Dynamic and static correlations in model Coulomb systems

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The classical method of moments is applied to express the dynamic structure factor of model two-component plasmas in terms of static correlations. The latter are studied using an original algorithm based on the temperature-Green's-function method and including the local-field corrections to the random-phase approximation.

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I. INTRODUCTION

Extensive molecular-dynamics (MD) computations of time-independent and dynamic correlation functions for model Coulomb systems over a wide range of thermodynamic conditions characterized by dimensionless parameters

$$\Gamma=\beta e^2/a \ , \quad \Theta=(\beta E_F)^{-1} \tag{1.1}$$

have been carried out by Hansen *et al.* [1,2]. Here $\beta^{-1}=k_B T$ is the plasma temperature in energy units, a is the Wigner-Seitz radius, E_F is the Fermi energy, k_B is the Boltzmann constant, and $-e$ is the electron charge. Note that the Gell-Mann and Brueckner parameter

$$r_s=a/a_B=1.84159\Gamma\Theta \ ,$$

a_B being the Bohr radius.

Hansen and his collaborators studied the properties of one- and two-component plasmas and binary ionic systems. Classical (by definition) one-component plasmas (OCP) and binary ionic mixtures (BIM) were characterized only by the parameter Γ.

Earlier we applied the results of the approach [3] based on exact relations and sum rules to the calculation of dynamical characteristics of OCP and BIM [4]. The dynamical properties and collective modes in strongly coupled plasmas have also been investigated within the quasilocalized-charge model and the mean-field theory (dynamical and static) [5], and the approach based on the representation of the Green's functions as continued fractions [6], but the implementation of the classical method of moments [4] proved to produce the best overall agreement with the MD data.

The aim of the present paper is to extend the results of Ref. [4] to the investigation of the "charge-charge" dynamic structure factor $S_{cc}(k,\omega)$ of the model semiquantal two-component plasma (TCP) [2].

II. THE STRUCTURE FACTOR

The structure factor $S_{cc}(k,\omega)$ is directly connected to the inverse longitudinal dielectric function $\epsilon^{-1}(k,\omega)$ of the plasma via the fluctuation-dissipation theorem (FDT)

$$S_{cc}(k,\omega)=-\frac{\hbar \mathrm{Im}\epsilon^{-1}(k,\omega)}{\pi\Phi(k)[1-\exp(-\beta\hbar\omega)]} \ , \tag{2.1}$$

where $\Phi(k)=4\pi e^2/k^2$, and $\hbar$ is the reduced Planck constant.

In order to construct the inverse dielectric function it is useful to consider the frequency moments of the loss function $[-\mathrm{Im}\epsilon^{-1}(k,\omega)/\omega]$ [3],

$$C_\nu(k)=-\pi^{-1}\int_{-\infty}^{\infty}\omega^{\nu-1}\mathrm{Im}\epsilon^{-1}(k,\omega)d\omega \ . \tag{2.2}$$

Notice the finiteness of the moment

$$C_0(k,\omega)=[1-\epsilon^{-1}(k,0)] \ . \tag{2.3}$$

Since the loss function is an even function of frequency, all odd-order moments are equal to zero; the second moment is the f-sum rule

$$C_2=\omega_p^2 \tag{2.4}$$

(ω_p is the plasma frequency); high-order even moments C_ν, $\nu>4$, diverge [3].

Since we deal here with the model system, there is no need to include the internal transverse electromagnetic field into its Hamiltonian and there is no high-frequency compensation of the Coulomb contribution to the fourth moment due to the magnetic interaction of charges [7]. Thus, the TCP fourth moment can be put as

$$C_4=\omega_p^4[1+Q(k)] \ . \tag{2.5}$$

In contrast to the OCP case, the TCP correction $Q(k)$ consists of three terms,

$$Q(k)=K(k)+L(k)+H \ , \tag{2.6}$$

where

$$K(k)=3(k/k_D)^2+\sqrt{\pi/18}(\lambda_T^3k^2/\lambda_L)+\lambda_T^2k^4k_D^2 \quad (2.7)$$

is the kinetic contribution involving quantal corrections; $k_D^2=k_{De}^2=k_{Di}^2=4\pi ne^2\beta$, $\lambda_T=(\hbar^2\beta/2m)^{1/2}$, $\lambda_L=3/2e^2\beta$, $n=n_e=n_i$ is the number density of charged particles, the indices e and i stand for electrons and ions, the hydrogenlike model with $n_e=n_i$ is considered, and m is the electron mass.

