PHYSICAL REVIEW E VOLUME 48, NUMBER 6 DECEMBER 1993

Positive and negative absorption by a plasma in an intense laser field

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(Received 27 August 1993)

Collisional absorption of a weak high-frequency linearly polarized electromagnetic wave by a plasma located in a strong laser field is studied. It is shown that the character of absorption essentially depends on the polarization of the weak laser wave relative to the intense one. Conditions for negative absorption are derived. Prospects for experimental demonstration of this effect are discussed.

PACS number(s): 52.40.–w, 52.25.Fi, 34.80.Kw

I. INTRODUCTION

With the advent of very intense short-pulse laser systems the interest to studies of short-pulse-laser–matter interactions and plasma properties in an intense laser field is renewed. There are now many papers in the literature related to this topic and it is reasonable to expect that their number will grow rapidly. In the experimental papers the part of the short-pulse-laser energy absorbed by the matter and the radiation spectra of the produced plasma are usually studied. Another way is to probe plasma properties in an intense laser field with the help of a weak test signal. This so-called pump-probe method allows a high temporal resolution and gives additional information about strongly nonequilibrium plasmas. It is well known that in the case of an equilibrium fully ionized plasma the weak test signal is absorbed. On the other hand, there are still some uncertainties with the fate of a weak signal in the case of a strongly nonequilibrium plasma, such as the plasma located in an intense laser field. The aim of the present paper is to remove these uncertainties.

The interest in plasma properties in an intense laser field arose many years ago. It was first shown [1] that in the strong laser field $\mathbf{E}(t) = \mathbf{E}\cos\omega t$ the dissipative part of the conductivity at the laser frequency ω and collisional absorption of the laser radiation are determined by the effective electron-ion collision frequency $\nu \sim v_E^{-3}$, where $v_E = eE/m\omega$ is the peak velocity of electon oscillations in the laser field. The corresponding nonlinear behavior of the inverse bremsstrahlung cross section $\sigma \sim v_E^{-4}$ was found in the studies of the elementary act of the electron-ion collision in presence of a strong laser field [2,3]. These results were confirmed later in classical [4–7] and quantum theories [8–10] of the inverse bremsstrahlung absorption.

Additional results in this field were obtained due to the studies of propagation of the weak electromagnetic wave $\mathbf{E}_1(t) = \mathbf{E}_1\cos\omega_1 t$ through a plasma in the strong linearly polarized laser field $\mathbf{E}(t)$ [11–13]. It was shown [11] that when $\mathbf{E}_1 \perp \mathbf{E}$, the conductivity at the frequency ω_1 and the effective electron-ion collision frequency describing the absorption of the weak wave vary as $\nu \sim v_E^{-1}v_T^{-2}$, where $v_T = (T_e/m)^{1/2}$ is the thermal velocity of electrons. In [12,13] the possibility for collisional amplification of a weak wave was discussed. Collisional amplification of a weak wave by a plasma in a strong laser field can be considered as the generalization of the well-known effect [14], where the amplification of the weak wave during electron-ion collision (when $\mathbf{v}_i \parallel \mathbf{E}_1$, $\mathbf{v}_i$ is the initial electron velocity) was manifested.

In the present paper we study positive and negative absorption of a weak high-frequency linearly polarized electromagnetic wave by a homogeneous fully ionized plasma located in a uniform strong laser field. We use our previous results [7] on nonstationary electron distribution functions in a strong laser field and assume the case of a transparent plasma for both waves ($\omega_p \ll \omega, \omega_1$), where ω_p is the plasma frequency. For $|\omega_1 - n\omega| > \nu$ the complete solution of the weak signal problem is given. The result [11] is confirmed. It is shown that when $\mathbf{E}_1 \parallel \mathbf{E}$ (linearly polarized fields) the weak signal is amplified. The effective collision frequency describing this amplification is negative $\nu \sim -v_E^{-3}\ln v_E/v_T$. When the strong laser field is circularly polarized in the plane $\mathbf{S}$, the effective collision frequencies describing absorption of the linearly polarized weak signal behave as $\nu \sim v_E^{-3}$ when $\mathbf{E}_1 \perp \mathbf{S}$ and $\nu \sim -v_E^{-3}$ when $\mathbf{E}_1 \parallel \mathbf{S}$.

