# Collective effects due to charge-fluctuation dynamics in a dusty plasma

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Highly charged dust grains immersed in a plasma can exhibit charge fluctuations in response to oscillations in the plasma currents flowing into them. This introduces a new physical effect, namely, the electric charge on the dust particles becomes time dependent and a self-consistent dynamical variable. The consequent modifications in the collective properties of a dusty plasma are investigated. It is shown that these effects lead to dissipative and instability mechanisms for ion waves in the plasma and can lead to interesting applications to many laboratory and astrophysical situations.

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Dusty plasmas are characterized by the presence of large-sized dust grains (in the range of 10 nm to 100 $\mu$m) immersed in a partially or fully ionized plasma. These dust grains are usually highly charged ($Q_d \sim 10^3 e - 10^4 e$) due to a variety of processes including plasma currents, photoelectric effects, secondary emission, etc. Dusty plasmas are the subject of much current interest [1–7] due to their occurrence in various space environments (e.g., the Earth's ionosphere, asteroid zones, planetary rings, cometary tails, interstellar clouds, etc.) as well as laboratory devices and industrial processes (e.g., plasmas in plasma processing, plasma etching, plasma furnace systems, edge plasmas in some magnetohydrodynamics power generators, rocket exhausts, fusion devices, etc.). The presence of these highly charged and massive particles can significantly influence the collective properties of the plasma in which they are suspended, and a number of recent studies have investigated this important question [8–10]. The primary emphasis in these studies has been on the delineation of modifications in collective properties arising from the inclusion of the dynamics of dust particles. The charge of the dust particles has been taken to be constant, and hence the analysis becomes analogous to earlier work on multispecies plasmas, particularly negative-ion plasmas, where the role of the negative-ion species is now played by the dust. In this paper we propose a different physical effect that can arise in a dusty plasma and investigate its consequences on wave propagation and instability phenomena. We suggest that dust particles immersed in a plasma with collective perturbations can exhibit self-consistent charge fluctuations in response to oscillations in the plasma currents flowing into them. The dust electric charge thereby becomes a time-dependent quantity and must be treated as a dynamical variable which is coupled self-consistently to other dynamical variables such as density, potential, current, etc. We show that this coupling leads to dissipative phenomena which can damp the usual ion waves and drive instabilities in certain negative-energy systems in which the ion species are streaming with respect to the dust. We also demonstrate how an anisotropic spectrum of excited ion waves can exert a mean quasilinear force on the dust particles thereby providing a mechanism for transport of dust—a problem of great interest in planetary atmospheres [3] and plasma-processing applications [7].

The basic charging equation for the dust particles immersed in a plasma is

$$\frac{dQ_d}{dt} = I_e + I_i \ , \tag{1}$$

where $Q_d$ is to be interpreted as the mean charge of the dust particles and $I_e, I_i$ are electron and ion currents collected by the particles. Since the dust particles have much smaller thermal velocities than electrons and ions, the electron and ion currents to the dust grain are given by [1]

$$I_e = -\pi a^2 e \left(\frac{8kT_e}{\pi m_e}\right)^{1/2} n_e \exp\left[\frac{e}{kT_e}(\phi_f - V)\right] \ , \tag{2}$$

$$I_i = \pi a^2 e \left(\frac{8kT_i}{\pi m_i}\right)^{1/2} n_i \left[1 - \frac{e}{kT_i}(\phi_f - V)\right] \ , \tag{3}$$

where $a$ is the grain radius, $T_{e,i}$ are the electron and ion temperatures, $n_{e,i}$ are to be interpreted as the local electron and ion densities, and $(\phi_f - V)$ is the potential difference between the dust grain and the bulk plasma potential [1]. Equations (2) and (3) are obtained by averaging the effective collision cross section $\sigma_j = \pi a^2[1 - 2Z_j e(\phi_f - V)/m_j w^2]$ for charged particles impacting the dust grains ($Z_j = -1, 1$ for electrons and ions, respectively) over the appropriate Maxwellian distribution of velocities ($w$). If the ions have a significant streaming velocity $v_0$, with respect to the dust, the current expressions are much more complicated [11]. However, in the limit, $v_0 \gg v_{\text{th}i}$ (where $v_{\text{th}i}$ is the thermal velocity of ions), we can write the approximate expression,

$$I_i \approx \pi a^2 e v_0 n_i \left[1 - \frac{2e}{m_i v_0^2}(\phi_f - V)\right] , \tag{4}$$

which may be readily obtained by integrating over $\delta$-function distributions. At equilibrium, the plasma currents add up to zero and yield the steady-state floating potential $\phi_{f0}$ and the equilibrium dust charge $Q_{0d} = C\phi_{f0}$, where $C$ is the grain capacitance. Note that in equilibrium, the total system of dust and plasma particles maintain an overall charge neutrality ($\sum_{j=e,i} e_j n_{0j} - Q_d^0 n_{0d} = 0$). Since the dust particles usually acquire a large negative equilibrium charge ($Q_d^0 \gg 1$), the plasma is highly non-neutral, with $n_{i0}/(n_{i0} - n_{e0}) \sim 1$.