The contribution

$$H=\tfrac{1}{3}h_{ei}(0)=\frac{1}{6\pi^2n}\int_0^\infty p^2S_{ei}(p)dp \quad (2.8)$$

is due to electron-ion Coulomb correlations, and

$$L(k)=\frac{1}{3\pi^2n}\int_0^\infty p^2[S_{ee}(p)-1]f(p,k)dp \quad (2.9)$$

takes into account the electronic correlations,

$$f(p,k)=\frac{5}{8}-\frac{3p^2}{8k^2}+\frac{3(k^2-p^2)^2}{16pk^3}\ln\left|\frac{p+k}{p-k}\right| . \quad (2.10)$$

In Eqs. (2.8) and (2.9)

$$h_{ab}(r)=\frac{1}{\sqrt{n_an_b}}\int[S_{ab}(p)-\delta_{ab}]\exp(i\mathbf{p}\cdot\mathbf{r})\frac{d\mathbf{p}}{(2\pi)^3} \quad (2.11)$$

$(a,b=e,i)$ are the correlation functions, and

$$S_{ab}(p)=\langle n_{\mathbf{p}}^a n_{-\mathbf{p}}^b\rangle$$

are the static structure factors, where $n_{\mathbf{p}}^a$ is the a-species (dimensionless) occupation number operator of the states with momentum $\hbar\mathbf{p}$.

The Nevanlinna formula of the classical theory of moments [8] expresses the response function

$$\epsilon^{-1}(k,z)=1+\frac{\omega_p^2(z+q)}{z(z^2-\omega_2^2)+q(z^2-\omega_1^2)} \quad (2.12)$$

in terms of an R function $q=q(k,z)$, analytic in the upper half-plane $\mathrm{Im}z>0$ and possessing there a positive imaginary part: $\mathrm{Im}q(k,\omega+i\eta)>0$, $\eta>0$; it also should satisfy the limiting condition: $(q(k,z)/z)\to 0$, as $z\to\infty$ within the sector $\vartheta<\arg(z)<\pi-\vartheta$ $(0<\vartheta<\pi)$. The frequencies $\omega_1(k)$ and $\omega_2(k)$ in Eq. (2.12) are defined as respective ratios of the moments C_ν,

$$\begin{aligned}\omega_1^2&=C_2/C_0=\omega_p^2[1-\epsilon^{-1}(k,0)]^{-1} ,\\ \omega_2^2&=C_4/C_2=\omega_p^2[1+Q(k)] .\end{aligned} \quad (2.13)$$

There is, obviously, no phenomenological basis for the choice of an unique $q(k,z)$, which would provide an exact expression for $\epsilon^{-1}(k,\omega)$. Nevertheless, to meet our goals it is sufficient to approximate $q(k,z)$ by its static value $q(k,0)=ih(k)$, connected to the static value $S_{cc}(k,0)$ of the dynamic structure factor through Eq. (2.1),

$$h(k)=(\omega_2^2-\omega_1^2)\omega_p^2[\pi\beta\Phi(k)\omega_1^4S_{cc}(k,0)]^{-1} , \quad (2.14)$$

so that the normalized dynamic factor

$$\frac{S_{cc}(k,\omega)}{S_{cc}(k,0)}=\frac{\beta\hbar}{[1-\exp(-\beta\hbar\omega)]}\times\frac{\omega h^2\omega_1^4}{[\omega^2(\omega^2-\omega_2^2)^2+h^2(\omega^2-\omega_1^2)^2]} . \quad (2.15)$$