II. DISSIPATIVE PROPERTIES OF HOMOGENEOUS PLASMA IN A LASER FIELD

The equation of evolution of the electron distribution function $f = f(\mathbf{v},t)$ in a homogeneous fully ionized plasma in the presence of two laser fields $\mathbf{E}(t)$ (strong field) and $\mathbf{E}_1(t)$ (weak field) is

$$\left(\frac{\partial}{\partial t} + \frac{e}{m}[\mathbf{E}(t) + \mathbf{E}_1(t)]\frac{\partial}{\partial \mathbf{v}}\right)f = S_{ei}(f) + S_{ee}(f,f), \tag{1}$$

where $S_{ei}(f)$ and $S_{ee}(f,f)$ are the electron-ion and electron-electron collision integrals. In the Fokker-Planck

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approximation they are given by

$$S_{ei}(f) = \frac{1}{2}\nu_{ei}(v)\frac{\partial}{\partial v_k}\left[(v^2\delta_{kj} - v_k v_j)\frac{\partial f}{\partial v_j}\right], \tag{2}$$

$$S_{ee}(f,f) = \frac{1}{2N}\frac{\partial}{\partial v_k}\int d\mathbf{v}'\,\nu_{ee}(\mid \mathbf{v}-\mathbf{v}'\mid) \times[(\mathbf{v}-\mathbf{v}')^2\delta_{kj} - (v_k - v_k')(v_j - v_j')]\left(\frac{\partial}{\partial v_j} - \frac{\partial}{\partial v_j'}\right)f(\mathbf{v},t)f(\mathbf{v}',t), \tag{3}$$

where $\nu_{ee}(v) = 4\pi e^4 N\Lambda m^{-2}v^{-3}$ and $\nu_{ei}(v) = Z\nu_{ee}(v)$ are the electron-electron and electron-ion collision frequencies, $Z \mid e \mid$ is the ion charge, N is the electron density, and Λ is the Coulomb logarithm, which is assumed to be constant. This accuracy is enough for our purposes. The strong laser field is described by the single-mode linearly ($\alpha = 0$) or circularly ($\alpha = 1$) polarized electric field of equal intensity

$$\mathbf{E}(t) = \frac{E}{\sqrt{1+\alpha}}(\mathbf{e}_x\cos\omega t - \mathbf{e}_y\alpha\sin\omega t). \tag{4}$$

The weak laser field is always linearly polarized.

It is useful to remove the rapid oscillations due to the strong laser field from Eqs. (1)–(3) by the transformation to the new variables $t = \tau$ and $\mathbf{u} = \mathbf{v} - v_E(\mathbf{e}_x\sin\omega t + \mathbf{e}_y\alpha\cos\omega t)/(1+\alpha)^{1/2}$ (where $v_E = eE/m\omega$). Then for the function $F(\mathbf{u},\tau) = f(\mathbf{V},\tau)$, where $\mathbf{V} = \mathbf{u} + v_E(\mathbf{e}_x\sin\omega\tau + \mathbf{e}_y\alpha\cos\omega\tau)/(1+\alpha)^{1/2}$, we get

$$\frac{\partial F}{\partial\tau} + \frac{e}{m}\mathbf{E}_1(\tau)\frac{\partial F}{\partial\mathbf{u}} = S_{ei}(F) + S_{ee}(F,F), \tag{5}$$

$$S_{ei}(F) = \frac{\partial}{\partial u_k}D_{kj}(\mathbf{V})\frac{\partial F}{\partial u_j}, \tag{6}$$

$$D_{kj}(\mathbf{V}) = \frac{1}{2}\nu_{ei}(V)(V^2\delta_{kj} - V_k V_j). \tag{7}$$

Here the electron-electron collision integral $S_{ee}(F,F)$ is given by the same expression (3), where f must be replaced by F and $\mathbf{v},\mathbf{v}'$ by $\mathbf{u},\mathbf{u}'$. This is due to the fact that the homogeneous electromagnetic field has no influence on the electron-electron collisions and vice versa.

