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Total backward reflection of electromagnetic radiation due to resonant excitation of surface waves

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We consider a p-polarized electromagnetic wave incident on a semi-infinite plasma covered by a dielectric layer. The plasma density is supposed to vary periodically along the plasma surface. The coefficients for specular as well as backward reflection are calculated. We show that, for certain parameter values, the specular reflection vanishes completely because all the energy is reflected in the backward direction due to the excitation of leaking surface waves.

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I. INTRODUCTION

The resonant excitation of leaking surface waves [1] in layered structures [2], surface-wave-produced plasmas [3], open plasma wave guides [4], and fusion plasmas [5] have been considered previously in connection with studies of total absorption [6] as well as of total transparency [7] of electromagnetic radiation. In the present paper we are going to point out a hitherto unseen phenomenon, namely, total backward reflection due to scattering of the incident electromagnetic wave by a periodic plasma structure. In contrast to the absorption phenomenon [6], where the wave energy is totally converted into heat due to dissipation mechanisms, we shall show that the incident energy in the present situation can be completely converted into a backscattered energy flux and that the dissipation mechanisms are unimportant in this case. Such a peculiar mirror effect can, of course, be of experimental interest.

II. DERIVATION OF THE RESULTS

Let us consider a semi-infinite ($z>z_0$) cold plasma with periodically varying permittivity $\epsilon_p=\epsilon_{p0}+\epsilon_{p1}\cos(2k_x x)$, where ϵ_{p0} and ϵ_{p1} are constants. The plasma is bounded by a dielectric, or plasma, layer with permittivity $\epsilon_{pd}<1$ in the region $0<z<z_0$, and by vacuum at $z<0$. A p-polarized wave with magnetic field $(B_0\hat{\mathbf{y}}/2)\exp(ik_x x+ik_z z-i\omega t)$+c.c, where c.c. denotes complex conjugate, B_0 is the wave amplitude, and $k_{x,z}>0$, is supposed to be incident on this structure. The angle of incidence θ is thus given by $k_x=(\omega/c)\sin\theta$ or $k_z=(\omega/c)\cos\theta$. Generally the interaction between the incident wave and the periodic density structure in the plasma will result in the generation of new waves corresponding to the sum and difference of the wave vectors. Here we have chosen the x component of the incident wave vector such that it is equal to half the wave number of the plasma inhomogeneity. This choice of incident angle θ means that the backscatter generation mechanism is resonant and thus much more important than the corresponding interaction mechanism for other choices of θ. Furthermore, it is obvious that, due to the effect of the ponderomotive force, the nonlinear interaction between the incident and the backward reflected wave will generate a density modulation in the plasma of the form assumed above.

The magnetic field in each of the three regions $z<0$, $0<z<z_0$, and $z>z_0$ is written in the form $\mathbf{B}=(\hat{\mathbf{y}}/2)B(x,z)\exp(-i\omega t)$+c.c. In the vacuum region it satisfies the equation

$$\partial_x^2 B+\partial_z^2 B+\frac{\omega^2}{c^2}B=0\ , \tag{1}$$

which has solutions

$$B=B_0 e^{ik_x x+ik_z z}+B_0(R_s e^{ik_x x}+R_b e^{-ik_x x})e^{-ik_z z}\ , \tag{2}$$

where R_s and R_b are constants which represent the specularly and backward reflected waves, respectively. Similarly, in the region $0<z<z_0$ we solve the equation

$$\partial_x^2 B+\partial_z^2 B+\frac{\omega^2\epsilon_{pd}}{c^2}B=0 \tag{3}$$

and write the solution in the form

$$B=\frac{B_0}{2}\left[1-\frac{ik_z\epsilon_{pd}}{\kappa_d}\right][e^{ik_x x}(e^{-\kappa_d z}+re^{\kappa_d z})+(R_s e^{ik_x x}+R_b e^{-ik_x x})\times(e^{\kappa_d z}+re^{-\kappa_d z})]\ , \tag{4}$$

where $\kappa_d=(\omega/c)(\sin^2\theta-\epsilon_{pd})^{1/2}$, and where $r=(1+ik_z\epsilon_{pd}/\kappa_d)/(1-ik_z\epsilon_{pd}/\kappa_d)$ represents the reflectivity at $z=0$. We note that the boundary conditions, namely, the continuity of B and $(1/\epsilon)\partial_z B$, are satisfied at $z=0$. The same boundary conditions shall, of course, be used below at $z=z_0$.

Inside the plasma, $z>z_0$, we have to solve the equation [1]

$$\partial_z\left[\frac{1}{\epsilon_p}\partial_z B\right]+\partial_x\left[\frac{1}{\epsilon_p}\partial_x B\right]+\frac{\omega^2}{c^2}B=0\ . \tag{5}$$