In the quantal TCP the static structure factor

$$S_{cc}(k)=(2n)^{-1}\int_{-\infty}^{\infty}S_{cc}(k,\omega)d\omega \quad (2.16)$$

is no longer directly determined by the static screening function

$$\epsilon^{-1}(k)=\mathrm{Re}\epsilon^{-1}(k,0)=\epsilon^{-1}(k,0) \quad (2.17)$$

like it was in the OCP,

$$S_{cc}^{\mathrm{OCP}}(k)=\frac{k^2}{k_D^2}[\epsilon_{\mathrm{OCP}}^{-1}(k)-1] , \quad (2.18)$$

and $\omega_1(k)$ and $h(k)$ should be calculated separately along with $\omega_2(k)$.

Finally, notice that the expression (2.12) with $q(k,z)$ substituted by $ih(k)$ [Eq. (2.14)] interpolates between $S_{cc}(k,0)$ and the asymptotic expansion

$$\epsilon^{-1}(k,\omega\to\infty)\simeq 1+\frac{\omega_p^2}{\omega^2}+\frac{\omega_2^2(k)}{\omega^4}+\cdots . \quad (2.19)$$

The specification of $q(k,\omega)$ including the Perel'-Eliashberg asymptotic value for $\epsilon(k,\omega\gg\beta^{-1}\hbar^{-1})$ [9] is discussed in Refs. [3,7].

III. CALCULATION OF THE TCP STATIC CHARACTERISTICS

As was already mentioned, the system under consideration is a hydrogenlike plasma with a partly degenerate electron subsystem. One can go beyond the random-phase approximation (RPA) to include the local-field corrections (LFC's) by putting

$$\epsilon(k)=1+\frac{\Phi\Pi_e^0(k)}{1-\Phi G_e(k)\Pi_e^0(k)}+\frac{k_D^2/k^2}{1-k_D^2G_i(k)/k^2} . \quad (3.1)$$

Here $\Pi_e^0(k)$ is the RPA static polarization operator of electrons [10], and $\Pi_i^0(k)$ is substituted by its classical (nondegenerate) value (βn_i); $G_a(k)$ are the static local-field corrections (SLFC's).

There are various (but still not applicable under arbitrary thermodynamic conditions) forms of $G_e(k)$ [11]. Here, in order to satisfy both long- and short-range limiting conditions, the electronic SLFC is cast in the Geldart-Vosko-form [12]

$$G_e(k)=k^2[\alpha k_s^2+\beta k^2]^{-1} . \quad (3.2)$$

In particular,

$$k_s=[\tfrac{1}{2}k_{\mathrm{TF}}^2\Theta^{1/2}F_{-1/2}(\eta)]^{1/2} \quad (3.3)$$

is the inverse screening length of the RPA electronic OCP (EOCP),

$$\epsilon_e^{\mathrm{RPA}}(k)=1+k_s^2/k^2 , \quad (3.4)$$

where $k_{\mathrm{TF}}=\sqrt{3}\omega_p m/\hbar k_f$, and

$$F_\nu(\eta)=\int_0^\infty \frac{x^\nu dx}{\exp(x-\eta)+1}$$

is the Fermi integral. The chemical potential (η/β) is to be calculated from the normalization condition

$$F_{1/2}(\eta)=\tfrac{3}{2}\Theta^{-3/2} \tag{3.5}$$

for the Fermi-Dirac distribution. The parameter α can be determined from the compressibility sum rule for the classical EOCP [12,13]

$$\alpha=\left\{-\frac{k_s^2}{k_D^2}[1+\tfrac{4}{9}A\Gamma+B/3+\tfrac{8}{27}C\Gamma^{-1/3}+\tfrac{11}{27}D\Gamma^{1/3}]+1\right\}^{-1}, \tag{3.6}$$