To solve Eq. (5) we expand $F(\mathbf{u},\tau)$ in the small parameter $(eE_1/m\omega_1)/\overline{u} \ll 1$, where $\overline{u}$ is the mean electron velocity. In a linear approximation over the weak field strength $F \approx F_0 + F_1$ we get

$$\frac{\partial F_1}{\partial\tau} + \frac{e}{m}\mathbf{E}_1(\tau)\frac{\partial F_0}{\partial\mathbf{u}} = S_{ei}(F_1) + S_{ee}(F_1,F_0) + S_{ee}(F_0,F_1). \tag{8}$$

Here F_0 is the electron distribution function in the strong laser field [the solution of Eq. (5) with $\mathbf{E}_1 = \mathbf{0}$], which has been recently found for arbitrary laser intensities [7]. For the high-frequency weak laser field we have the additional small parameter ν/ω_1, where ν is the effective electron-ion collision frequency. Using the expansion of F_1 in this parameter, we get $F_1 \approx F_1^{(1)} + F_1^{(2)}$, where from (8)

$$F_1^{(1)} = -\frac{e}{m}\int_0^\tau d\tau'\,\mathbf{E}_1(\tau')\frac{\partial}{\partial\mathbf{u}}F_0(\mathbf{u},\tau'), \tag{9}$$

$$F_1^{(2)} = -\frac{e}{m}\int_0^\tau d\tau'\,S_{ei}\left(\int_0^{\tau'} d\tau''\,\mathbf{E}_1(\tau'')\frac{\partial}{\partial\mathbf{u}}F_0(\mathbf{u},\tau'')\right). \tag{10}$$

Since we are interested in the absorption of the weak field, in Eq. (10) the electron-electron collision term is omitted. The energy exchange between the weak laser field and plasma per unit volume and unit time is given by $Q(\tau) = \mathbf{E}_1(\tau)\mathbf{j}$, where $\mathbf{j} = e\int d\mathbf{v}\,\mathbf{v}f(\mathbf{v},t) = e\int d\mathbf{u}\,\mathbf{V}F(\mathbf{u},\tau)$ is the electric current density. Averaging this value over time $T \gg 2\pi/\min(\omega,\omega_1)$, we get the average energy density transferred per unit time from the weak wave to the plasma

$$Q = \frac{1}{T}\int_0^T d\tau\,\mathbf{E}_1(\tau)e\int d\mathbf{u}\,\mathbf{V}F(\mathbf{u},\tau). \tag{11}$$

Using (9)–(11), it is easy to show that the nonzero contribution in (11) comes from $F_1^{(2)}$. This term gives a correction to the electron distribution function in the strong laser field F_0 due to the weak laser field and electron-ion collisions. Thus we get

$$Q = -\frac{e^2}{m}\int_0^T\frac{d\tau}{T}\mathbf{E}_1(\tau)\int d\mathbf{u}\,\mathbf{u}\int_0^\tau d\tau'\,S_{ei}\left[\int_0^{\tau'} d\tau''\,\mathbf{E}_1(\tau'')\frac{\partial F_0(\mathbf{u},\tau'')}{\partial\mathbf{u}}\right]. \tag{12}$$

Since in the present paper we use the simplified expressions (2) and (3) for the collision integrals, which could be not appropriate in the case of resonances $\omega_1 = n\omega$, we assume later that $\mid \omega_1 - n\omega \mid > \nu$. Taking into account that $F_0(\mathbf{u},\tau)$ depends on τ only due to the relatively slow time dependence of the electron temperature ($T_e \sim \nu\tau$, see [7]), and $\mathbf{E}_1(\tau)$ rapidly oscillates in time, we can rewrite Eq. (12) as

$$Q = -\frac{e^2}{2m\omega_1^2}\int d\mathbf{u}\,E_{1k}\left[\int_0^{2\pi/\omega}\frac{\omega d\tau}{2\pi}D_{kj}(\mathbf{u},\tau)\right]\frac{\partial}{\partial u_j}\left(\mathbf{E}_1\frac{\partial F_0}{\partial\mathbf{u}}\right). \tag{13}$$

Here Eq. (6) for the electron-ion collision integral with $D_{kj}(\mathbf{u},\tau) \equiv D_{kj}(\mathbf{V})$ was used. In Secs. II A and II B we consider the rate (13) of the energy loss from the weak wave for the different polarizations of the strong laser field.

