Effects of negative ions on nonlinear wave propagation in a magnetized rotating plasma

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Nonlinear dispersion relations of the obliquely propagating left circularly polarized and right circularly polarized waves in a magnetized rotating plasma, composed of electrons, positive ions, and negative ions, have been derived. Effects of negative ions on the refractive indices including their shifts have been investigated with graphical representation. Stability criteria, stop bands, pass bands, and cut-off conditions of the propagating waves are obtained which generalize the results of earlier workers.

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I. INTRODUCTION

In recent years, much attention has been paid by physicists to the investigation of wave propagation in multicomponent plasmas having electrons, positive ions, and negative ions [1–8]. The negative ions have influence on the charge equilibrium, on formation of sheaths, and on wave dispersion. According to D'Angelo, Goeler, and Ohe [9] ion waves have two modes of propagation in plasmas having negative ions. One of these is the "fast-ion mode," observed by Wong, Mamas, and Arnush [10]. The other is a slow-ion mode, observed by Sato and Amemiya [1]. From satellite observations Branscomb [11] showed that the negative ions in the ionosphere may be of the type O^- or O_2^-. Goeler, Ohe, and D'Angelo [12] reported that thermally ionized plasmas consist of appreciable percentages of negative ions. Plasmas containing negative ions appear in dc discharges of halogens and oxygens [13] and in the lower atmosphere *D* layer [14]. In the *F* region of the ionosphere, negative ions can be produced artificially by injecting SF_6 vapor [15]. It has been found that these artificially produced negative ions change the characteristics of the plasma and have dominating roles in various aspects of wave propagation in the ionosphere [16–18]. In the laboratory negative ions are produced by electron attachment of neutral particles when an electronegative gas [gas with a high electron-attachment cross section, e.g., halogens, hexafluoride (SF_6), iodine, and oxygen] is admitted into electric gas discharges. Negative ions are also produced in direct discharge in electronegative gases.

In the past two decades several authors have theoretically and experimentally investigated the various aspects of wave propagation in a plasma having negative ions. Uberoi and Das [19] showed that the presence of negative ions in the lower ionosphere has significant effects on the diagnosis of the plasma. Later, Uberoi [20] discussed the crossover frequencies in multicomponent plasmas, and investigated the conditions of stop bands and pass bands. Subsequently, Sur [21] and others [22,23] showed that the negative ions affect the group travel time of whistlers at the midlattitude and at the equator, and the cyclotron damping of whistlers at the midlattitude of the ionosphere. Instability associated with the negative ions or a decrease of wavelength and velocity striations are very interesting for study [24]. Paul, Sur, and Pakira [25] and others [26,27] showed that the negative ions introduce a contribution to the instability of the wave and the shift of the wave parameters. It is also seen that the dispersion of ion waves and the formation of solitons in plasmas in the presence of negative ions are important in laboratory and in space plasmas [28–33]. If negative ions are present in a plasma as contamination, a rarefactive and negative potential soliton may exist there [34–38].

It should be mentioned that the works of the above authors are restricted to a nonrotating plasma. But Chandrasekhar [39–41], Lehnert [42], and other authors [43–44] showed that the nature of wave propagation in rotating plasmas including the Coriolis force is very important in cosmic phenomena. Recently, Paul, Kashyapi, and Chakraborty [45] have studied the wave propagation in a magnetized rotating plasma and the characteristics of variation of the refractive indices with the rotational frequency. They obtained the cutoff and resonance frequencies from both left circularly polarized (LCP) and right circularly polarized (RCP) waves in the presence of the Coriolis force of rotation. Subsequently, Kashyapi, Paul, and Chakraborty [46] and Chakraborty *et al.* [47] investigated the instability of the wave and the shifts of refractive indices in a magnetized rotating plasma. Paul, Kashyapi, and Chakraborty [48] have also theoretically investigated the induced magnetization due to the inverse Faraday effect (IFE) [49–52] in a rotating magnetized plasma. The effects of ion motion in a rotating plasma have been considered by Kashyapi, Chakraborty, and Paul [53] for the investigation of nonlinear refractive indices, instability of the waves, and the shifts of refractive indices, cutoffs, and resonances of the waves. In continuation of our earlier work, we are here motivated to include the influence of negative ions on wave propagation in a plasma in the presence of the Coriolis force for study of nonlinear instability of waves, shifts of refractive indices, etc. In Sec. II, the nonlinear dispersion relations for the LCP and the RCP waves in a negative-ion plasma have been derived. In Sec. III, nonlinear shifts of refractive indices of the waves have been found and the effects of negative ions on it have been investigated. In Sec. IV, nonlinear instability of the LCP and the RCP waves in

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the presence of both negative ions and the Coriolis force in plasmas have been discussed and limiting values of the power of the waves for such instabilities have been obtained. In Sec. V, the results are discussed, and some concluding remarks are given for the wave propagation in magnetized rotating multicomponent plasmas.

II. BASIC EQUATIONS AND DISPERSION RELATIONS

We consider multicomponent plasmas as having electrons, positive ions, and negative ions. The plasma assumed here is cold, collisionless, and homogeneous. Moreover, the plasma is rotating around the magnetic-field lines with a constant angular velocity $\boldsymbol{\omega}_0$ $(=0,0,\omega_0)$, and the wave vector $\mathbf{K}$ $(=k_x,0,k_z)$ makes an angle θ with the ambient static magnetic field $\mathbf{H}^{(0)}$ $(=0,0,H^{(0)})$, i.e., $\mathbf{K}$ $(=k\sin\theta,0,k\cos\theta)$. Therefore, in this rotating frame of reference when other forces have been neglected and only the Coriolis force is taken into consideration, the momentum-transfer equation can be written as

$$\frac{\partial \mathbf{v}_\alpha}{\partial t}+(\mathbf{v}_\alpha\cdot\nabla)\mathbf{v}_\alpha=\frac{q_\alpha}{m_\alpha}\left[\mathbf{E}+\frac{\mathbf{v}_\alpha\times\mathbf{H}}{c}\right]+2(\mathbf{v}_\alpha\times\omega_0)\ , \qquad (1)$$

