

Mechanism of the electric-field-induced vortex drift in excitable media

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A vortex behavior in an electric field is shown to depend strongly on the elongation and contraction of the vortex tip. The wave width of the rotating vortex tip is periodically modulated: it increases (decreases) when the tip propagates to the positive (negative) electrode. This affects the elongation of the vortex tip, resulting in a perpendicular component of the drift defined by chirality of the vortex. A kinematic model is formulated where the elongation of a wave break is explicitly defined by the wave width, and its behavior is compared with the numerical calculations of reaction-diffusion equations.

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Studies of the electric-field influence on chemical active media have shown [1,2] that a wave propagating to the positive (negative) electrode is accelerated (decelerated). It is quite clear that this effect would give rise to a drift of a rotating vortex toward the positive electrode. The effect of electric field on a vortex [3,4] and the drift [5,6] have really been found experimentally. Unexpectedly, a component of the drift, perpendicular to the electric-field direction, was found too. This component was comparable to, or even greater than, the parallel component of the drift.

In this Rapid Communication, we propose a mechanism underlying these observations. It is based on the influence of electric field on the elongation and contraction of the vortex tip. From a topological point of view, vortices in excitable media are equivalent to dislocation-type defects in striped patterns. Elongation and contraction of these defects are known to be strongly dependent on the parameters of the system [7].

The electric field affects not only the velocity V but the width w of a wave as well, since $w = \tau V$, where τ is the wave duration. For a rotating vortex, it means that the width of its tip should be periodically modulated, increasing (decreasing) when it moves to the positive (negative) electrode. This in turn should affect the wave propagation. In particular, the increase of the width of the vortex tip should result in increasing the elongation velocity of the tip and in a perpendicular component of the drift defined by the chirality of the rotating spiral.

This idea is explained below in more detail. Movement of the vortex tip (free edge or wave break) is described by its normal V_n and tangential (elongation or growth) V_g velocities. V_g is usually modeled as determined by the curvature of the wave front near the tip [8], and normal velocity as $V_n = c + \text{curvature term} + \text{free-edge term}$ [9]. Here c depends on the medium properties, curvature term $= aK / (1 - wK)$, where K is the front curvature and w is the wave width, free-edge term $= -b \exp(-s/d)$, where s is the distance from the free edge; and a , b , and d are parameters.

The electric field should affect not only the normal velocity (as shown in one-dimensional experiments [1,2]) but the tangential velocity also, which is not so evident.

Changes of the wave width affect the diffusion flux into an area element, and thus influence the wave velocity. The standard kinematic model [9,10] ignores this effect; the model [9] describes it for the normal velocity only. We will take this effect into account to formulate a kinematic model where tangential velocity is explicitly influenced by the wave width, and to investigate vortex behavior in the electric field.

The behavior of a vortex in an excitable medium is determined by the dynamics of the vortex tip. So, we are interested in the description of the vortex tip velocities only. We model the tangential growth velocity as proportional to wave width w ,

$$V_g = \gamma(w - w_0), \quad (1)$$

and the normal velocity as dependent upon the angle ϑ

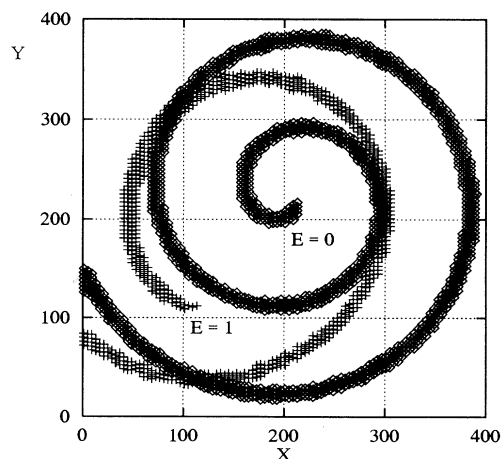


FIG. 1. Drift of the vortex in an electric field parallel to the X axis. Black, the vortex with $E=0$; gray, the vortex 67 time units (t.u.) after switching on the electric field ($E=+1$, positive electrode at the left). The two-variable Oregonator model (1) was simulated by the Euler method with null flux boundary conditions on a grid of 400×400 elements with $\Delta t = 10^{-3}$ and $\Delta x = 0.2$. The parameters are $\epsilon = 0.1$, $f = 1.4$, $q = 0.002$, $D_u = 1$, and $D_v = 0.6$.