where $A=0.899\,374\,9$, $B=-0.224\,469\,9$, $C=-0.017\,874\,6$, $D=0.517\,575\,3$ are the parameters of the MD-fitted OCP equation of state [14], and $\not{\alpha}$ stems from the self-consistency condition for $G_e(k\to\infty)$,

$$\lim_{k\to\infty} G_e(k)=1-g_e(0)\ ,\quad \text{i.e.,}\quad \not{\alpha}=[1-g_e(0)]^{-1}\ , \tag{3.7}$$

and $g_e(0)$ is the EOCP radial distribution function,

$$g_e(0)=1+(2\pi^2 n)^{-1}\int_0^\infty p^2[S_e(p)-1]dp\ , \tag{3.8}$$

with $S_e(k)$ being the EOCP static structure factor.

The ionic SLFC and the static structure factor follow from the algorithm of Ichimaru *et al.* [11]:

$$G_i(k)=k^2[\alpha k_s^2+\not{\alpha}k^2\epsilon_e(k)]^{-1}\ , \tag{3.9}$$

$$S_{ii}(k)=\{1+(k_D^2/k^2)[\epsilon_e^{-1}(k)-G_i(k)]\}^{-1}\ , \tag{3.10}$$

$$S_{ei}(k)=[1-\epsilon_e^{-1}(k)]S_{ii}(k)\ , \tag{3.11}$$

$$S_{ee}(k)=S_e(k)-[1-\epsilon_e^{-1}(k)]S_{ei}(k)\ , \tag{3.12}$$

where

$$\epsilon_e(k)=1+\Phi\Pi_e^0(k)[1-\Phi G_e(k)\Pi_e^0(k)]^{-1} \tag{3.13}$$

is the EOCP static dielectric function.

Now, to compute the frequencies $\omega_1(k)$ and $\omega_2(k)$ according to Eqs. (2.13), (2.6), (2.7), (2.8), and (2.9) it is sufficient to determine the static structure factor of the quantal EOCP and to carry out the self-consistency procedure over $g_e(0)$. Keep in mind that the MD results for the static value of the dynamic structure factor $S_{cc}(k,0)$ can be used for the determination of $S_{cc}(k,\omega)$ according to Eqs. (2.14) and (2.15).

IV. THE EOCP STRUCTURE FACTOR

To take into account the quantal corrections, one can calculate the EOCP structure factor $S_e(k)$ using the Green's-function method,

$$S_e=\frac{1}{\beta n}\sum_{l=-\infty}^{\infty}\frac{\Pi_e(k,l)}{1+\Phi\Pi_e(k,l)}\ , \tag{4.1}$$

where

$$\Pi_e(k,l)=\Pi_e^0(k,l)[1-\Phi G_e(k)\Pi_e^0(k,l)]^{-1}\ , \tag{4.2}$$

$$\Pi_e^0(k,l)=\frac{mk_F^2}{k\pi^2\hbar^2}\int_0^\infty y\,\frac{dy}{\exp(\Theta^{-1}y^2-\eta)+1}\times\ln\left|\frac{z+y+iv_l}{z-y+iv_l}\right|\ , \tag{4.3}$$

and the frequency dependence of the local-field correction is neglected. In Eq. (4.3) $k_F=(3\pi^2 n)^{1/3}$ is the Fermi wave number, $z=k/2k_F$, $v_l=2\pi lm(\beta\hbar^2 k k_F)^{-1}$, and, in what follows, $\Pi_e^0(k)=\Pi_e^0(k,0)$.