where, $\mathbf{v}_\alpha$, m_α, q_α are the velocity, mass, and charge of the α-type particles. $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields, respectively. c is the velocity of light. The related basic systems of equations governing the plasma dynamics are

$$\frac{\partial N_\alpha}{\partial t}+\nabla\cdot(N_\alpha\mathbf{v}_\alpha)=0\ , \qquad (2)$$

$$\nabla\times\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}\ , \qquad (3)$$

$$\nabla\times\mathbf{H}=\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}+\frac{4\pi}{c}\sum_\alpha N_\alpha\mathbf{v}_\alpha q_\alpha\ , \qquad (4)$$

$$\nabla\cdot\mathbf{E}=4\pi\sum_\alpha N_\alpha q_\alpha\ , \qquad (5)$$

$$\nabla\cdot\mathbf{H}=0\ , \qquad (6)$$

where N_α is the number density of the α-type particles, $\alpha=i,j,e$ for positive ions, negative ions, and electrons, respectively. q_α represent the charges of αth species with proper sign. $q_\alpha=(e)$ when $\alpha=i$, $q_\alpha=(-e)$ when $\alpha=j$ and e. All other symbols have their usual meanings.

We assume the following scheme of perturbation expansion of the field variables:

$$\begin{vmatrix}\mathbf{v}_\alpha\\ N_\alpha\\ \mathbf{E}\\ \mathbf{H}\end{vmatrix}=\begin{vmatrix}0\\ N_\alpha^{(0)}\\ 0\\ \mathbf{H}^{(0)}\end{vmatrix}+\epsilon^1\begin{vmatrix}\mathbf{v}_\alpha^{(1)}\\ N_\alpha^{(1)}\\ \mathbf{E}^{(1)}\\ \mathbf{H}^{(1)}\end{vmatrix}+\epsilon^2\begin{vmatrix}\mathbf{v}_\alpha^{(2)}\\ N_\alpha^{(2)}\\ \mathbf{E}^{(2)}\\ \mathbf{H}^{(2)}\end{vmatrix}+\epsilon^3\begin{vmatrix}\mathbf{v}_\alpha^{(3)}\\ N_\alpha^{(3)}\\ \mathbf{E}^{(3)}\\ \mathbf{H}^{(3)}\end{vmatrix}+\cdots\ , \qquad (7)$$

where the terms on the right-hand side having the superscript zero represent the equilibrium values and with 1,2,3, etc., represent the first-order, second-order, third-order, etc., values of their respective parameters. ϵ is an arbitrary expansion parameter.

Moreover, we assume that the wave is transverse, quasicircular, and represented by

$$E_\pm=(ae^{i\theta_\pm}+\bar{a}e^{-i\theta_\mp})\ , \qquad (8)$$

where $\theta_\pm=\mathbf{K}_\pm\cdot\mathbf{r}-\omega t$, a is the amplitude of the wave, and $\bar{a}$ is the complex conjugate of amplitude of the wave. The plus and minus signs in the subscripts represent the left circularly polarized and right circularly polarized waves, respectively, while ω and K are the frequency and the wave number of the wave.

Now substituting (7) and (8) into (1)–(6) we obtain the following values for the first-order quantities:

$$H_\pm^{(1)}=\pm\frac{ica}{\omega}(k_\pm e^{i\theta_\pm}+k_\mp e^{-i\theta_\mp})\ , \qquad (9)$$

$$v_{\alpha\pm}^{(1)}=\frac{iq_\alpha q}{m_\alpha}\left[\frac{e^{i\theta_\pm}}{\omega\mp\epsilon_\alpha\pi_\alpha}-\frac{e^{-i\theta_\mp}}{\omega\pm\epsilon_\alpha\pi_\alpha}\right]\ , \qquad (10)$$

$$E_z^{(1)}=0\ ,\quad H_z^{(1)}=0\ ,\quad v_{\alpha z}^{(1)}=0\ ,\quad N_\alpha^{(1)}=0\ , \qquad (11)$$

where $\Omega_\alpha=|q_\alpha H^{(0)}/m_\alpha c|$ and $\pi_\alpha=[\Omega_\alpha+(2\omega_0)\epsilon_\alpha]$,

$$\epsilon_\alpha=\begin{cases}+1\ , & \text{when } \alpha=i\\ -1\ , & \text{when } \alpha=j \text{ and } \alpha=e\ .\end{cases}$$

The first-order dispersion relations for the left circularly polarized and the right circularly polarized waves are

$$n_\pm^2=1-\frac{\omega_{p_e}^2}{\omega(\omega\pm\pi_e)}-\frac{\omega_{p_i}^2}{\omega(\omega\mp\pi_i)}-\frac{\omega_{p_j}^2}{\omega(\omega\pm\pi_j)}\ , \qquad (12)$$

where n_+ $(=ck_+/\omega)$ and n_- $(=ck_-/\omega)$ are the refractive indices for the LCP and the RCP waves, respectively,

$$\omega_{p_e}=\left[\frac{4\pi e^2N_e^{(0)}}{m_e}\right]^{1/2}\ ,\quad \omega_{p_i}=\left[\frac{4\pi e^2N_i^{(0)}}{m_i}\right]^{1/2}\ ,$$

$$\omega_{p_j}=\left[\frac{4\pi e^2N_j^{(0)}}{m_j}\right]^{1/2}\ ,$$

$$\pi_e=\Omega_e-2\omega_0\ ,\quad \pi_i=\Omega_i+2\omega_0\ ,\quad \pi_j=\Omega_j-2\omega_0\ .$$