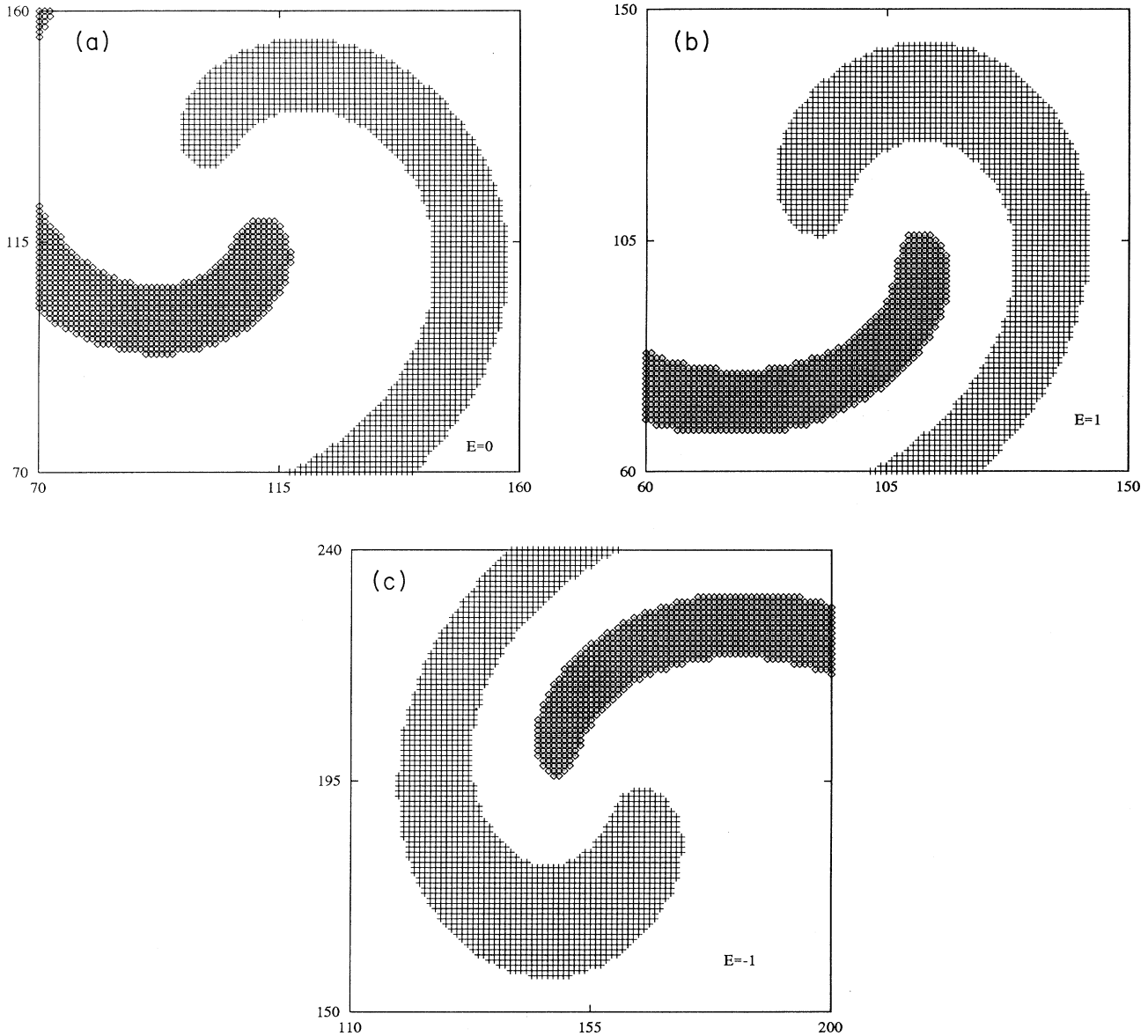


FIG. 2. The vortex tip. (a) No electric field, $E=0$; (b) $E=1$; (c) $E=-1$. Two positions of the rotating vortex tip are depicted, when it is moving to positive (shown in gray) or negative (shown in black) electrodes. Note that the wave width is increased when the wave is moving to the positive electrode (same parameters as in Fig. 1; in black or gray are shown the values of $u > 0.5u_{\max}$).

between E and the vortex tip: $V_n = h(V_0 + \beta \sin \vartheta)$, where h is the chirality of the vortex ($h = \pm 1$). Then the wave width is $w = w_0 + \alpha h \sin \vartheta$, where $\alpha = \tau \beta$. Coefficient β can be easily estimated for small E , when a linear approximation of the velocity is acceptable: $V = V_0 + V_1 E + \dots$; then $\beta = V_1 E$. The growth velocity V_g is known to increase with the excitability of the medium, so the same is valid for γ ; but because no quantitative investigations are published, we will not specify γ here.

The position (X, Y) of the vortex tip is described as

$$\dot{x} = V_g \cos \vartheta + h V_n \sin \vartheta, \quad \dot{y} = -V_g \sin \vartheta + h V_n \cos \vartheta, \quad (2)$$

where ϑ is the angle between E and the vortex tip direc-