It is clear that fluctuations in plasma currents associated with collective plasma perturbations can induce charge fluctuations on the dust grains, given by

$$\frac{d\tilde{Q}_d}{dt} = \tilde{I}_e + \tilde{I}_i \ , \tag{5}$$

where expressions for $\tilde{I}_e$,$\tilde{I}_i$ are to be obtained by averaging over the perturbed distribution functions $f_0 + \tilde{f}$. We consider the dust particles to have a size which is much smaller than typical wavelengths of the self-consistent perturbations as well as the typical Debye lengths. This is the appropriate regime for the collective participation of dust particles [4,8–10]. We may thus assume that the mean charge fluctuation $\tilde{Q}_d$ faithfully follows the spatiotemporal variations of the plasma perturbations $\tilde{f}$. We can now write model expressions for the current fluctuations as

$$\tilde{I}_e = I_{e0}\left(\frac{\tilde{n}_e}{n_{e0}} + \frac{e\tilde{\phi}_f}{kT_e}\right) \tag{6}$$

and

$$\tilde{I}_i = I_{i0}\left(\frac{\tilde{n}_i}{n_{i0}} - \frac{e}{w_0}\tilde{\phi}_f\right) , \tag{7}$$

where $w_0 = T_i - e\phi_{f0}$ or $m_i v_0^2/2 - e\phi_{f0}$ depending on whether (3) or (4) describes the equilibrium ions adequately and $I_{i0} = -I_{e0}$ because of equilibrium constraints. Noting that $\tilde{\phi}_f = \tilde{Q}_d/C$, we may now combine Eqs. (5)–(7) to give the charge dynamics equation as

$$\frac{d\tilde{Q}_d}{dt} + \eta\tilde{Q}_d = \mid I_{e0} \mid \left(\frac{\tilde{n}_i}{n_{i0}} - \frac{\tilde{n}_e}{n_{e0}}\right) , \tag{8}$$

where $\eta = e(\mid I_{e0} \mid /C)(1/T_e + 1/w_0)$. Equation (8) shows that charge fluctuations on dust particles are driven by the difference in relative density fluctuations of ions and electrons and have a natural decay rate of order $\eta$. Physically, the charge fluctuations decay because any deviation of the grain potential from the equilibrium floating potential is opposed by electron and/or ion currents into the grain. The typical time scale of decay $\eta^{-1} \approx 10^2 \omega_p^{-1}(n_{i0}/n_{e0})(\lambda_D/a)$ can be fairly rapid (where $\omega_p$ and $\lambda_D$ are the plasma frequency and Debye length, respectively).

Equation (8) is the additional dynamical equation that is coupled to the plasma equations through the plasma density fluctuations. The plasma dynamical equations also get modified with new particle loss terms due to electron (ion) attachment to dust particles and Maxwell's equations have additional source terms arising from $\tilde{Q}$. To study the consequences of this new dynamical variable on the plasma collective properties we have examined the linear propagation of low-frequency electrostatic modes in a dusty plasma. The self-consistent fluctuations are then described by the Poisson's equation

$$\nabla^2\tilde{\phi} = 4\pi \sum_\alpha \tilde{n}_\alpha e_\alpha + 4\pi n_{d0}\tilde{Q}_d \ , \tag{9}$$

where $\alpha = e, i, d$ refer to electrons, ions, and charged dust grains and we have explicitly written the extra term arising due to dust charge fluctuations. Assuming a $\exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t)$ dependence and using standard linear perturbation theory it is easy to obtain the dispersion relation

$$1+\chi_e\left(1 + \frac{i\beta}{\omega + i\eta}\right)+\chi_i\left(1 + \frac{i\beta}{\omega + i\eta}\frac{n_{e0}}{n_{i0}}\right)+\chi_d = 0 \ , \tag{10}$$

where $\chi_\alpha$ are the particle susceptibility functions, $\beta = (\mid I_{e0} \mid /e)(n_{0d}/n_{e0})$ and the $i\beta$ terms are the new terms arising through coupling to dust charge fluctuations which result as a response to collective plasma perturbations. Note that the coupling parameter $\beta \approx 10^{-1}\pi a^2 n_{d0} v_{\text{the}}$ (where $v_{\text{the}}$ is the thermal velocity of electrons) is like an effective collision frequency of electrons with the dust grains; the numerical factor arises because of the difference between the equilibrium floating potential of the grain and the bulk plasma potential.