Similarly, the expressions for the second-order field variables are

$$E_z^{(2)}=\sum_\alpha\frac{\omega_{p_\alpha}^2q_\alpha a^2}{2im_\alpha\omega}\left[\frac{k_+}{\omega+\epsilon_\alpha\pi_\alpha}+\frac{k_-}{\omega-\epsilon_\alpha\pi_\alpha}\right]\left[\frac{e^{i(\theta_++\theta_-)}-e^{-i(\theta_++\theta_-)}}{\omega_p^2}\right]\ , \qquad (13)$$

$$v_{\alpha z}^{(2)}=\left[\frac{q_\alpha^2 a^2}{m_\alpha^2}\left[\frac{k_+}{\omega+\epsilon_\alpha\pi_\alpha}+\frac{k_-}{\omega-\epsilon_\alpha\pi_\alpha}\right]-\frac{q_\alpha^2 a^2(\omega_p^2+4\omega^2-\omega_{p_\alpha}^2)}{4m_\alpha^2\omega^2}\left[\frac{k_+}{\omega+\epsilon_\alpha\pi_\alpha}+\frac{k_-}{\omega-\epsilon_\alpha\pi_\alpha}\right]\right.$$
$$\left.+\sum_\beta \frac{\omega_{p_\beta}^2 q_\alpha q_\beta a^2}{4m_\alpha m_\beta\omega^2}\left[\frac{k_+}{\omega+\psi_\beta\pi_\beta}+\frac{k_-}{\omega-\psi_\beta\pi_\beta}\right]\right]\left[\frac{e^{i(\theta_++\theta_-)}+e^{-i(\theta_++\theta_-)}}{\omega_p^2}\right], \tag{14}$$

$$\sum_\alpha N_\alpha^2\epsilon_\alpha=\sum_\alpha\frac{\omega_{p_\alpha}^2 a^2\epsilon_\alpha(k_++k_-)}{8\pi m_\alpha\omega}\left[\frac{k_+}{\omega+\epsilon_\alpha\pi_\alpha}+\frac{k_-}{\omega-\epsilon_\alpha\pi_\alpha}\right]\left[\frac{e^{i(\theta_++\theta_-)}+e^{-i(\theta_++\theta_-)}}{\omega_p^2}\right], \tag{15}$$

$$E_\pm^{(2)}=0\ ,\quad v_{\alpha\pm}^{(2)}=0\ ,\quad H_\pm^{(2)}=0\ ,\quad H_z^{(2)}=0\ , \tag{16}$$

where $\omega_p^2=(\sum_\alpha\omega_{p_\alpha}^2-4\omega^2)$, $\psi_\beta=+1$, when $\beta=i$ and $\psi_\beta=-1$, when $\beta=j,e$ and $\alpha\neq\beta$ at the same time.

For the third-order excited fields of the transverse components we derive two equations:

$$\left[\frac{\partial}{\partial t}\pm i\epsilon_\alpha\pi_\alpha\right]v_{\alpha\pm}^{(3)}+\left[\frac{\partial(v_{\alpha\pm}^{(1)})}{\partial z}\mp\frac{iq_\alpha H_\pm^{(1)}}{m_\alpha c}\right]v_{\alpha z}^{(2)}-\frac{q_\alpha}{m_\alpha}E_\pm^{(3)}=0 \tag{17}$$

and

$$\left[\frac{\partial^2}{\partial t^2}-c^2\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right]\right]E_\pm^{(3)}+4\pi\frac{\partial}{\partial t}\left[\sum_\alpha N_\alpha^{(0)}q_\alpha v_{\alpha\pm}^{(3)}+\sum_\alpha N_\alpha^{(2)}q_\alpha v_{\alpha\pm}^{(1)}\right]=0\ . \tag{18}$$

To consider the first-harmonic part of $E_\pm^{(3)}$, the values of $v_{\alpha\pm}^{(3)}$ are first calculated from (17). Therefore,

$$v_{\alpha\pm}^{(3)}=\pm\frac{1}{\omega_P^2}\left[\frac{q_\alpha^2 a^2}{m_\alpha^2}\left[\frac{k_+}{\omega+\epsilon_\alpha\pi_\alpha}+\frac{k_-}{\omega-\epsilon_\alpha\pi_\alpha}\right]-\frac{(\omega_p^2+4\omega^2-\omega_{p_\alpha}^2)q_\alpha^2 a^2}{4m_\alpha^2\omega^2}\left[\frac{k_+}{\omega+\epsilon_\alpha\pi_\alpha}+\frac{k_-}{\omega-\epsilon_\alpha\pi_\alpha}\right]\right.$$
$$\left.+\sum_\beta\frac{\omega_{p_\beta}^2 a^2 q_\alpha q_\beta}{4m_\alpha m_\beta\omega^2}\left[\frac{k_+}{\omega+\psi_\beta\pi_\beta}+\frac{k_-}{\omega-\psi_\beta\pi_\beta}\right]\right]$$
$$\times\left[\frac{aq_\alpha\epsilon_\alpha\pi_\alpha}{i\omega m_\alpha[\omega^2-(\epsilon_\alpha\pi_\alpha)^2]}(k_\pm e^{-i\theta_\mp}+k_\mp e^{i\theta_\pm})\right]-\frac{aq_\alpha}{im_\alpha}\left[\frac{e^{i\theta_\pm}}{\omega\mp\epsilon_\alpha r_\alpha}-\frac{e^{-i\theta_\mp}}{\omega\pm\epsilon_\alpha\pi_\alpha}\right]. \tag{19}$$

