

Electrostatic oscillations in the presence of grain-charge perturbations in dusty plasmas

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The properties of electrostatic oscillations and instability phenomena are studied, accounting for the time-dependent variation of the grain electric charge due to wave motions in an unmagnetized dusty plasma. It is found that charge fluctuations on the dust grains give rise to two purely damped modes, in addition to causing a collisionless damping of the existing normal modes. Furthermore, some interesting electrostatic instabilities arise in the presence of an equilibrium drift of charged fluids.

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I. INTRODUCTION

Recently, the physics of dusty plasmas is receiving a great deal of attention [1–3]. Dusty plasmas are characterized as a low-temperature multispecies ionized gas comprising electrons, protons, and negatively (or positively) charged grains of micrometer or submicrometer size. In fact, the dust particles collect electrons and acquire an electric charge, which can be thousands of electron charges. The negative charging of the grains could be due to field emission, plasma attachments, plasma currents, radiation field, etc. The importance of dusty plasmas has been recognized [1–3] in the study of space environments, such as asteroid zones, planetary rings, cometary tails, interstellar clouds, and Earth's noctilucent clouds. Dusty plasmas are also found [4–7] in low-temperature radio-frequency and direct-current glow discharges, the plasma-aided manufacturing of semiconductors, as well as near solid objects such as artificial satellites and the container-wall region of magnetically confined fusion plasmas.

The dust particles in a plasma generally acquire a negative charge determined by the capacity of the grain and the electron and ion thermal current balance to the grain. When an equilibrium charge has been attained by the dust grains, the plasma with the negatively charged dust grains may be regarded as simply a multispecies plasma for processes with a time scale shorter than the characteristic grain-charging time. Many of the interesting investigations pertaining to the dusty plasma fall in this category. However, an important distinctive feature of a dusty plasma is the grain-charge fluctuations arising due to the wave-motion-induced oscillations in the plasma currents that flow to the grains. Clearly, the dust electric charge is a time-dependent quantity and one has to treat it as a dynamical variable in the plasma.

The study of collective effects in dusty plasmas is of significant interest. Charged dust grains are found to modify or even dominate wave propagation [8–12], wave scattering [13–16], wave instability [17], self-similar plasma expansion [18], velocity modulation [19], charged-particle transport [20], and ion trapping [21]. However, most of the studies on wave motions [8–19] in dusty plasmas assume constant charge on the dust grains. Since the

dust particles can exhibit charge fluctuations due to wave motions, it is physically relevant to investigate the effect of charge fluctuations on electrostatic oscillations and instabilities in dusty plasmas. Indeed, one would expect a host of interesting effects to follow from the inclusion of the grain-charge evolution.

In this paper, we study the properties of electrostatic oscillations and instability phenomena, taking into account the temporal evolution of the grain charge in an unmagnetized dusty plasma. It is found that the grain-charge perturbation gives rise to two purely damped modes, in addition to causing a collisionless damping of the existing normal modes. Some interesting electrostatic instabilities appear when the equilibrium streaming of charged fluids is included in combination with the grain-charge perturbation.

Our manuscript is organized in the following fashion. In Sec. II, we discuss our three-component dusty-plasma model and present the particle-number-density perturbations for different frequency regimes. The first-order total dust charge density in Poisson's equation contains parts associated with the dust-number-density and the dust-charge-number perturbations. The latter are determined from the dust charge evolution equation that accounts for the oscillations in the plasma currents owing to wave motions. Following standard procedures, we derive and analyze the three dispersion relations. Furthermore, the latter are also generalized to include the equilibrium drifts of charged fluids. The two-stream instabilities in the presence of the grain-charge perturbation are examined. In Sec. III, we briefly summarize our findings and discuss possible applications of our investigation to wave activity in laboratory as well as space and astrophysical plasmas.

II. WAVE SPECTRA AND INSTABILITIES

We consider a multicomponent unmagnetized dusty plasma whose constituents are electrons, singly charged positive ions, and negatively charged dust grains. The latter is regarded as just another component of the plasma, and all the plasma particles are assumed to be point charges. The quasineutrality condition at equilibrium is thus given by

$$n_{e0} + Z_{d0}n_{d0} - n_{i0} = 0, \quad (1)$$

where n_{j0} is the equilibrium number density of the particle species j (equals e for electrons, i for ions, and d for grains), and Z_{d0} is the unperturbed charge number on the grain. Although the present analysis considers the ion species as protons, it could easily be generalized for any relevant multiply charged ion species.