A. Absorption in the strong circularly polarized laser field

Since the homogeneous plasma in the strong laser field has axial symmetry, it is natural to write the electron-ion collision integral in cylindrical coordinates

$$
\begin{aligned}
S_{ei}(F) = {} & \frac{1}{u_\perp}\frac{\partial}{\partial u_\perp}\left[u_\perp\left(D_{\perp\perp}\frac{\partial F}{\partial u_\perp} + \frac{1}{u_\perp}D_{\perp\phi}\frac{\partial F}{\partial \phi} + D_{\perp z}\frac{\partial F}{\partial u_z}\right)\right] \\
& + \frac{1}{u_\perp}\frac{\partial}{\partial \phi}\left[D_{\phi\perp}\frac{\partial F}{\partial u_\perp} + \frac{1}{u_\perp}D_{\phi\phi}\frac{\partial F}{\partial \phi} + D_{\phi z}\frac{\partial F}{\partial u_z}\right] + \frac{\partial}{\partial u_z}\left[D_{z\perp}\frac{\partial F}{\partial u_\perp} + \frac{1}{u_\perp}D_{z\phi}\frac{\partial F}{\partial \phi} + D_{zz}\frac{\partial F}{\partial u_z}\right].
\end{aligned} \tag{14}
$$

In the circularly polarized laser field for the diffusion tensor D we choose the directions parallel (z) and perpendicular ($\perp$) to the direction of propagation of the laser beam

$$
\begin{pmatrix} D_{\perp\perp} \\ D_{\perp\phi} \\ D_{\perp z} \\ D_{\phi\phi} \\ D_{\phi z} \\ D_{zz} \end{pmatrix} = \frac{1}{2}\nu_{ei}(V)\begin{pmatrix} u_z^2 + \frac{1}{2}v_E^2\cos^2\psi \\ -\frac{1}{\sqrt{2}}v_E\cos\psi(u_\perp + \frac{1}{\sqrt{2}}v_E\sin\psi) \\ -u_z(u_\perp + \frac{1}{\sqrt{2}}v_E\sin\psi) \\ u_z^2 + (u_\perp + \frac{1}{\sqrt{2}}v_E\sin\psi)^2 \\ -\frac{1}{\sqrt{2}}v_E u_z\cos\psi \\ u_\perp^2 + \frac{1}{2}v_E^2 + \sqrt{2}u_\perp v_E\sin\psi \end{pmatrix}. \tag{15}
$$

Here $\psi = \omega\tau + \phi$, where ϕ is azimuthal angle of vector $\mathbf{u}$, $D_{\alpha\beta} = D_{\beta\alpha}$ for $\alpha,\beta = \perp, z, \phi$, and $V = (u^2 + \sqrt{2}v_E u_\perp\sin\psi + v_E^2/2)^{1/2}$.

Introducing the standard definition of the effective electron-ion collision frequency ν from the energy balance condition $Q = \nu N m v_{E_1}^2/2$ (where $v_{E_1} = eE_1/m\omega_1$) we can rewrite Eq. (13) as

$$
Q = \frac{1}{8\pi}\frac{\omega_p^2}{\omega_1^2}(\nu_z E_{1z}^2 + \nu_\perp E_{1\perp}^2). \tag{16}
$$