Then using the values of $v_{\alpha\pm}^{(3)}$, $E_\pm^{(3)}$, $\sum_\alpha N_\alpha^{(2)}q_\alpha v_{\alpha\pm}^{(1)}$ in (18), and equating the coefficients of $e^{i\theta_+}$ and $e^{i\theta_-}$, the nonlinear dispersion relations for the third-order electric fields, correct up to the first-harmonic part of the LCP and the RCP waves are

$$n_\pm^2=1-\frac{\omega_{p_\alpha}^2}{\omega(\omega\mp\epsilon_\alpha\pi_\alpha)}\pm\frac{\cos^2\theta}{\omega_p^2}\sum_{\alpha,\beta}\epsilon_\alpha\left[\frac{(\omega_{p_\alpha}^2-\omega_p^2)\omega_{p_\alpha}^2 q_\alpha^2 a^2\pi_\alpha k_\mp}{4m_\alpha^2\omega^4(\omega^2-\pi_\alpha^2)}+\frac{\omega_{p_\alpha}^2\omega_{p_\beta}^2 q_\alpha^2 a^2\pi_\beta k_\mp}{4m_\alpha m_\beta\omega^4(\omega^2-\pi_\beta^2)}\right]\left[\frac{k_+}{\omega+\epsilon_\alpha\pi_\alpha}+\frac{k_-}{\omega-\epsilon_\alpha\pi_\alpha}\right]$$
$$+\frac{a^2(k_++k_-)\cos^2\theta}{2\omega_p^2}\sum_\alpha\frac{q_\alpha^2\omega_{p_\alpha}^2(\omega_{p_\alpha}^2-\omega_p^2)}{4m_\alpha^2\omega^4}\left[\frac{k_\pm}{(\omega\pm\epsilon_\alpha\pi_\alpha)^2}+\frac{k_\mp}{(\omega^2-\pi_\alpha^2)}\right]$$
$$+\frac{a^2(k_++k_-)\cos^2\theta}{2\omega_p^2}\sum_{\alpha,\beta}\epsilon_\alpha\psi_\alpha\frac{\omega_{p_\alpha}^2\omega_{p_\beta}^2 q_\alpha^2}{4m_\alpha m_\beta\omega^4(\omega\pm\epsilon_\alpha\pi_\alpha)}\left[\frac{k_+}{(\omega+\psi_\beta\pi_\beta)}+\frac{k_-}{(\omega-\psi_\beta\pi_\beta)}\right], \tag{20}$$

where θ is the angle between ambient static magnetic field and the wave-propagation vector.

When $\omega\approx\pi_i$ the relation (20) is simplified to

$$n_+^2=\frac{\omega_{p_i}^2}{\pi_0(\Delta\omega)}-\frac{3\alpha_i^2\omega_{p_i}^2c^2k_-^2\cos^2\theta}{8\pi_0^2(\Delta\omega)^2}-\frac{\alpha_i^2c^2k_+k_-\omega_{p_i}^2\cos^2\theta}{8\pi_0^3(\Delta\omega)}$$
$$+\frac{\chi\alpha_i^2\omega_{p_i}^2c^2k_+k_-\cos^2\theta}{8\pi_0^2(\Delta\omega)^2}+\frac{\chi\alpha_i^2\omega_{p_i}^2c^2k_-^2\cos^2\theta}{4\pi_0^2(\Delta\omega)^2}+\frac{\chi\alpha_i^2\omega_{p_i}^2c^2k_+^2\cos^2\theta}{16\pi_0^3(\Delta\omega)}\ , \tag{21}$$

and

$$n_-^2=\frac{\chi\omega_{p_i}^2}{\pi_0(\Delta\omega)}-\frac{\alpha_i^2\omega_{p_i}^2c^2k_+^2\cos^2\theta}{8\pi_0^3(\Delta\omega)}-\frac{\chi\alpha_i^2\omega_{p_i}^2c^2k_+^2\cos^2\theta}{8\pi_0^2(\Delta\omega)^2}+\frac{\chi\alpha_i^2\omega_{p_i}^2c^2k_-^2\cos^2\theta}{32\pi_0^4}-\frac{\alpha_i^2\omega_{p_i}^2c^2k_-^2\cos^2\theta}{8\pi_0^2(\Delta\omega)^2}\ , \tag{22}$$

where α_i ($=ea/m_i\omega c$) denotes the dimensionless amplitude of the wave, $\chi=n_j^{(0)}/n_i^{(0)}$, and $\Delta\omega=\pi_0-\omega$. Moreover, the masses of negative and positive ions have been assumed to be the same, i.e., $\pi_i=\pi_j=\pi_0$.

We now substitute the following values of k_+ and k_- on the right-hand side of (21) and (22), obtained from linearized approximations,

$$k_+=\omega^{1/2}\omega_{p_i}/c(\Delta\omega)^{1/2}\ , \tag{23}$$

$$k_-=\chi^{1/2}\omega^{1/2}\omega_{p_i}/c(\Delta\omega)^{1/2}\ , \tag{24}$$

where $\omega\approx\pi_0$, and then using (23) and (24) in (21) and (22), we get

$$n_+^2=\frac{\omega_{p_i}^2}{\pi_0(\Delta\omega)}-\frac{\alpha_i^2(\chi^{1/2}-2\chi^2)\omega_{p_i}^4\cos^2\theta}{2(\Delta\omega)^2(\pi_0+\omega)^2}+\frac{\alpha_i^2(\chi^{1/2}-3\chi)\omega_{p_i}^4\cos^2\theta}{4(\Delta\omega)^3(\pi_0+\omega)}\ , \tag{25}$$

$$n_-^2=\frac{\chi\omega_{p_i}^2}{\pi_0(\Delta\omega)}-\frac{\alpha_i^2(1-\chi^2)\omega_{p_i}^4\cos^2\theta}{2(\Delta\omega)^2(\pi_0+\omega)^2}+\frac{\alpha_i^2\chi^{1/2}(1+\chi)\omega_{p_i}^4\cos^2\theta}{8\pi_0^2(\Delta\omega)(\pi_0+\omega)}-\frac{\alpha_i^2\chi\omega_{p_i}^4\cos^2\theta}{4\pi_0(\Delta\omega)^3}\ . \tag{26}$$

III. NONLINEAR REFRACTIVE-INDEX SHIFT

To obtain the nonlinear refractive-index shifts we substitute $n_+=n_{0_+}+\partial n_+$ and $n_-=n_{0_-}+\partial n_-$ in Eqs. (25) and (26) and, neglecting the higher-order terms of ∂n_+ and ∂n_-, we obtain

$$\partial n_+=-\frac{\alpha_i^2\omega_{p_i}^3\cos^2\theta}{4c\omega^2(y-1)^{3/2}(y+1)}\left[\frac{(3\chi-\chi)^{3/2}}{2(y-1)}+\frac{(\chi^{1/2}-2\chi^2)}{y+1}\right]\ , \tag{27}$$

$$\partial n_-=-\frac{\alpha_i^2\omega_{p_i}^3\cos^2\theta}{4c\chi^{1/2}(y-1)^{3/2}}\left[\frac{(1-\chi^2)}{\omega^2(y+1)^2(y-1)}-\frac{\chi^{1/2}(1+\chi)}{4\pi_0^2(y+1)}+\frac{\chi}{2\pi_0\omega(y-1)^2}\right]\ , \tag{28}$$

where $y=\pi_0/\omega$.