Direct application of Eq. (4.1) is complicated by the fact that the main summands of the $l\to\infty$ expansion of $\Pi_e^0(k,l)$ behave like

$$\Pi_e^0(k,l\to\infty)\simeq\frac{\alpha}{l^2+l_0^2}\ , \tag{4.4}$$

with $\alpha=(\beta/\pi)^2 nE_k$, $E_k=\hbar^2k^2/2m$,

$$l_0=(\beta E_k/2\pi)\left[1+\frac{3}{2}\frac{\Theta^{5/2}}{z^2}F_{3/2}(\eta)\right]^{1/2}.$$

To improve the convergence of Eq. (4.1), one can rewrite it as

$$S_e(k)=\frac{\pi\alpha}{\beta n l_2}\coth(\pi l_2)+\frac{1}{\beta n}\sum_{l=-l_1}^{l_1}\left\{\frac{\Pi_e(k,l)}{1+\Phi\Pi_e(k,l)}-\frac{\alpha}{l^2+l_2^2}\right\}, \tag{4.5}$$

where

$$l_2=\{l_0^2+(\hbar\beta\omega_p/2\pi)^2[1-G_e(k)]\}^{1/2}\ ,$$

and the number l_1 of summands in Eq. (4.5) is determined by the precision of the self-consistency procedure [over $g_e(0)$] and the computation itself.

The results for the parameter $g_e(0)$ for various values of Γ and Θ are given in Table I. In contrast to the RPA results, our $g_e(0)$ is always positive.

V. DISCUSSION OF RESULTS

The results for all static structure factors are presented in Tables II and III. Since no adjustable parameters were used, an agreement with the results of Hansen and McDonald [2] confirms the applicability of our algorithm to the computation of static characteristics of strongly coupled hydrogenlike two-component plasmas.

The "molecular-dynamics" simulations of Ref. [2] were performed for a model hot Boltzmann plasma. Quantum effects were taken into account only through the use of $\hbar$-corrected effective pair potentials [15]; at short distances these differ significantly from the bare Coulomb potential. Thus the collapse characteristic of purely classical systems of particles of opposite charge was prevented [2].

Within our quantum-statistical *ab initio* approach there was no need to care for the collapse prevention and

TABLE I. Results of self-consistent computation of the EOCP $g_e(0)$ according to Eqs. (3.8) and (4.1); the thermodynamic conditions of Ref. [2] are marked by the asterisk. Numbers in brackets indicate the power of 10 by which the entry is to be multiplied.

Γ	Θ	T (K)	n_e (cm^{-3})	$g_e(0)$
0.100	0.1000	0.1715[9]	0.2579[30]	0.73242
0.100	2.0000	0.8573[7]	0.3224[26]	0.41309
0.500	0.4344*	0.1579[7]	0.2517[26]	0.38457
0.500	1.0860*	0.6315[6]	0.1611[25]	0.25996
1.000	0.1000	0.1715[7]	0.2579[27]	0.67383
1.000	1.0000	0.1715[6]	0.2579[24]	0.15088
2.000	0.2715*	0.1579[6]	0.1611[25]	0.28574
54.10	0.0586	0.1000[4]	0.8099[22]	0.35822
64.35	0.0518	0.8000[3]	0.6980[22]	0.37045
194.0	0.0114	0.4000[3]	0.2390[23]	0.46888

TABLE II. Results for the partial and charge-charge static structure factors (nominators) vs the corresponding MD data (denominators) for $\Gamma=0.5$, $\Theta=0.4344$ $(r=0.4)$.

$q=ka$	$\frac{S_{ii}(k)}{S_{ii}^{\mathrm{MD}}(k)}$	$\frac{S_{ie}(k)}{S_{ie}^{\mathrm{MD}}(k)}$	$\frac{S_{ee}(k)}{S_{ee}^{\mathrm{MD}}(k)}$	$\frac{S_{cc}(k)}{S_{cc}^{\mathrm{MD}}(k)}$
0.767	$\frac{0.6604}{0.5803}$	$\frac{0.4373}{0.4386}$	$\frac{0.6074}{0.6590}$	$\frac{0.1966}{0.1811}$
1.074	$\frac{0.7160}{0.6256}$	$\frac{0.3319}{0.3600}$	$\frac{0.6198}{0.7390}$	$\frac{0.3360}{0.3223}$
1.381	$\frac{0.7770}{0.6823}$	$\frac{0.2484}{0.2813}$	$\frac{0.6572}{0.8117}$	$\frac{0.4687}{0.4657}$
1.534	$\frac{0.8041}{0.7117}$	$\frac{0.2147}{0.2454}$	$\frac{0.6811}{0.8424}$	$\frac{0.5272}{0.5317}$

TABLE III. Same as in Table II, but for $\Gamma=2.0$, $\Theta=0.2715$ $(r_s=1)$.