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Inserting the expression

$$B = b_+(z)e^{ik_x x} + b_-(z)e^{-ik_x x} \tag{6}$$

into (5) and assuming that ϵ_{p1} is much smaller than ϵ_{p0} we obtain the two coupled equations

$$\partial_z^2 b_\pm - \left[\kappa_p^2 - \frac{\epsilon_{p1}^2\omega^2}{c^2\epsilon_{p0}}\right] b_\pm = -\frac{\epsilon_{p1}}{2\epsilon_{p0}}[\partial_z^2 b_\mp - (k_x^2 + 2\kappa_p^2) b_\mp] \tag{7}$$

with the boundary conditions

$$(b_+)_{z=z_0} = \tilde{B}_0[r + R_s + (1 + rR_s)e^{-2\kappa_d z_0}] , \tag{8}$$

$$(b_-)_{z=z_0} = \tilde{B}_0 R_b (1 + re^{-2\kappa_d z_0}) , \tag{9}$$

$$(\partial_z b_+)_{z=z_0} = \tilde{B}_0 \frac{\kappa_d}{\epsilon_{pd}} \left[\epsilon_{p0}[r + R_s - (1 + rR_s)e^{-2\kappa_d z_0}] + \frac{\epsilon_{p1}}{2} R_b (1 - re^{-2\kappa_d z_0})\right] , \tag{10}$$

and

$$(\partial_z b_-)_{z=z_0} = \tilde{B}_0 \frac{\kappa_d}{\epsilon_{pd}} \left[\epsilon_{p0} R_b (1 - re^{-2\kappa_d z_0}) + \frac{\epsilon_{p1}}{2}[r + R_s - (1 + rR_s)e^{-2\kappa_d z_0}]\right] , \tag{11}$$

where $\kappa_p = (\omega/c)(\sin^2\theta - \epsilon_{p0})^{1/2}$ and $\tilde{B}_0 = (B_0/2)(1 - ik_z\epsilon_{pd}/\kappa_d)\exp(\kappa_d z_0)$. The solution of (7) can be found by expanding $b_\pm$ in powers of the small parameter ϵ_{p1}. After lengthy but straightforward calculations we obtain the solution

$$b_\pm \approx C_\pm e^{-\kappa_p(z - z_0)} - \frac{\epsilon_{p1}(k_x^2 + \kappa_p^2)z}{4\kappa_p\epsilon_{p0}} C_\mp e^{-\kappa_p(z - z_0)} , \tag{12}$$

where $C_\pm$, as well as R_b and R_s, are complex constants that can be found from (8)–(11). As $\epsilon_{p1}/\epsilon_{p0} \ll 1$, it appears that only weakly damped leaking surface waves are of interest here. Thus we shall from now on focus our interest on the regime $\exp(-\kappa_p z_0) \ll 1$. We then obtain the simple approximate solution

$$R_s \approx -r\frac{|D|^2}{D^2}\left[1 + \frac{\epsilon_{p1}^2\omega^4(1 - r^2)e^{-2\kappa_d z_0}}{8\kappa_p^4 c^4 r|D|^2 D}\right] \tag{13}$$

and

$$R_b \approx \epsilon_{p1}\frac{\omega^2}{2\kappa_p^2 c^2}\frac{(1 - r^2)}{D^2}e^{-2\kappa_d z_0} , \tag{14}$$

where

$$D \approx 1 + \frac{\kappa_d\epsilon_{p0}}{\kappa_p\epsilon_{pd}} + 2re^{-2\kappa_d z_0} . \tag{15}$$

In the absence of plasma density modulations, i.e., if $\epsilon_{p1} = 0$, we recover, from (13), the well-known formula $R_s^{(0)} = -r|D|^2/D^2$, i.e., $|R_s^{(0)}| = 1$. Obviously $R_b^{(0)} = 0$ for this case. However, from (13) and (14), we also find the interesting result that $R_s \approx 0$ and $R_b \approx ir$ if the real part of the dielectric function (15) is equal to zero, i.e.,

$$1 + \frac{\kappa_d\epsilon_{p0}}{\kappa_p\epsilon_{pd}} + \frac{2(1 - k_z^2\epsilon_{pd}^2/\kappa_d^2)}{(1 + k_z^2\epsilon_{pd}^2/\kappa_d^2)}e^{-2\kappa_d z_0} \approx 0 \tag{16}$$

and if

$$\frac{|\epsilon_{p1}|\omega^2}{8\kappa_p^2 c^2} = \frac{k_z\epsilon_{pd}}{\kappa_d(1 + k_z^2\epsilon_{pd}^2/\kappa_d^2)}e^{-2\kappa_d z_0} . \tag{17}$$

When $R_s \approx 0$ it also follows that $|R_b| \approx 1$. Equation (16) means that the incident wave resonantly excites a leaking plasma surface wave, in which most of the energy is concentrated near the plasma-dielectric interface. Equation (17) then defines the condition for the backward energy flux to be equal to the incident flux.

Finally, we remind the reader that in order to be able to study the present phenomenon it is necessary to consider configurations with suitable density profiles so that weakly damped leaking waves can exist [1]. In the absence of the boundary region (i.e., if $z_0 = 0$ or if $\epsilon_{pd} \to 1$) there are no such waves. In our simple model that includes a layer with permittivity $\epsilon_{pd} = 1 - \omega_{pd}^2/\omega^2 < 1$, it is possible to investigate leaking waves, however [1]. Similar studies can naturally be performed for other bounded plasmas but the mathematics would then be much more involved.

III. CONCLUSIONS

In the present Brief Report we have shown that specular reflection is absent ($R_s = 0$) if the frequency of the incident wave satisfies the dispersion relation (16), and if the width z_0 of the dielectric layer is connected to the amplitude ϵ_{p1} of the plasma permittivity modulation by means of the relation (17). All the incident wave energy is then reflected in the backward direction. This peculiar effect could be of use in mirror constructions. Stimulated Brillouin scattering phenomena in the ionosphere [8] may also be influenced by the present mechanism.

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