Cutoff frequency

To obtain the cutoff frequencies ($k_\pm^2=0$) Eqs. (25) and (26) are simplified to

$$\omega^4(4\omega_{p_i}^2)-\omega^2(8\pi_0^2\omega_{p_i}^2)+\omega[2\alpha_i^2(\pi_0)(\chi^{1/2}-2\chi^2)\omega_{p_i}^4\cos^2\theta+\alpha_i^2(\chi^{1/2}-3\chi)\omega_{p_i}^4\cos^2\theta(\pi_0)+16\pi_0^3\omega_{p_i}^2]$$
$$+[4\omega_{p_i}^2\pi_0^4-2\alpha_i^2(\chi^{1/2}-2\chi^2)\omega_{p_i}^4\cos^2\theta(\pi_0^2)+\alpha_i^2(\chi^{1/2}-3\chi)\omega_{p_i}^4\cos^2\theta(\pi_0^2)]=0 \tag{29}$$

and

$$\omega^4[8\chi\omega_{p_i}^2(\pi_0)]+\omega^3[16\pi_0^2\chi\omega_{p_i}^2+\alpha_i^2\chi^{1/2}(1+\chi)\omega_{p_i}^4\cos^2\theta]-\omega^2[16\chi\pi_0^3\omega_{p_i}^2+\alpha_i^2\chi^{1/2}(1+\chi)\omega_{p_i}^4(\pi_0)+2\alpha_i^2\chi\omega_{p_i}^4\pi_0]$$
$$+\omega[4\pi_0^2\alpha_i^2(1+\chi^4)\omega_{p_i}^4\cos^2\theta+3\alpha_i^2\chi^{1/2}(1+\chi)\omega_{p_i}^4\cos^2\theta(\pi_0^2)-4\pi_0^2\alpha_i^2\chi\omega_{p_i}^4\cos^2\theta]$$
$$+[8\chi(\pi_0^5\omega_{p_i}^2)-4\pi_0^3\alpha_i^2(1+\chi^4)\omega_{p_i}^4\cos^2\theta+\alpha_i^2\chi^{1/2}(1+\chi)\omega_{p_i}^4\cos^2\theta\pi_0^3-2\alpha_i^2\chi\omega_{p_i}^4\pi_0^3]=0\ . \tag{30}$$

From (29) it is seen that there are two real positive roots when $\pi_0>0$. So, the LCP wave has only two cutoff frequencies. But when $\pi_0<0$, Eq. (30) for the RCP wave can have only two real positive roots; so there are two cutoff frequencies.

IV. INSTABILITY OF THE WAVE

To investigate the nonlinear instability of the LCP and the RCP waves Eqs. (21) and (22) are written in the following form:

$$x_1k_+^2-y_1k_++z_1=0\ , \tag{31}$$

$$x_2k_-^2-z_2=0\ , \tag{32}$$

where

$$x_1=\frac{c^2}{\omega^2}-\frac{\chi\alpha_i^2c^2\omega_{p_i}^2\cos^2\theta}{16\pi_0^3(\Delta\omega)}\ ,$$

$$y_1=c\left[\frac{\chi\alpha_i^2\omega_{p_i}^2\cos^2\theta}{8\pi_0^2(\Delta\omega)^2}-\frac{\alpha_i^2\omega_{p_i}^2\cos^2\theta}{8\pi_0^3(\Delta\omega)}\right]\left[\frac{\omega^{1/2}\chi^{1/2}\omega_{p_i}}{(\Delta\omega)^{1/2}}\right],$$

$$z_1=\left[-\frac{\chi\alpha_1^2\omega_{p_i}^2\cos^2\theta}{4\pi_0^2(\Delta\omega)^2}+\frac{3\alpha_i^2\omega_{p_i}^2\cos^2\theta}{8\pi_0^2(\Delta\omega)^2}\right]\left[\frac{\chi\omega\omega_{p_i}^2}{\Delta\omega}\right]-\frac{\omega_{p_i}^2}{\pi_0(\Delta\omega)}\ ,$$

$$x_2=\frac{c^2}{\omega^2}-\frac{\chi\alpha_1^2c^2\omega_{p_1}^2\cos^2\theta}{32\pi_0^4}+\frac{c^2\alpha_i^2\omega_{p_i}^2\cos^2\theta}{8\pi_0^2(\Delta\omega)^2}\ ,$$

$$z_2=\left[-\frac{\alpha_i^2\omega_{p_i}^2\cos^2\theta}{8\pi_0^3(\Delta\omega)}-\frac{\chi\alpha_i^2\omega_{p_i}^2\cos^2\theta}{8\pi_0^2(\Delta\omega)^2}\right]\left[\frac{\omega\omega_{p_i}^2}{\Delta\omega}\right]+\frac{\chi\omega_{p_i}^2}{\pi_0(\Delta\omega)}\ .$$

