Comment II on "Possible experiment to check the reality of a nonequilibrium temperature"

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Recently, Jou and Casas-Vázquez [Phys. Rev. A **45**, 8371 (1992)] proposed an experiment aimed to verify the physical reality of the nonequilibrium temperature Θ they introduced in their earlier works. I argue here—completely within the framework and philosophy of their theory—that this experiment cannot yield the desired result.

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To investigate the physical reality of the nonequilibrium temperature Θ introduced in their earlier works, Jou and Casas-Vázquez [1] suggest checking whether there arises a heat flux q_x between two connected thermodynamic systems (Fig. 1): The system on the left is in equilibrium, at the temperature T. To the system on the right, the steady heat flux q_y is applied; at the connection point, the local-equilibrium temperature T shall equal the temperature of the equilibrium system.

The theory of Jou and Casas-Vázquez gives for the nonequilibrium temperature Θ

$$\frac{1}{\Theta}=\frac{1}{T}-\gamma\mathbf{q}\cdot\mathbf{q} \tag{1}$$

(cf. Ref. [1] for the definition of γ). It follows also from their theory that a heat flux is driven by a gradient in Θ rather than in T [Eq. (3) of Ref. [1]]. Hence, in the experiment, with T adjusted, the heat flux q_x will arise in the connecting link:

$$q_x=\lambda\frac{T-\Theta}{L}\ , \tag{2}$$

where λ is the thermal conductivity of the link, and L is its length.

The existence of the heat flux q_x shall, according to Jou and Casas-Vázquez, be interpreted as evidence for the physical reality of the nonequilibrium temperature Θ.

For this experiment to work, it is obviously crucial to prepare the system such that, at the connection point in the nonequilibrium system, T (and not Θ) is equal to T in the equilibrium system. Note that the local-equilibrium temperature T cannot be checked *in situ:* Again within the framework of the theory of Jou and Casas-Vázquez, a thermometer measures Θ, not T. So all depends on the appropriate preparation of the system.

This is done, of course, via an adjustment of the temperatures T'',T' in the two reservoirs (cf. Fig. 1). The temperature difference $(T''-T')$ then gives rise to a heat flux in the y direction. Assuming a linear temperature distribution, we simply have

$$q_y=\lambda^*\frac{T''-T'}{D}\ . \tag{3}$$

Here, λ^* is the effective heat conductivity of the nonequilibrium system, and D is its length in the y direction.

One is tempted to conclude that, at the connection point (put in the middle),

$$T=\tfrac{1}{2}(T''-T') \tag{4}$$

with $\frac{1}{2}(T''-T')$ set equal to the temperature in the equilibrium system.

However, this conclusion stands in contradiction to the theory of Jou and Casas-Vázquez: Since the heat flux is driven by the gradient in Θ rather than in T, one expects Θ rather than T to be the temperature that has a linear distribution. This observation results in

$$\Theta=\tfrac{1}{2}(T''-T') \tag{5}$$

at the connection point. Hence, if the preparation is such that $\frac{1}{2}(T''-T')$ equals the temperature in the equilibrium system, then there will not be a Θ gradient in the link, and there will not arise a heat flux q_x.

Actually, $\frac{1}{2}(T''-T')$ is not too well defined, either. Let us examine the reservoirs a little closer. A thermal reservoir is characterized by an infinite heat capacity C, guaranteeing the constancy of the temperature in time. Now, in the experiment, the reservoirs sustain the heat flux q_y, driven by temperature gradients, and are therefore not in equilibrium. So, in principle, all ambiguities associated with the definition of a temperature in a nonequilibrium system apply to T'',T' as well.

But fortunately, the coefficient γ is proportional to $1/C$ so that, for a thermal reservoir, $T=\Theta$. The difference between T and Θ starts, and ends, at the reservoir walls: From Eq. (2) of Ref. [1] follows

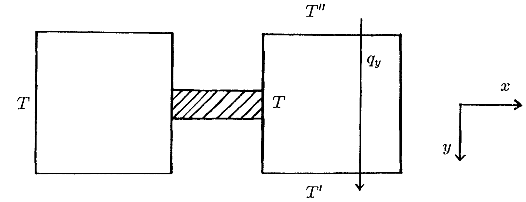

FIG. 1. Experiment proposed by Jou and Casas-Vásquez. The left system is in equilibrium, at the temperature T. The right system is exposed to the steady heat flux q_y. The two systems are connected by a good thermal link.

$$\mathbf{F}=\frac{\mathbf{q}}{\Theta}\ , \qquad (6)$$

where $\mathbf{F}$ is the entropy flux. With no other dissipative processes present, and without the consideration of surface entropy production, $\mathbf{F}$ as well as $\mathbf{q}$ are continuous at the interfaces. This leads to the boundary condition

$$\Theta^{[1]}=\Theta^{[2]}\ . \qquad (7)$$

The local-equilibrium temperature T, on the other hand, generally is *not* continuous [cf. Eq. (1): γ is material-specific]. Hence, if one connects one thermodynamic system to another, better known, one applies the value of Θ to the system, rather than the value of T (Fig. 2).

So, obviously, if the theory of Jou and Casas-Vázquez is correct, their experiment in the suggested form (Ref. [1]) cannot work. If their theory is not correct, their experiment cannot work either: Quite generally, that temperature whose gradient drives the heat flux is also continuous at interfaces, and is such the *only* temperature accessible by thermometry.

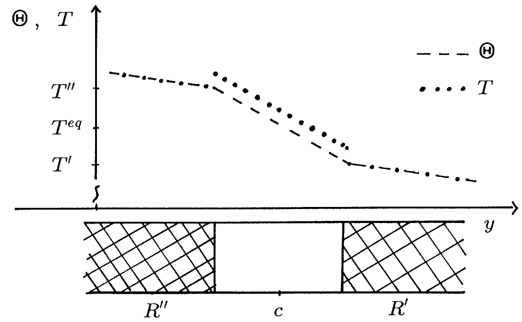

FIG. 2. Slope of the temperatures Θ and T in the nonequilibrium system. R'', R' are the reservoirs. c is the connection point to the equilibrium system that has the temperature $T=T^{\text{eq}}$.

Physical reality can be attributed to more than one temperature only if, for these temperatures, additional equations are available that are independently measurable.

[1] D. Jou and J. Casas-Vázquez, Phys. Rev. A **45**, 8371 (1992).