In order to examine effects arising due to dust charge fluctuations, we investigate the dispersion relation (10) in the simplest possible limit, namely that of low-frequency waves in an unmagnetized plasma. We first consider almost static perturbations $\omega \ll kv_d, kv_i, kv_e$. In this case $\chi_\alpha = 1/(k^2\lambda_{D\alpha}^2)$, where $\lambda_{D\alpha}$ is the Debye length of the species $\alpha$, for all the species and we get the dispersion relation

$$\omega = -i(\eta + \beta_1) \ , \tag{11}$$

where $\beta_1 = \beta[(\lambda_{Dd}^2/\lambda_{De}^2) + (\lambda_{Dd}^2/\lambda_{Di}^2)(n_{e0}/n_{i0})]/[1 + (\lambda_{Dd}^2/\lambda_{De}^2) + (\lambda_{Dd}^2/\lambda_{Di}^2)] \approx \beta(n_{e0}/n_{i0})$ (for $T_e = T_i = T$). Equation (11) shows that a low-frequency damped mode exists in this limit. This is a purely dust-related mode and may be understood as follows. When the mean grain charge is disturbed by a very low-frequency fluctuation, it disturbs the floating potential of the grain self-consistently which causes a damping at a decay rate $\eta$ discussed above. It is interesting to ask what happens to the energy lost by the decay of the low-frequency perturbations. It is clear that the energy is given to the plasma particles by the $\mathbf{j}\cdot\mathbf{E}$ power to the current carriers which drain the charge of the dust grains. In a weakly collisional plasma, this may lead to non-Maxwellian components in the electron or ion velocity distributions.

Let us next consider the regime of dust acoustic waves [9] $kv_d \ll \omega \ll kv_i, kv_e$. In this limit $\chi_{e,i} = 1/(k^2\lambda_{De,i}^2)$ and $\chi_d \approx -\omega_{pd}^2/\omega^2$ and we get the dispersion relation

$$(\omega^2 - k^2C_D^2)(\omega + i\eta + i\beta_2) = -i\beta_2 k^2 C_D^2 \ , \tag{12}$$

where $C_D^2 = [(T_e/m_d)(n_d/n_{e0})]/[1 + (\lambda_{De}^2/\lambda_{Di}^2)]$ and $\beta_2 = \beta[1 + (n_{e0}/n_{i0})(\lambda_{De}^2/\lambda_{Di}^2)]/[1 + (\lambda_{De}^2/\lambda_{Di}^2)] \approx 2\beta(n_{e0}/n_{i0})$. The dispersion relation (12), which is a cubic equation in $\omega$, has been written in a form which makes it easy to identify the physical origin of the three roots. The first bracket on the left-hand side describes the two dust acoustic waves, the second bracket indicates the damped low-frequency dust fluctuation mode (discussed above), and the right-hand side can be interpreted as a coupling term which is proportional to the dust charge fluctuation. The dust acoustic wave is the very low-frequency analog of an ion acoustic wave, in which the dust mass provides the inertia and the ions and electrons constitute the light fluid with finite pressure. A perturbative solution of Eq. (12) (treating $\eta, \beta_2$ as small), yields the three roots

$$\omega = \mp k C_D - i\frac{\beta_2}{2} \ ,$$
$$\omega = -i\eta\left(1 + \eta\frac{\beta_2}{k^2 C_D^2}\right) \ . \tag{13}$$

It is seen that the charge fluctuation terms lead to an unusual dissipation of the dust acoustic modes. Physically, the dust acoustic modes consume some energy in sustaining the self-consistent dust charge fluctuation, which is constantly draining away due to the equilibriation effects discussed after Eqn. (11).