$q=ka$	$\frac{S_{ii}(k)}{S_{ii}^{\mathrm{MD}}(k)}$	$\frac{S_{ie}(k)}{S_{ie}^{\mathrm{MD}}(k)}$	$\frac{S_{ee}(k)}{S_{ee}^{\mathrm{MD}}(k)}$	$\frac{S_{cc}(k)}{S_{cc}^{\mathrm{MD}}(k)}$
0.767	$\frac{0.7206}{0.5638}$	$\frac{0.6583}{0.5716}$	$\frac{0.7790}{0.7191}$	$\frac{0.0915}{0.0598}$
1.074	$\frac{0.6697}{0.5131}$	$\frac{0.5327}{0.5000}$	$\frac{0.7372}{0.7382}$	$\frac{0.1708}{0.1257}$
1.381	$\frac{0.6608}{0.5065}$	$\frac{0.4303}{0.4274}$	$\frac{0.7291}{0.7766}$	$\frac{0.2647}{0.2142}$
1.534	$\frac{0.6679}{0.4171}$	$\frac{0.3875}{0.3937}$	$\frac{0.7364}{0.7989}$	$\frac{0.3146}{0.2643}$

TABLE IV. Frequency moments of $S_{cc}(k,\omega)$ in units in which $\omega_p=1$, $\Gamma=0.5$, $\Theta=0.4344$ $(r_s=0.4)$. Compare $S_0(k)$ to the corresponding $S_{cc}(k)$ and $S_{cc}^{\mathrm{MD}}(k)$.

$q=ka$	$S_{-1}(k)$	$S_0(k)$	$S_1(k)$	$S_2(k)$	$S_3(k)$	$S_4(k)$
0.767	0.115	0.195	0.134	0.282	0.419	0.935
1.074	0.192	0.333	0.263	0.641	1.306	3.338
1.381	0.262	0.464	0.434	1.251	3.354	10.220
1.534	0.291	0.521	0.534	1.687	5.121	17.049

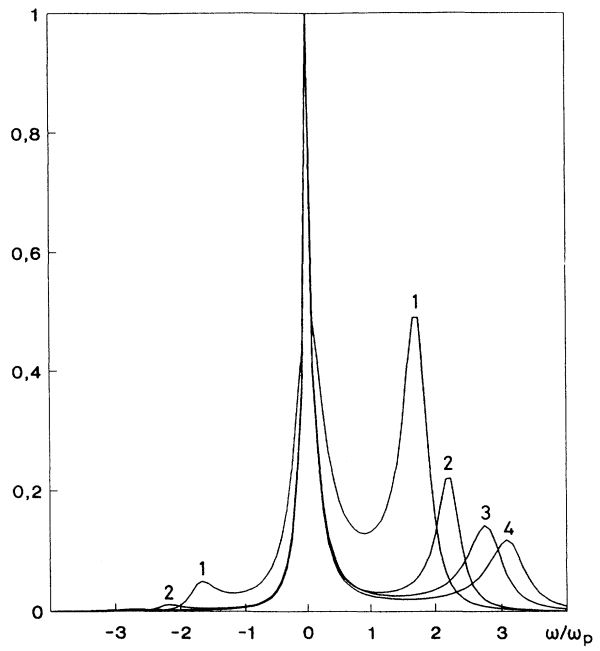

FIG. 1. Results for the normalized dynamic structure factor charge-charge $S_{cc}(k,\omega)/S_{cc}(k,0)$ of Eq. (2.15) vs ω/ω_p for $1-k=0.767/a$, $2-k=1.074/a$, $3-k=1.381/a$, $4-k=1.534/a$, and $\Gamma=0.5$, $\Theta=0.4344$ ($r_s=0.4$).

