}^{N} a_i \sin^2(\theta_i)\cos(2\phi_i) \tag{3}$$

[12]. The situation is shown schematically in Fig. 2.

The exponent obtained from the simulation for the scaling of the first 1.5 frequency decades of the power spectrum is shown as a function of the number of modes

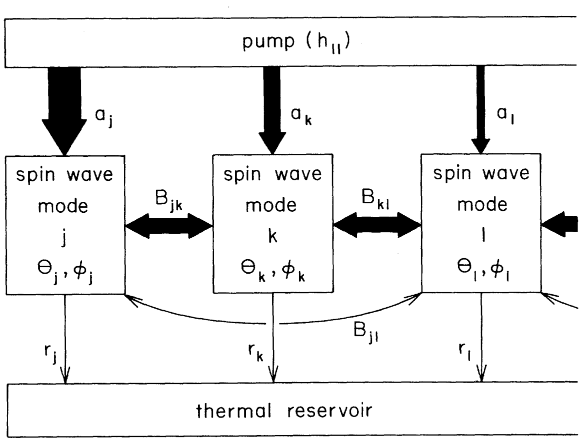

FIG. 2. Schematic diagram of the multidimensional model [Eq. (2)] of interacting spin-wave modes driven by a field $h_\parallel$ which is parallel to the static field. Only three modes are shown. The constants used for the simulations were an exponentially reducing coupling a_k of $h_\parallel$ to the modes, strong neighbor mode coupling $B_{i,i\pm1}$ and weak next neighbor mode coupling $B_{i,i\pm2}$, and identical weak damping for all modes.

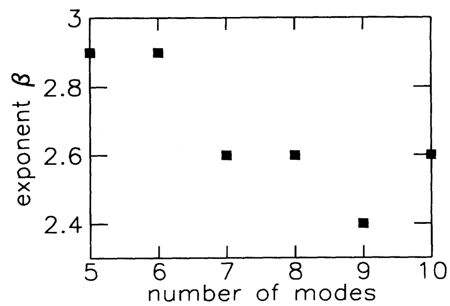

FIG. 3. Scaling exponent $\beta_{\text{th}} = -\log_{10}p/\log_{10}f$, where p and f are the power spectral density of the signal obtained from Eq. (3) and the frequency, respectively, vs the number of modes N used in Eq. (1).

in Fig. 3. In the case of two, three, and four modes, the exponent β is not defined because the system evolved after the initial transient to a fixed point. The general trend up to $N=9$ is that β decreases with N. For $N=9$ the simulation gave the same exponent as the experiment. Coupling further modes to the system led to an increased β for $N=10$, and for 11 modes the system lost its (low-frequency) time-scale invariance and evolved on a limit cycle after a long transient. This change is behavior for $N=11$ demonstrates how sensitive the model is to changes in the parameter space. The further dependence

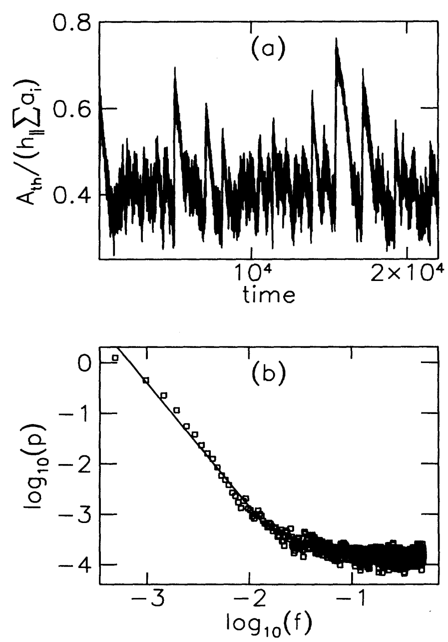

FIG. 4. (a) Simulated absorption in dimensionless units normalized to the maximum possible value [Eq. (3)] obtained by numerical integration of the spin-wave model [Eq. (2)] with nine modes vs reduced time. Similarly to the experimental signal, a large range of time scales was obtained. (b) Logarithm of the power spectral density vs logarithm of frequency. The line is a linear fit over 1.5 decades of the low-frequency part of the power spectrum with an exponent $\beta_{\text{th}} = -\log_{10}p/\log_{10}f \approx 2.4$, which was very close to the experimental value suggesting that the $1/f$ behavior observed in the experiment is possibly of deterministic origin.

of β on the mode number has not yet been explored and cannot be readily predicted due to the sensitivity of the system to changes in parameter space and parameter values. In order to obtain the same scaling as in the experiment, a fine tuning of N was necessary. Consequently the model cannot provide a generic explanation of $1/f$ noise, but for a specific parameter set and nine modes an excellent agreement between experiment and simulation can be obtained.

An example of a calculation considering nine modes is shown in Fig. 4(a). The absorption A_{th} was normalized to the maximum value $(h_{\parallel}\sum_i a_i)$. The power spectral density was obtained by averaging 25 data segments of 2048 points. In comparison with the experimental observation [Fig. 1(a)] the simulation also displayed a large range of time scales. The low-frequency scaling was dominated by strong irregular pulses with a fast rise time followed by a slow relaxation. Because of the exponential reduction of the coupling with mode number the absorption is controlled mainly by the dynamics of the first mode [see Eq. (3)]. This model implies that the experimental observation is governed only by a small number of variables, but a large number of "hidden" variables are implicated as in the case of the on-off intermittency discussed recently [13]. Due to the restricted data sets and low-frequency deviations from power-law scaling which began to occur for nine and ten modes, exact values for β were difficult to obtain. On the other hand, experiment and model data were analyzed with the same method described above, allowing a direct comparison. The theoretical value was close to the value obtained from experiment for nine modes supporting the presumption of a deterministic origin of the $1/f$ noise observed in the experiment.

IV. CONCLUSIONS

In conclusion, high-power magnetic resonance experiments reveal $(1/f)^{\beta}$ dependence of the power spectral density where $\beta=2.4$. The measurements were compared to the results of a multimode model of spin-wave dynamics. For nine modes, the simulation revealed an exponent $\beta=2.4$. The numerical results imply that the $(1/f)^{\beta}$ behavior observed in this experiment is of deterministic origin and can be obtained by considering a few modes, granted that the pumping power is strong enough. These results suggest that deterministic $1/f$ noise can develop from a situation of high-dimensional chaos as the pumping power is increased.

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