Here ν_z and $\nu_\perp$ are the longitudinal and transverse effective collision frequencies, which are given by

$$
\nu_z = -\frac{2\pi}{N}\int_0^\infty du_\perp\, u_\perp \int_{-\infty}^\infty du_z \left(\overline{D}_{z\perp}\frac{\partial}{\partial u_\perp} + \overline{D}_{zz}\frac{\partial}{\partial u_z}\right)\frac{\partial F_0}{\partial u_z}, \tag{17}
$$

$$
\nu_\perp = -\frac{\pi}{N}\int_0^\infty du_\perp\, u_\perp \int_{-\infty}^\infty du_z \left(\overline{D}_{\perp\perp}\frac{\partial}{\partial u_\perp} + \frac{1}{u_\perp}\overline{D}_{\phi\phi} + \overline{D}_{\perp z}\frac{\partial}{\partial u_z}\right)\frac{\partial F_0}{\partial u_\perp}, \tag{18}
$$

and $\overline{D}_{\alpha\beta} = \int_0^{2\pi} d\psi\, D_{\alpha\beta}(u_\perp, u_z, \psi)/2\pi$.

The absorption coefficient of the weak wave is

$$
\alpha = \frac{8\pi Q}{cE_1^2} = \frac{\omega_p^2}{c\omega_1^2}\frac{(\nu_z E_{1z}^2 + \nu_\perp E_{1\perp}^2)}{E_1^2}. \tag{19}
$$

In the limit when the basic laser field, which is taken into account in the tensor $\overline{D}$ and in the electron distribution function $F_0(u,\tau)$, is weak ($v_E \ll v_T$), Eqs. (17) and (18) give the well-known result [15,16]

$$
\nu_z = \nu_\perp = \frac{\sqrt{2}\,U}{3\sqrt{\pi}}\nu_{ei}(v_T), \tag{20}
$$

where $U = (2\pi)^{3/2}v_T^3 N^{-1}F_0(u=0,\tau)$ and v_T is the thermal velocity of electrons.

When $F_0(u,\tau)$ is the Maxwellian electron distribution function then the cases of a strong and weak basic laser field can be described by the same formula. With the help of Eqs. (14)–(18) we can write

$$
\begin{aligned}
\nu_z &= \sqrt{8}\nu_{ei}(v_E)[\mathrm{erf}(\gamma/2) - \gamma\exp(-\gamma^2/4)/\sqrt{\pi}], \\
\nu_\perp &= \sqrt{2}\nu_{ei}(v_E)[-\mathrm{erf}(\gamma/2) \\
&\quad + \gamma(1+\gamma^2/2)\exp(-\gamma^2/4)/\sqrt{\pi}],
\end{aligned} \tag{21}
$$

where $\gamma = v_E/v_T$ and $\mathrm{erf}(x) = (2/\sqrt{\pi})\int_0^x dt\, \exp(-t^2)$ is the error function. The corresponding absorption coefficient is given by Eq. (19).

In the limit of the weak basic laser field $\gamma \ll 2$ Eqs. (21) give the same result as (20) with $U = 1$. In the strong laser field $\gamma \gg 2$

$$
\begin{aligned}
\nu_z &\simeq \sqrt{8}\nu_{ei}(v_E)[1 - \gamma\exp(-\gamma^2/4)/\sqrt{\pi}], \\
\nu_\perp &\simeq -\sqrt{2}\nu_{ei}(v_E)[1 - \gamma^3\exp(-\gamma^2/4)/(2\sqrt{\pi})].
\end{aligned} \tag{22}
$$

Thus, when the weak radiation wave is polarized along the direction of propagation of the strong laser radiation (z axis), the wave is absorbed by the plasma. When the polarization of the weak wave is perpendicular to the direction of propagation of the strong laser radiation, the wave is amplified. In Fig. 1 the dependence of the effective collision frequencies (21) on the strength of the basic field is illustrated. As can be seen, the negative absorption arises when $(v_E/v_T) \geq 3$. From this condition, taking into account the time dependence [7] of the electron temperature in the strong laser field $T(\tau) \sim 4(3\sqrt{2})^{-1}\tau m v_E^2 \nu_{ei}(v_E)$, we can find the time duration of the negative absorption $\tau_n = \sqrt{2}[12\nu_{ei}(v_E)]^{-1} \sim 0.1\nu_{ei}^{-1}(v_E)$.