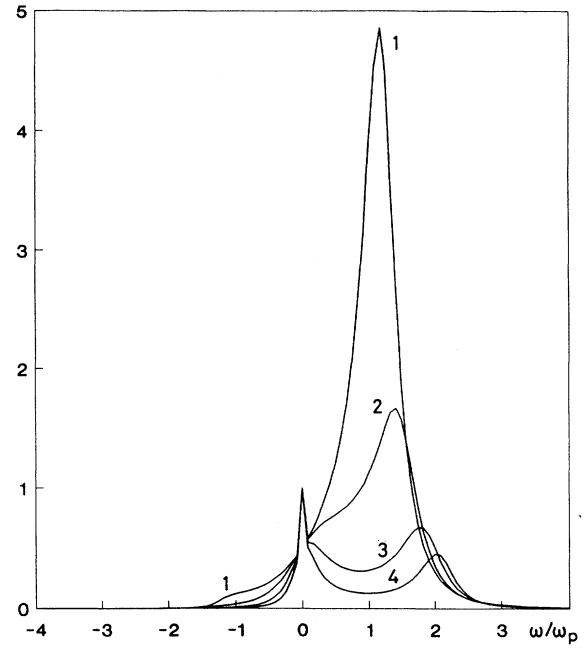

FIG. 2. Results for the normalized dynamic structure factor charge-charge $S_{cc}(k,\omega)/S_{cc}(k,0)$ of Eq. (2.15) vs ω/ω_p for $1-k=0.767/a$, $2-k=1.074/a$, $3-k=1.381/a$, $4-k=1.534/a$, and $\Gamma=2.0$, $\Theta=0.2715$ ($r_s=1.0$).

to modify the screened interaction potential $[-4\pi e^2/k^2\epsilon(k)]$ beyond the employed approximation for the LFC's. We believe that this approach could be applied to cold and dense plasmas as well.

There is a distinction between our results for the dynamic "charge-charge" structure factor (2.15) (see Figs. 1 and 2) and the corresponding MD data [2]. In particular, the high-frequency Langmuir plasma mode manifests itself in our results much stronger than in Ref. [2]: a sharp plasmon peak persists for all wave numbers considered in Ref. [2]. Low damping of the propagating mode could be attributed to our negligence of the frequency dependence of the interpolation function $q(k,\omega)$ substituted by its zero-frequency value $ih(k)$ (2.14). To test our dynamic results we calculated the dimensionless zeroth-frequency moment of our $S_{cc}(k,\omega)$,

$$S_0(k)=(2n)^{-1}\int_{-\infty}^{\infty} S_{cc}(k,\omega)d\omega \ , \tag{5.1}$$

which should be equal to the static structure factor of Eq. (2.16),

$$S(k)=[S_{ee}(k)+S_{ii}(k)-2S_{ei}(k)]/2 \tag{5.2}$$

computed independently using our "static" algorithm. One can notice from Tables II–V that the disagreement among (5.1), (5.2), and the corresponding MD quantity $S^{\mathrm{MD}}(k)$ is within numerical errors.

In addition, the frequency moments

$$S_\nu(k)=(2n)^{-1}\int_{-\infty}^{\infty}\omega^\nu S_{cc}(k,\omega)d\omega \ , \quad \nu=-1,1,2,3,4 \tag{5.3}$$

were computed using our data for the dynamic structure factor $S_{cc}(k,\omega)$ [see Tables IV and V]. The odd moments S_{-1},S_1,S_3 proved to have nonzero values. This is due to the fact that within our quantum-statistical model the dynamic structure factor $S_{cc}(k,\omega)$ possesses no parity. Within the effectively classical model, employed by Hansen and McDonald [2] to compute $S_{cc}^{\mathrm{MD}}(k,\omega)$, its odd-frequency moments are all equal to zero, and even moments are directly connected to the classical limiting values of the moments (2.2)

$$S_{2\nu}^{\mathrm{cl}}(k)=(k^2/2k_D^2)C_{2\nu}^{\mathrm{cl}}(k) \ , \quad \nu=0,1,2 \ . \tag{5.4}$$