tion ($\vartheta = \omega t$). The X and Y components of drift velocity V_x and V_y can be readily estimated by averaging (2) with respect to fast rotations:

$$V_x = \beta/2, \quad V_y = -h\gamma\alpha/2. \quad (3)$$

TABLE I. Influence of the electric field on the wave-break characteristics. V_n is the normal velocity, V_g is the tangential growth velocity, and w is the thickness of the wave.

| | V_N (s.u./t.u.) | V_g (s.u./t.u.) | w (s.u.) |
|--------|-------------------|-------------------|------------|
| $E=0$ | 3.3 | 1.8 | 2.2 |
| $E=-1$ | 2.5 | 1.7 | 1.7 |
| $E=+1$ | 4.4 | 1.9 | 2.6 |

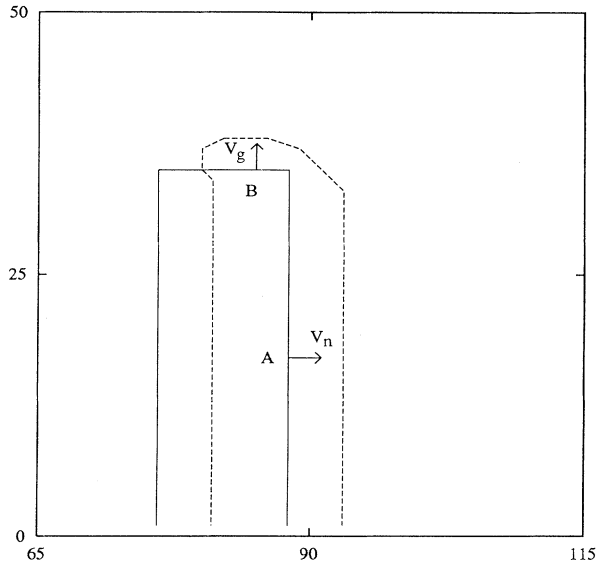


FIG. 3. Numerical experiment for measuring the tangential growth V_g and the normal V_n velocities. Solid line, initial position of a wave break ($t=0$); dashed line, its position at $t=0.25$. The wave break was created as follows. The wave was moving to the right in a medium of 200×35 space units (s.u.). Then at $t=0$, the size of the medium was increased to 50 s.u. in the Y direction, thus forming the free end of the wave (the wave break). The free end started propagating into the newly added medium. The tangential growth velocity V_g was measured as the velocity of the wave-break elongation (same parameters as in Fig. 1, but $D_v=0$).