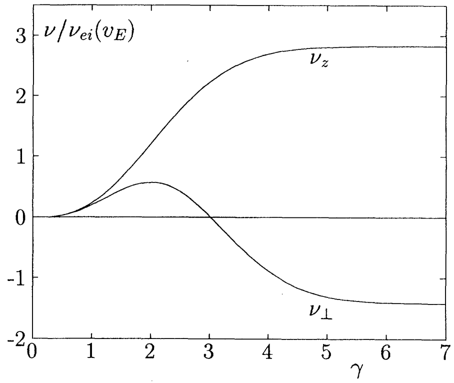

FIG. 1. Dependence of the effective electron-ion collision frequencies (21) on the parameter $\gamma = v_E/v_T$ for the circularly polarized basic laser field.

The absorption coefficient of the weak wave is

$$\alpha = \frac{\sqrt{2}\nu_{ei}(v_E)}{c}\left(\frac{\omega_p}{\omega}\right)^2\left(\frac{\omega}{\omega_1}\right)^2(3\cos^2\theta - 1), \tag{23}$$

where $\cos\theta = E_{1z}/E_1$ and z is directed along the direction of propagation of the strong laser field. The weak-wave amplification exists for $125^\circ \geq \theta \geq 55^\circ$ for arbitrary ω_1. Note that the classical approach used in this paper is valid only when the condition $\hbar\omega_1 \ll mv_E^2/2$ is fulfilled.

To estimate the value of the gain coefficient ($\theta = 90^\circ$) we need to make some assumptions. First, to overcome the problems connected with the refraction of the strong laser radiation and with the excitation of electromagnetic instabilities, we choose $(\omega_p/\omega)^2 = 0.1$. Second, to be observed, the amplification should exist for a sufficient period of time. We assume for this time $\tau_n = 100$ fs. This corresponds to $\nu_{ei}(v_E) = (\omega_p/\omega)^2(e^2\omega^2/mv_E^3)\Lambda Z = 10^{12}$ s^{-1}. This value of the electron-ion collision frequency in the case of a Nd glass laser ($\lambda = 1.06\mu$m) corresponds to the laser intensity $\sim 4 \times 10^{15}$ W/cm^2 and in the case of a KrF laser ($\lambda = 268$ nm) to the laser intensity $\sim 3 \times 10^{17}$ W/cm^2. For these estimates we used $\Lambda = \ln\{[v_T/\max(\omega,\omega_1)]/[2Ze^2/mv_E^2]\}$ and assumed $Z = 5$.

With $\nu_{ei}(v_E) = 10^{12}$ s^{-1} the expression for the gain coefficient reduces to $|\alpha| = 4.7 \times (\omega/\omega_1)^2$ cm^{-1}. When $\omega_1 \gg \omega$ the gain coefficient is negligible, but when $\omega_1 \sim \omega$ the amplification can be observed.

Note that the duration of amplification will be longer if the strong-laser-pulse intensity grows in time as $I \sim \tau^{2/3}$. In this case the ratio $\gamma = v_E(\tau)/v_T(\tau)$ remains constant and the gain coefficient is given by $|\alpha(\tau)| = (\omega_p/\omega)^2(\omega/\omega_1)^2/(c\gamma^2\tau)$. This expression is valid for $\tau > \gamma\nu_{ei}^{-1}[v_T(0)]$, where $v_T(0)$ is the initial electron thermal velocity. In the time interval from 10 fs to 1 ps for $\gamma = 4$, $(\omega_p/\omega)^2 = 0.1$ and $\omega \sim \omega_1$, as an example, the gain coefficient will vary from 20 to 0.2 cm^{-1}.