For a quantal system one can deduce from the FDT

TABLE V. Same as in Table IV, but for $\Gamma=2.0$, $\Theta=0.2715$ ($r_s=1$).

$q=ka$	$S_{-1}(k)$	$S_0(k)$	$S_1(k)$	$S_2(k)$	$S_3(k)$	$S_4(k)$
0.767	0.082	0.084	0.085	0.122	0.183	0.327
1.074	0.157	0.159	0.167	0.268	0.459	0.888
1.381	0.249	0.248	0.276	0.516	1.023	2.207
1.534	0.299	0.294	0.340	0.697	1.496	3.403

(2.1), using the relation

$$S_{cc}(k,-\omega)=\exp(-\beta\hbar\omega)S_{cc}(k,\omega)\ , \tag{5.5}$$

an analogous expression for the odd moments,

$$S_{2\nu-1}(k)=(\hbar\beta/4)(k/k_D)^2C_{2\nu}(k)\ ,\quad \nu=0,1,2\ . \tag{5.6}$$

It follows from Eq. (2.6) that for a quantal system

$$\begin{aligned} &S_3(k)/S_1(k)=\omega_2^2(k)\ ,\\ &S_1(k)/S_{-1}(k)=\omega_1^2(k)\ . \end{aligned} \tag{5.7}$$

In the classical approximation Eqs. (5.7) are replaced by

$$\begin{aligned} &S_4^{\rm cl}(k)/S_2^{\rm cl}(k)=(\omega_2^2(k))^{\rm cl}\ ,\\ &S_2^{\rm cl}(k)/S_0^{\rm cl}(k)=(\omega_1^2(k))^{\rm cl}\ . \end{aligned} \tag{5.8}$$

Equations (5.7), but not Eqs. (5.8), are directly verified by the data of Tables IV–VI.

Thus, we believe that in quantal strongly coupled Coulomb systems the collisional damping of plasmons is sufficiently low so that the Langmuir mode persists at least until quite high values of the wave number $k\simeq a^{-1}$, and its high damping demonstrated by the MD computations was just due to the neglect of quantum-statistical effects.

We compared positions of high-frequency peaks on the graphs of Figs. 1 and 2 to the data for the frequency $\omega_2(k)$ [Table VI], and concluded that at least for the conditions considered the dispersion law of the plasma mode is very well approximated by the k dependence of $\omega_2(k)$ [Eq. (2.13)].

Finally, since higher-order frequency moments of the energy-loss function $C_{2\nu}(k)$ diverge for $\nu>2$ [9,3,5], the presented results can be improved only by a specification of the interpolation function $q(k,\omega)$ [7]. In conclusion, the present approach can also be used to calculate other dynamic properties of nonideal plasmas, like the stopping power, etc.

TABLE VI. Values of normalized frequencies $\omega_1(k)/\omega_p$ (nominators) and $\omega_2(k)/\omega_p$ (denominators) for various values of k and thermodynamics conditions. Compare $\omega_2(k)/\omega_p$ to the positions of peaks (near ω_p) of $S_{cc}(k,\omega)/S_{cc}(k,0)$ plots at Figs. 1 and 2.

	$\Gamma;\Theta$	
$q=ka$	0.5;0.4344	2.0;0.2715
0.767	1.081 / 1.735	1.016 / 1.509
1.074	1.160 / 2.092	1.031 / 1.701
1.381	1.263 / 2.524	1.052 / 1.970
1.534	1.322 / 2.764	1.066 / 2.134

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