The dusty-plasma model used here is valid if the grain radius (a) as well as the average intergrain spacing [$d = (3/4\pi n_{d0})^{1/3}$] are much smaller than the plasma Debye length λ_D . This model is appropriate in interstellar clouds, cometary tails, planetary rings, as well as noctilucent clouds at an altitude of about 80 km in the Earth's lower ionosphere. Note that for $a \ll d \ll \lambda_D$, the charged dust particulates can be considered as massive point particles similar to multiply charged negative (or positive) ions in multispecies plasmas. On the other hand, for $d < \lambda_D$, the effect of neighboring particles can be significant, whereas for $d \gg \lambda_D \gg a$, the grains are completely isolated from its neighbors.

The equilibrium is disturbed in the presence of electrostatic oscillations with the electric field $\mathbf{E} = -\nabla\phi$, where ϕ is the corresponding wave potential. The wavelengths of oscillations are assumed to be much larger than the Debye length of the plasma particles and the intergrain distance. Furthermore, we suppose that the time scales of the phenomena under consideration are much shorter than the attachment rate of the charged particles to the grains. The large negative charge that a dust particle acquires also offers the possibility of the plasma (positive) ions being trapped by the electrostatic field of the dust. It is like the recombination process in a partially ionized plasma. Accordingly, the consideration of attachment and trapping of charged particles in dusty plasmas would cause a damping similar to that of recombinational damping in weakly ionized gases.

In order to study collective wave motions in plasmas, one usually requires the plasma response to wave fields. In the following, we present the number-density perturbations of the various plasma components on different time scales.

We start with electrostatic waves with $|\omega - \mathbf{k} \cdot \mathbf{v}_{e0}| \gg v_{te} = (T_e/m_e)^{1/2}$. Here, the inertial force is balanced by the electric force and the linearized continuity and momentum equations give the electron-number-density perturbation

$$n_{e1} = -n_{e0}ek^2\phi/m_e(\omega - \mathbf{k} \cdot \mathbf{v}_{e0})^2, \quad (2)$$

where ω is the wave frequency, $\mathbf{k}$ is the wave vector, $\mathbf{v}_{e0}$ is the equilibrium electron drift velocity, T_e (m_e) is the electron charge (mass), and e is the magnitude of the electron charge.

On the other hand, when $|\omega - \mathbf{k} \cdot \mathbf{v}_{e0}| \ll v_{te}$, the electron inertial force is negligible in comparison with the pressure-gradient force. The electron-number-density perturbation is then obtained from the balance of the pressure gradient and the electric forces, yielding the Boltzmann relation

$$n_{e1} = n_{e0}e\phi/T_e, \quad (3)$$

where we have assumed isothermal compression for the electron fluid.

In the presence of an equilibrium drift, the ion-number-density perturbation can be derived from the linearized Vlasov equation. The result is

$$n_{i1}(\omega, \mathbf{k}) = -\frac{k^2}{4\pi e}\chi_i\phi, \quad (4)$$

where

$$\chi_i = \frac{1}{k^2\lambda_{Di}^2} \left[1 + \frac{\omega}{kv_{ti}} Z \left(\frac{\omega - \mathbf{k} \cdot \mathbf{v}_{i0}}{kv_{ti}} \right) \right] \quad (5)$$

is the ion susceptibility, and the ion Debye length is denoted by $\lambda_{Di} = (T_i/4\pi n_{i0}e^2)^{1/2}$. Furthermore, $v_{ti} = (T_i/m_i)^{1/2}$ is the ion thermal velocity, T_i (m_i) is the ion temperature (mass), $\mathbf{v}_{i0}$ is the equilibrium ion drift velocity, and Z is the standard plasma dispersion function.