We next consider the case of usual ion acoustic waves $kv_d, kv_i \ll \omega \ll kv_e$. In order to include the situation where possible instabilities may arise, we allow for a streaming of the ion fluid with respect to the dust with a speed $v_0$. We may now write the dispersion relation as

$$\left(\frac{\bar{\omega}^2}{k^2\lambda_{De}^2} - \omega_{pi}^2\right)(\omega + i\eta + i\beta_3) - \omega_{pd}^2\left(\frac{\bar{\omega}^2}{\omega^2}\right)(\omega + i\eta) = -i\omega_{pi}^2\left(1 - \frac{n_{e0}}{n_{i0}}\right) \ , \tag{14}$$

where $\bar{\omega} = \omega - \mathbf{k}\cdot\mathbf{v_0}$. We first consider the limit $v_0 = 0$, for which the dispersion relation can be written as

$$(\omega^2 - k^2 C_{ds}^2)(\omega + i\eta) = -i\beta_3 k^2 C_{ds}^2 \ , \tag{15}$$

where $C_{ds}^2 = \lambda_{De}^2\omega_{pi}^2 + \lambda_{De}^2\omega_{pd}^2$ is the modified ion-acoustic speed in the presence of dust particles and $\beta_3 = \beta[\omega_{pi}^2(1 - n_{e0}/n_{i0}) + \omega_{pd}^2]/(\omega_{pi}^2 + \omega_{pd}^2)$. This dispersion relation may be investigated as before and yields three roots corresponding to damped ion-acoustic modes and the purely damped dust mode. It is interesting that the coupling coefficient $\beta_3 \approx \beta$ in contrast to $\beta_1, \beta_2 \approx \beta(n_{e0}/n_{i0})$. The basic reason for this difference is that the contribution from $\chi_e$ is negligible for the two very-low-frequency modes and hence $\beta$ only enters in the combination $\beta(n_{e0}/n_{i0})$ [Eq. (12)]. However, for ion acoustic waves the basic balance is between $|\chi_e|$ and $|\chi_i|$ so that $\chi_e$ effects are important and coupling effects of order $\beta$ survive.

Finally, we consider the case when $v_0 \neq 0$. For simplicity, we ignore the effects due to dust inertia ($\omega_{pd} \to 0$). The dispersion relation now takes the form

$$(\bar{\omega}^2 - k^2 C_{ds}^2)(\omega + i\eta) = -i\beta k^2 C_{ds}^2 \ . \tag{16}$$

A perturbative analysis shows that the above equation has an unstable root for $v_0 > C_{ds}$, given approximately by

$$\omega = kv_0 - kC_{ds} + i\frac{\beta}{2}\frac{C_{ds}}{v_0 - C_{ds}} \ . \tag{17}$$

This streaming instability has its origin purely in the unusual dissipation introduced in the plasma by the fluctuations in the dust charge. The critical streaming velocity $v_0 > C_{ds}$ is such that the ion wave may be viewed as a negative-energy wave from the ion frame so that dissipative effects associated with nonstreaming dust charge fluctuations lead to amplification. Note the contrast with a dust-free plasma where a streaming in the ions merely shifts the real part of the wave frequency in the laboratory frame. It is of interest to compare the growth term resulting from Eq. (17) with additional ion dissipation arising through ion-dust attachment and momentum loss effects (which enter as a consequence of particle and momentum conservation in the dynamical equations for the ions). The critical condition is $\mathrm{Im}(\omega) \sim \beta/2 > \nu_{id}$, where $\nu_{id} \approx n_{0d}(\pi a^2)v_i$ is an estimate of the effective ion-dust attachment rate. This condition may be readily satisfied. We may also quantitatively assess the importance of this instability by comparing the growth rate with the real part of the frequency. One may show that $\mathrm{Im}(\omega)/\mathrm{Re}(\omega) \sim \beta/(kC_{ds}) \sim 10^{-1}(\lambda a^2 n_{0d})\sqrt{(n_{0e}/n_{0i})(m_i/m_e)}$, a quantity which can readily be of order unity, thus leading to rapid growth of the ion waves. The possibility that ion wave instabilities may be responsible for structures observed in planetary magnetospheres has been considered by several authors [3]. The mechanism discussed here gives a fairly robust fluid instability which may have a relevance to such situations (magnetic-field effects can be shown to be unimportant for $\omega \gg \omega_{ci}$). It is also of interest to extend the above treatment to consider collective effects in a magnetized and inhomogeneous plasma. Such an investigation is in progress and will be reported elsewhere.