Now, we substitute $k_+=k_{r_+}+ik_{i_+}$ and $k_-=k_{r_-}+ik_{i_-}$ in (31) and (32), then evaluating the imaginary and real parts, obtain

$$k_{r_+}=\frac{\left[\frac{\chi\alpha_i^2c\omega_{p_i}^2\cos^2\theta}{8\pi_0^2(\Delta\omega)^2}-\frac{\alpha_i^2c\omega_{p_i}^2\cos^2\theta}{8\pi_0^3(\Delta\omega)}\right]\left[\frac{\omega^{1/2}\chi^{1/2}\omega_{p_i}}{(\Delta\omega)^{1/2}}\right]}{2\left[\frac{c^2}{\omega^2}-\frac{\chi\alpha_i^2c^2\omega_{p_i}^2\cos^2\theta}{16\pi_0^3(\Delta\omega)}\right]}\ , \tag{33}$$

$$k_{i_+}=\pm\left[\frac{c^2}{\omega^2}-\frac{\chi\alpha_i^2\omega_{p_i}^2c^2\cos^2\theta}{16\pi_0^3(\Delta\omega)}\right]^{-1}$$
$$\times\left\{\left[\left[\frac{c^2}{\omega^2}-\frac{\chi\alpha_i^2\omega_{p_i}^2c^2\cos^2\theta}{16\pi_0^3(\Delta\omega)}\right]\left[-\frac{\chi\alpha_i^2\omega_{p_i}^2\cos^2\theta}{4\pi_0^2(\Delta\omega)^2}+\frac{3\alpha_i^2\omega_{p_i}^2\cos^2\theta}{8\pi_0^2(\Delta\omega)^2}\right]\left[\frac{\omega\chi\omega_{p_i}^2}{\Delta\omega}\right]-\frac{\omega_{p_i}^2}{\pi_0(\Delta\omega)}\right]\right.$$
$$\left.-\left[\left[\frac{\chi\alpha_i^2c\omega_{p_i}^2\cos^2\theta}{8\pi_0^2(\Delta\omega)^2}-\frac{\alpha_i^2c\omega_{p_i}^2\cos^2\theta}{8\pi_0^3(\Delta\omega)}\right]^2\left[\frac{\omega\chi\omega_{p_i}^2}{4(\Delta\omega)}\right]\right]\right\}^{1/2}\ , \tag{34}$$

and

$$k_{r_-}=\frac{\left[\left[-\frac{\alpha_i^2\omega_{p_i}^2\cos^2\theta}{8\pi_0^3(\Delta\omega)}-\frac{\chi\alpha_i^2\omega_{p_i}^2\cos^2\theta}{8\pi_0^2(\Delta\omega)^2}\right]\left[\frac{\omega\omega_{p_i}^2}{\Delta\omega}\right]+\frac{\chi\omega_{p_i}^2}{\pi_0(\Delta\omega)}\right]}{2\left[\frac{c^2}{\omega^2}-\frac{\chi\alpha_i^2c^2\omega_{p_i}^2\cos^2\theta}{32\pi_0^4}+\frac{\alpha_i^2c^2\omega_{p_i}^2\cos^2\theta}{8\pi_0^2(\Delta\omega)^2}\right]}\ , \tag{35}$$

$$k_{i_-}=0\ . \tag{36}$$

From (33) and (34) it is clear that k_{r_+}, k_{i_+} both are real and positive in a magnetized plasma in the presence of negative ions due to nonlinear effects. So k_+ is complex and the LCP wave is unstable; but from (35) and (36), k_{r_-} is real and positive, $k_{i_-} \approx 0$, so the RCP wave is stable. Moreover, in a LCP wave when $\chi = 0$, the value of k_{r_+} is zero and k_{i_+} is positive. So k_+ becomes positive and imaginary which implies that the LCP wave is attenuated.

V. RESULTS AND DISCUSSIONS

From relations (25)–(28) it is observed that the refractive indices and their shifts for the LCP and the RCP waves depend on (i) the negative-ion concentration (χ), (ii) the rotational frequency (ω_0), (iii) gyrofrequencies (Ω_e, Ω_i), (iv) the angle of propagation (θ), and (v) the amplitude of the wave (α_i). To have a clear idea about the role of these parameters on the refractive indices and their shifts as well as on cutoff and resonance frequencies of the wave, we have drawn Figs. 1–3, for a model plasma. Figure 1 shows the variation of the average of the square of the refractive indices n_+^2 of (25) and n_-^2 of (26), i.e., $\bar{n}^2 = (n_+^2 + n_-^2)/2$ with the variation of negative-ion concentration (χ) for different values of angle of propagation (θ) and the rotational frequency (ω_0) when $(\omega_{p_i}/\omega)^2 = 1\times 10^{-1}$, $\Omega_e/\omega = 0.2$, $\Omega_i/\omega = 0.1$, $\alpha_i^2 = 1\times 10^{-1}$. It is observed that $\bar{n}^2$ increases with the increase of negative-ion concentration (χ) for different values of rotational frequency ($2\omega_0/\omega = 0.1$, 0.3, 0.5, 0.7) when $\theta = 0°$, 45°, and 60°. Moreover, it is seen that when $2\omega_0/\omega = 0.1$, $\theta = 0°$, $\bar{n}^2$ is much higher than the values of $\bar{n}^2$ when $\theta = 45°$ and 60°. It is also observed that $\bar{n}^2$ increases with the increase of rotational frequencies.