To calculate the effective electron-ion collision frequencies (17) and (18) when the electron distribution function is non-Maxwellian, we can use the expansion of the tensor $\overline{D}$ in $\overline{u}/v_E \ll 1$. The results of such expansion up to the second order in the small parameter are given in Appendix A. Using these expressions for the case when $F_0(u,\tau)$ is the anisotropic Maxwellian electron distribution (see [7])

$$F_0 = \frac{Nm^{3/2}}{(2\pi)^{3/2}T_\perp\sqrt{T_z}}\exp\left(-\frac{mu_\perp^2}{2T_\perp} - \frac{mu_z^2}{2T_z}\right), \tag{24}$$

where $T_\perp$ and T_z are the transverse and longitudinal electron temperatures in the strong circular polarized laser field, we get

$$\nu_z = -2\nu_\perp = 2^{3/2}\nu_{ei}(v_E)\left\{1 - \frac{9}{mv_E^2}(T_z - T_\perp) + \frac{1}{2}\left[\frac{15}{mv_E^2}(T_z - T_\perp)\right]^2\right\}. \tag{25}$$

Thus, independent of the electron distribution, the effective collision frequencies describing the weak-field absorption are determined by the electron-ion collision frequency in the strong laser field $\nu_{ei}(v_E)$.

B. Absorption in the strong linearly polarized laser field

Using Eq. (14) for the electron-ion collision integral in cylindrical coordinates with directions (z) parallel and ($\perp$) perpendicular to the strong laser field, we can write for the nonzero components of the diffusion tensor D

$$\begin{pmatrix} D_{\perp\perp} \\ D_{\perp z} \\ D_{zz} \\ D_{\phi\phi} \end{pmatrix} = \frac{1}{2}\nu_{ei}(V)\begin{pmatrix} V_z^2 \\ -u_\perp V_z \\ u_\perp^2 \\ u_\perp^2 + V_z^2 \end{pmatrix}, \tag{26}$$

$V_z = u_z + v_E\sin\omega\tau$, and $V^2 = u_\perp^2 + V_z^2$.

In the case when $F_0(u,\tau)$ is the Maxwellian electron distribution function, we have from (17) and (18)

$$\nu_\perp = \nu_{ei}(v_E)(I_1 + I_2),$$
$$\nu_z = -2\nu_{ei}(v_E)(I_2 + I_3). \tag{27}$$

Expressions for the integrals I_k ($k = 1,2,3$) together with the discussion of integration procedure are given in Appendix B. After integration, for $v_E \gg v_T$, we get

$$\nu_\perp = \frac{v_E^2}{\pi v_T^2}\nu_{ei}(v_E),$$

$$\nu_z = -\frac{\nu_{ei}(v_E)}{\pi}(2\ln v_E/v_T + 3\ln 2 + C - 4), \tag{28}$$

where $C = 0.577$ is Euler's constant. The expression for $\nu_\perp$ coincides with the expression, which has been derived earlier [11].

The corresponding absorption coefficient is

$$\alpha = \frac{\omega_p^2}{\pi c \omega_1^2} \nu_{ei}(v_E)\{(1-\cos^2\theta)v_E^2/v_T^2 - \cos^2\theta[2\ln(v_E/v_T) + 3\ln 2 + C - 4]\}, \quad (29)$$

where $\cos\theta = E_{1z}/E_1$ with z directed along the strong laser field $\mathbf{E}$. According to this expression the amplification of the weak wave in the linearly polarized strong laser field is possible in the narrow region of angles $\theta \leq v_T/v_E$.

III. CONCLUSION

We studied collisional absorption and amplification of a high-frequency weak electromagnetic wave by a nonequilibrium plasma located in a strong laser field. The effective electron-ion collision frequencies determining the absorption or amplification of the weak wave are derived. It is shown that the collisional amplification of the weak wave by a plasma located in a strong circularly polarized laser field can be observed.

IV. ACKNOWLEDGMENTS

We would like to express our gratitude to Professor P. Mulser for discussions, support, and hospitality. This work was perforemed under the auspices of the Bundesministerium für Forschung und Technologie under Contract No. 06DA331.