A topic of considerable interest, both from the point of view of planetary atmospheres [3] and the dust clouds observed in laboratory plasmas [7] is the possible anomalous transport of the dust grains due to a turbulent spectrum of ion waves. As the dust grain charge fluctuates in response to the ion waves, one obtains a quasilinear mean force on the dust grains whose magnitude is given by

$$m_d\frac{d\mathbf{v}_d}{dt} = -\frac{|I_{e0}|}{4\pi e n_{e0}}\sum\frac{\mathbf{k}\,|\tilde{E}_k|^2}{(\omega + i\eta)}\left(\chi_e + \frac{n_{e0}}{n_{i0}}\chi_i\right) . \tag{18}$$

It is clear that an isotropic spectrum of ion waves produces no mean force on the dust particles. On the other hand, an anisotropic spectrum such as the one which may be generated by streaming ions, can readily produce a mean dc acceleration of dust particles. Balancing the turbulent force against the dust ion drag $m_d\nu_{di}\mathbf{v}_d$, one gets typical mean velocities of order $v_d \sim F/(m_d\nu_{di}) \sim T n_{i0}\pi a^2 \,|\, e\tilde{\phi}/T\,|^2 /(m_d\nu_{di}) \sim v_{\mathrm{th}i} \,|\, e\tilde{\phi}/T\,|^2$. For ion acoustic fluctuations of even a few percent, this gives a very effective method of sweeping dust grains using collective effects. This mechanism may also prove useful in explaining the transport of dust grains in planetary environments.

We now briefly discuss some of the major assumptions that have gone into our model calculations and their impact on our results. The basic charging model for the dust grains that we have used is the so-called standard model [6] or the point model in which particle size and shape effects have been neglected. Such a model is reasonable in the limit $a \ll \lambda_D \ll \lambda$ where $a$ is the dust radius, $\lambda_D$ is the Debye radius, and $\lambda$ is the wavelength of the collective fluctuations provided that one can also assume that the spread in $Q_d/m_d$ for the dust particles in the equilibrium plasma may be neglected. In a way, this approach of replacing a whole distribution of mass (radii) and charges by an averaged mass and mean charge is similar to that of using a "hydrodynamic" rather than a "kinetic" description for the corresponding evolution equations and should be reasonable as long as the spread in $Q_d/m_d$ is small and we can effectively use $\delta$-function distributions for the effective charge. This is a physically valid regime for the cases we have studied, e.g., low-frequency long-wavelength electrostatic waves such as ion acoustic waves. On the scale length of the wave motion the dimension of the dust particle is extremely small and any microscopic effects arising from the finite size of the dust grain is negligible. The relevant time scales of our problem, namely the wave period $\omega^{-1}$, the charging time $\eta^{-1}$, and the collision time $\nu^{-1}$, satisfy the relation $\nu^{-1} > (\omega^{-1} \sim \eta^{-1})$, i.e., the collision times are long compared to wave period and the charging time for the dust particle. As may be seen from the estimates of the time scales [see discussion after Eq. (8)], these conditions can be readily satisfied over a wide range of interest. Our charging model for dust particles, which is based on the classical probe theory, is strictly valid only for conducting dust particles. It should be emphasized, however, that a large class of physical systems indeed involve metallic dust grains (such as those of graphite, magnetite, aluminium, etc.) for which the conducting grain model is directly applicable. Examples are interstellar and circumstellar clouds, terrestrial aerosols, laboratory plasmas (tokamaks), plasma-processing furnaces, etc. In the case of dielectric dust particles, one may in general have to incorporate effects due to nonuniform charge distribution and polarization effects. However, it seems likely that in the point-particle model where the size of the particle is negligible compared to all characteristic lengths (the Debye length, wavelength of collective plasma modes, etc.), the multipolar contributions due to nonuniform charge distributions should have a negligible effect and the conducting-grain model with strictly monopolar charge distributions gives a good lowest-order estimate.

In conclusion, we have pointed out an unusual effect of essential importance in the study of self-consistent fluctuations in dusty plasmas, viz. effects of fluctuations in dust charge. In the case of low-frequency ion waves in an unmagnetized dusty plasma it leads to dissipation and instability effects with important consequences. Plasmas in which the particle charge of a dynamical species fluctuates self-consistently have been considered in the past in connection with the physics of quark gluon plasmas [12]. Although the charge fluctuation dynamics in such plasmas is basically nonlinear and reactive (modifying dispersive rather than dissipative properties), it is still of interest to investigate whether some aspects of QCD plasmas could be mocked up in the laboratory with dusty plasmas having self-consistent charge fluctuations.

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