It is seen that the transverse component of the drift V_y is not zero; its sign is determined by the chirality h of the vortex, and the ratio $V_y/V_x = -h\gamma\tau$ is governed by the excitability of the medium (as is γ).

We compared the behavior of the model [Eqs. (1), and (2)] with the numerical simulations of the reaction-diffusion model (the two-variable Oregonator model [11]):

$$\frac{\partial u}{\partial t} = \frac{F}{\epsilon} + D_u \Delta u + E_x \nabla_x u, \quad \frac{\partial v}{\partial t} = \Phi + D_v \Delta v. \quad (4)$$

Here the variables u and v represent the dimensionless concentrations of HBrO_2 and of ferrin, respectively; D_u and D_v are the dimensionless diffusion coefficients of u and v ; and E is the electric field. The functions $F = u - u^2 - fv(u-q)/(u+q)$, and $\Phi = u - v$; q and ϵ are parameters.

In Fig. 1, a vortex is shown. It is seen that the electric field initiates the vortex drift in both parallel and perpendicular directions. In Fig. 2, enlarged images of the tip are displayed. The width of the tip is seen to be increased (decreased) when it moves to the positive (negative) electrode.

We studied normal V_n and tangential growth V_g veloc-

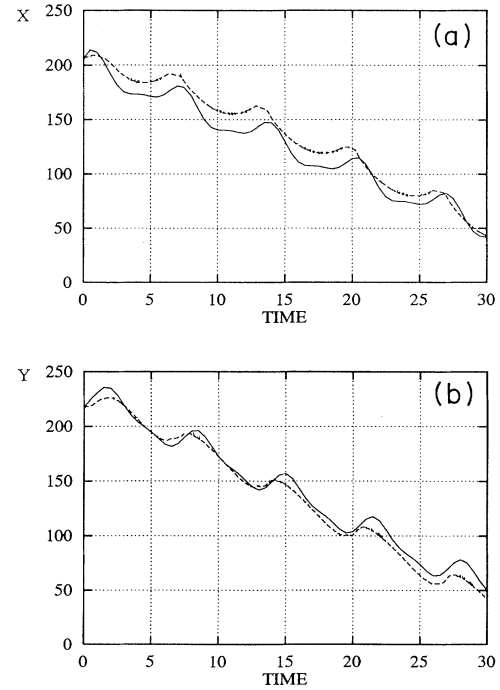


FIG. 4. X and Y positions of the vortex tip in the electric field. Solid line, the Oregonator model; dashed line, the kinematic model based on the dependence of the tangential growth velocity on the wave width.

ities of a broken wave front by comparing two positions of the wave break, as shown in Fig. 3. An electric field was applied along the X axis, so that it directly affected the normal velocity only, and not the tangential growth velocity. We observed, nevertheless (Table I), that not only V_n but also V_g was changed as well as the wave width, in accordance with the model prediction.

Note that the characteristic time scale τ of the chemical kinetics as estimated from Table I ($\tau = 0.26 \pm 0.02$) is not influenced by the intensity and direction of the electric field. The independence of τ of E results in the simplicity of the model [Eqs. (1) and (2)].

Figure 4 compares the X and Y positions of the vortex tip obtained by numerical integration of the kinematic model [(1) and (2)] and the reaction-diffusion equations (4); a reasonable agreement is seen.

Let us finally mention that the mechanism of governing the vortex tip elongation described in this Rapid Communication indicates alternative experimental approaches to control vortices in excitable media [12].

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