APPENDIX A: EXPRESSIONS FOR THE TENSOR COMPONENTS $\overline{D}_{\alpha\beta}$

The tensor components for the circular polarized strong laser field used in the calculations of the electron-ion collision frequencies (17) and (18) are

$$\begin{aligned}
\overline{D}_{\perp\perp} &= \frac{1}{\sqrt{8}}\nu_{ei}(v_E)\left[v_E^2 + u_z^2 + \frac{3}{4}u_\perp^2 + \frac{3}{16v_E^2}(5u_\perp^4 + 36u_\perp^2u_z^2 - 24u_z^4) + \frac{25}{128v_E^4}(64u_z^6 - 432u_z^4u_\perp^2 + 120u_z^2u_\perp^4 + 7u_\perp^6)\right],\\
\overline{D}_{\perp z} &= \frac{1}{\sqrt{8}}\nu_{ei}(v_E)u_z u_\perp\left[2 - \frac{3}{2v_E^2}(12u_z^2 - 3u_\perp^2) + \frac{75}{8v_E^4}(8u_z^4 - 12u_z^2u_\perp^2 + u_\perp^4\right],\\
\overline{D}_{\phi\phi} &= \frac{1}{\sqrt{8}}\nu_{ei}(v_E)\left[v_E^2 + u_z^2 + \frac{1}{4}u_\perp^2 + \frac{3}{16v_E^2}(u_\perp^4 + 12u_\perp^2u_z^2 - 24u_z^4) + \frac{25}{128v_E^4}(64u_z^6 - 144u_z^4u_\perp^2 + 24u_z^2u_\perp^4 + u_\perp^6)\right],\\
\overline{D}_{zz} &= \frac{1}{\sqrt{8}}\nu_{ei}(v_E)\left[2v_E^2 - 6u_z^2 + u_\perp^2 + \frac{3}{8v_E^2}(40u_z^4 - 72u_\perp^2u_z^2 + 3u_\perp^4) + \frac{5}{16v_E^4}(-112u_z^6 + 600u_z^4u_\perp^2 - 270u_z^2u_\perp^4 + 5u_\perp^6)\right].
\end{aligned} \quad \text{(A1)}$$

APPENDIX B: EXPRESSIONS FOR THE INTEGRALS I_K

In the strong linearly polarized laser field the effective electron-ion collision frequencies determining the absorption of the weak laser wave depend on the integrals I_k $(k = 1,2,3)$

$$I_k = \frac{\gamma^3}{2\sqrt{2\pi}v_T^2}\int_{-\infty}^{\infty}du_z\int_0^{\infty}du_\perp\int_0^{2\pi}\frac{d\psi}{2\pi}\Phi_k(u_\perp,u_z,\psi) \times \exp[-(u_\perp^2 + u_z^2 - 2u_zv_E\sin\psi + v_E^2\sin^2\psi)/2v_T^2], \quad \text{(B1)}$$

where

$$\Phi_1 = u_\perp/\sqrt{u_\perp^2 + u_z^2},$$

$$\Phi_2 = -\frac{u_\perp^3(v_T^2 + u_zv_E\sin\psi)}{2v_T^2(u_\perp^2 + u_z^2)^{3/2}},$$

$$\Phi_3 = \frac{\gamma^2u_\perp^3\sin^2\psi}{2(u_\perp^2 + u_z^2)^{3/2}}. \quad \text{(B2)}$$

Introducing spherical variables $u = (u_\perp^2 + u_z^2)^{1/2}$, $\cos\theta = u_z/u$ and integrating first over θ, we get the double integrals, which can then be easily reduced to

$$\begin{aligned}
I_1 &= \frac{2\gamma^2}{\pi^{3/2}}\int_0^{\gamma/\sqrt{2}}dt\,\exp(-t^2)L(t,\gamma),\\
I_2 &= -\frac{2}{\pi^{3/2}}\int_0^{\gamma/\sqrt{2}}dt\exp(-t^2)\{2t^2\gamma^2L(t,\gamma) + (1-2t^2)[\gamma\sqrt{\gamma^2-2t^2}/2 + t^2L(t,\gamma)]\},\\
I_3 &= \frac{4\gamma^2}{\pi^{3/2}}\int_0^{\gamma/\sqrt{2}}dt\,t^2\exp(-t^2)L(t,\gamma).
\end{aligned} \quad \text{(B3)}$$

Here $L(t,\gamma) = \ln[(\gamma + \sqrt{\gamma^2 - 2t^2})/\sqrt{2}t]$. Then in the limit $\gamma = v_E/v_T \gg 1$ we get Eqs. (28).

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