In Fig. 2(a) we have shown the variation of an average shift of the refractive indices, $\partial n = (\partial n_+ + \partial n_-)/2$, of (27) and (28) for different values of the angle of propagation (θ) when $(\omega_{p_i}/\omega)^2 = 1\times 10^{-1}$, $\alpha_i^2 = 1\times 10^{-1}$, $\chi < 1$,

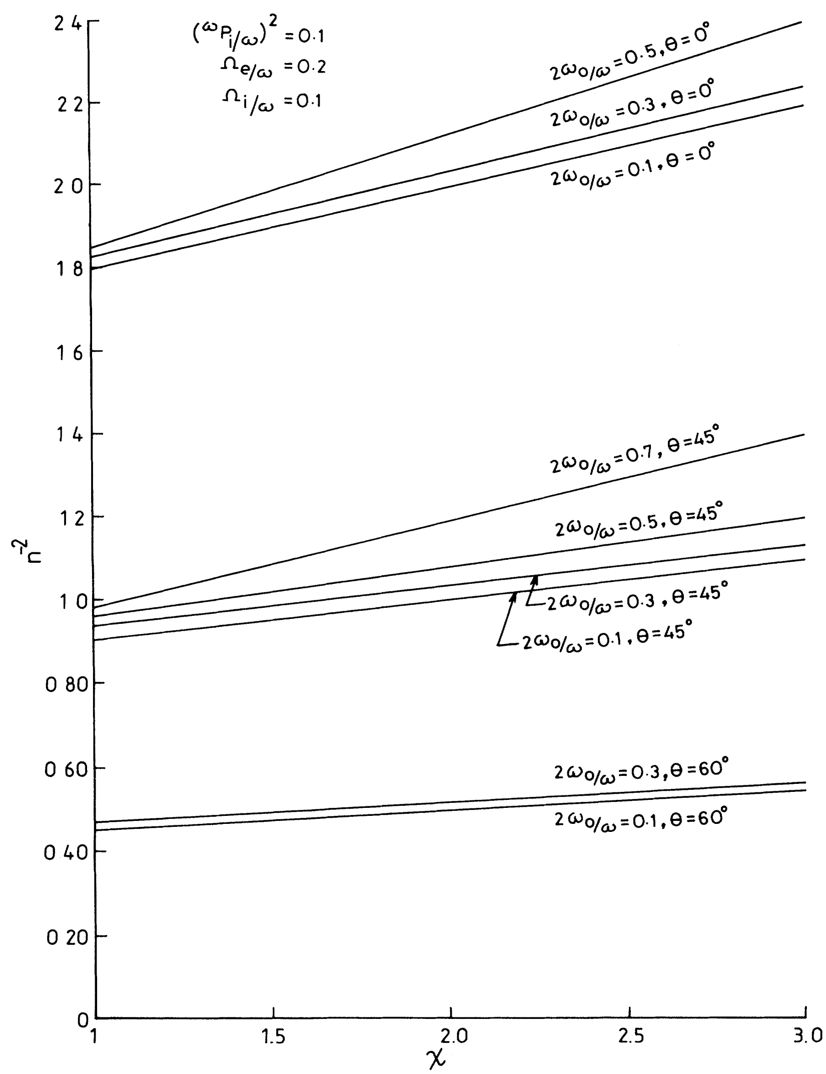

FIG. 1. The effect of negative-ion concentration on the nonlinear refractive index of the wave.

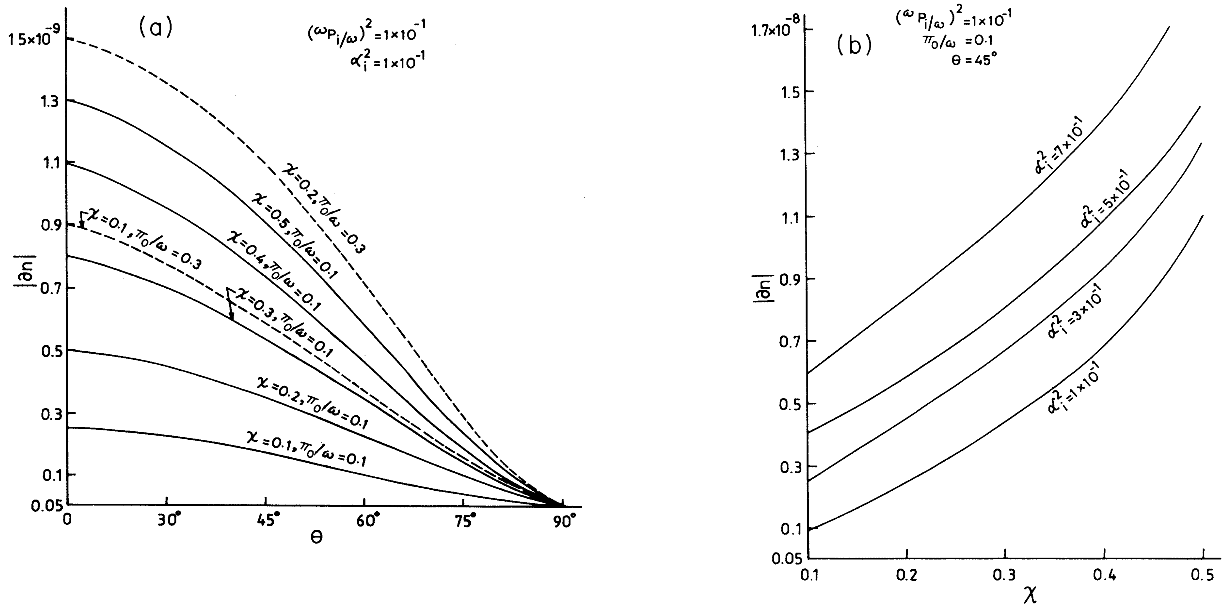

FIG. 2. (a) The effect of angle of propagation on the shift of refractive index of the wave. (b) The effect of negative-ion concentration on the nonlinear refractive-index shift of the wave.

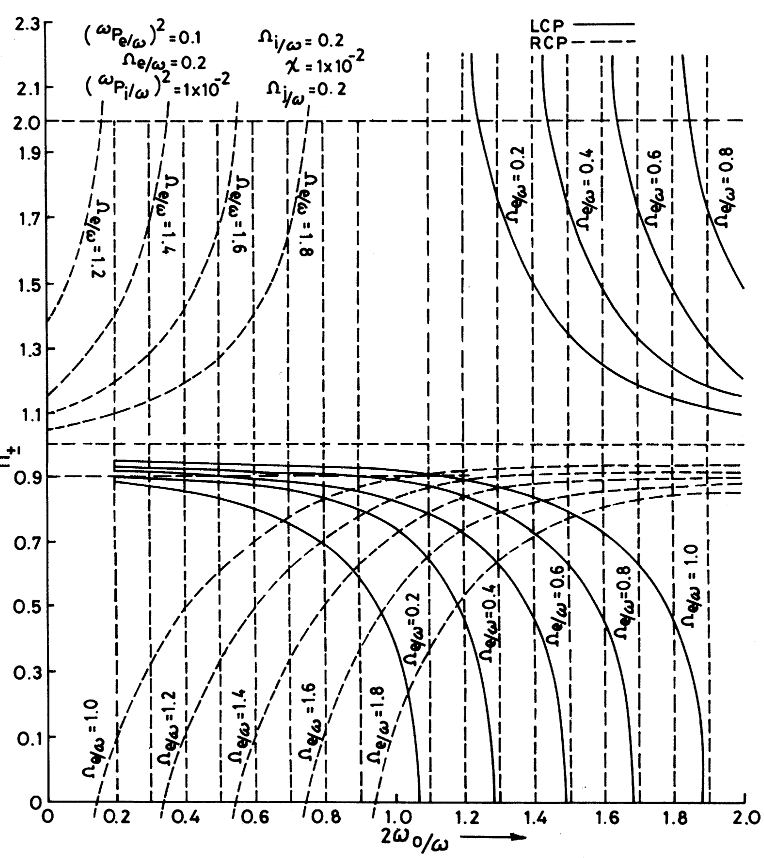

FIG. 3. The effect of the angular frequency of rotation on the cutoff and resonance frequencies of the wave.

$\pi_0/\omega<1$. The dotted lines and solid lines indicate the variation of ∂n for (i) $\chi=0.1$ and 0.2, $\pi_0/\omega=0.3$, (ii) $\chi=0.1$, 0.2, 0.3, 0.4, and 0.5, $\pi_0/\omega=0.1$, respectively. It is interesting to note that as θ increases ∂n decreases and ultimately it becomes almost zero for $\theta=90°$. The shifts of the refractive indices are maximum when the wave propagates along the magnetic-field lines (i.e. for $\theta=0$).

Figure 2(b) shows the average shift of refractive indices (∂n) with the increase of negative-ion concentration (χ) in the plasma when $(\omega_{p_i}/\omega)^2=1\times10^{-1}$, $\pi_0/\omega=0.1$, $\theta=45°$, and $\alpha_i^2<1$. From this figure it is observed that ∂n increases with an increase in negative-ion concentration. Moreover, any increase of the amplitude of the wave increases the wave-number shift.

In Fig. 3 we have shown the variation of the square of the refractive indices (n_+^2 and n_-^2) of (25) and (26) with variation of angular frequency of rotation for the LCP and the RCP waves when $(\omega_{p_e}/\omega)^2=1\times10^{-1}$, $(\omega_{p_i}/\omega)^2=1\times10^{-2}$, $\Omega_e/\omega=0.2$, $\Omega_i/\omega=0.2$, $\Omega_j/\omega=0.2$, and $\chi\ll1$. From this figure the stop bands for the LCP and the RCP waves are obtained for different values of the rotational frequency ($2\omega_0/\omega$), gyrofrequency (Ω_e/ω) when the angle of propagation $\theta=45°$. It is known that the wave does not propagate at the cutoff frequencies, because $k_\pm=0$ and resonances occur when $k_\pm\to\infty$. The cutoff points are found to be at $2\omega_0/\omega=1.1$, 1.3, 1.5, 1.7 and the resonance at $2\omega_0/\omega=1.2$, 1.4, 1.6, 1.8 for $\Omega_e/\omega=0.2$, 0.4, 0.6, 0.8, respectively. So, the widths of the stop bands for the LCP wave are given by the inequalities $1.1\le\delta\le1.2$, $1.3\le\delta\le1.4$, $1.5\le\delta\le1.6$, and $1.7\le\delta\le1.8$. But, for the RCP waves, the cutoff points are for $2\omega_0/\omega=0.3$, 0.5, 0.7, 0.9 and the resonances are for $2\omega_0/\omega=0.2$, 0.4, 0.6, 0.8 and for $\Omega_e/\omega=1.2$, 1.4, 1.6, 1.8, respectively. Therefore, the widths of the stop bands are $0.2\le\delta\le0.3$, $0.4\le\delta\le0.5$, $0.6\le\delta\le0.7$, and $0.8\le\delta\le0.9$, respectively.

VI. SUMMARY AND CONCLUDING REMARKS

We have here investigated the effects of negative ions on the refractive indices of LCP and RCP waves including the stability criteria, stop bands, pass bands, cutoff conditions, and resonances. The negative ions are found to have important contributions to the characteristics of the wave, particularly on the stability of the wave, when the power of the wave is much below the threshold power for generating nonlinear phenomena such as filamentation, self-focusing, etc. The effects of rotation on the nonlinear refractive indices and their shifts are important when the cyclotron frequency is very close to twice the rotational frequency. Since both the Coriolis-force term and the centrifugal-force term appear in a rotating frame, the centrifugal force has a considerable influence on characteristics of nonlinear wave propagation in plasmas.

Negative ions are present in the ionosphere, but there the effect of the Coriolis force is insignificant. So, our results will be applicable only in laboratory experiments where the negative ions can be produced and rotation can be introduced or generated. Recently, some authors tried to produce high-density negative ions in the plasma by experiments. Interesting properties are observed when negative-ion density is higher than the electron density [1]. In such plasmas no appreciable contributions are made by the electrons and new features of plasma phenomena are expected to appear due to a lack of electron shielding of potential variation. Actually, an increase of negative ions will decrease the phase velocity of the fast mode of ion waves. But the phase velocity of the slow mode of ion waves will be increased due to an increase of negative ions in the plasma [54]. Investigations of negative-ion plasmas are important in reactive-plasma processing where many negative ions exist and also in negative-ion-beam production for fusion plasma heating as well.

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