For $|\omega - \mathbf{k} \cdot \mathbf{v}_{i0}| \ll kv_{ti}$, Eq. (4) yields the Boltzmann distribution

$$n_{i1} \approx -n_{i0} \frac{e\phi}{T_i}. \quad (6)$$

On the other hand, in the opposite limit, viz., $|\omega - \mathbf{k} \cdot \mathbf{v}_{i0}| \gg kv_{ti}$, Eq. (4) gives

$$n_{i1} \approx \frac{en_{i0}k^2\phi}{m_i[(\omega - \mathbf{k} \cdot \mathbf{v}_{i0})^2 - 3k^2v_{ti}^2]}. \quad (7)$$

The number-density perturbation of negatively charged cold dust fluid is determined from the linearized continuity and momentum equations, which read, respectively,

$$(\partial_t + \mathbf{v}_{d0} \cdot \nabla)n_{d1} + n_{d0}\nabla \cdot \mathbf{v}_{d1} = 0 \quad (8)$$

and

$$(\partial_t + \mathbf{v}_{d0} \cdot \nabla)\mathbf{v}_{d1} = (Q_{d0}/m_d)\nabla\phi, \quad (9)$$

where n_{d1} is the number-density perturbation of the dust fluid, $\mathbf{v}_{d1}$ is the dust velocity perturbation in the equilibrium dust drift speed $\mathbf{v}_{d0}$, $Q_{d0} = Z_{d0}e$, and m_d is the mass of the dust particles.

The equations are closed with the help of Poisson's equation

$$\nabla^2\phi = 4\pi(en_{e1} - en_{i1} + Q_{d0}n_{d1} + n_{d0}Q_{d1}), \quad (10)$$

where Q_{d1} is the first-order dust charge perturbation due to the wave-motion-induced oscillations in the plasma currents flowing to the grains. In fact, the time-dependent variation in the grain charge sets in when the balance between the electron and ion currents flowing to the grains is disturbed by the wave potential ϕ . The dust charge perturbation is determined from the equation [22]

$$d_t Q_{d1} = I_{e1} + I_{i1}, \quad (11)$$

ignoring the photoelectric effect. Furthermore, we observe that the first-order dust charge density in (10) consists of the terms $Q_{d0}n_{d1}$ and $n_{d0}Q_{d1}$, which represent the dust-particle-number density and the dust-charge-number perturbations, respectively. Thus, the sum of both the terms can be regarded as an effective dust charge-density

perturbation.

For negatively charged particulates, the first-order thermal electron current to the grain is given by

$$I_{e1} \approx -4\pi a^2 n_{e0} (T_e/2\pi m_e)^{1/2} [\exp(e\phi_0/T_e)] e\phi/T_e, \quad (12)$$

where the dust particle is assumed to be spherical with a geometrical radius a , and $\phi_0 (=eZ_{d0}/a)$ is the equilibrium (stationary) uniform potential at the surface of the dust particle. Clearly, for a given size and the equilibrium charge number, ϕ_0 is a predetermined potential of a grain.

The first-order ion current entering the grain's sheath is

$$I_{i1} = -4\pi a^2 n_{i0} (T_i/2\pi m_i)^{1/2} (1 - e\phi_0/T_i) e\phi/T_i. \quad (13)$$

We note that the first-order oscillating electron and ion currents are directly proportional to the wave potential ϕ , and an imbalance among the two currents causes the dust charge fluctuations.

Assuming that the perturbations vary as $\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$, we obtain from (8) and (9)

$$n_{d1} = -\frac{n_{d0} Q_{d0} k^2 \phi}{m_d (\omega - \mathbf{k}\cdot\mathbf{v}_{d0})^2}. \quad (14)$$

The grain-charge perturbation, found from (11), (12), and (13), is

$$Q_{d1} = -i(\beta/\omega)\phi, \quad (15)$$

where

$$\beta = \frac{1}{\sqrt{2\pi}} \left[\left(\frac{a}{\lambda_{De}} \right)^2 v_{te} \exp(\Phi_0) + \left(\frac{a}{\lambda_{Di}} \right)^2 v_{ti} (1 - T_e \Phi_0/T_i) \right], \quad (16)$$

with $\lambda_{De} = (T_e/4\pi n_{e0} e^2)^{1/2}$ being the electron Debye length and $\Phi_0 = e\phi_0/T_e$.

The Poisson equation (10) can be expressed as

$$k^2 \phi = -4\pi (en_{e1} - en_{i1} + Q_{d0} n_{d1} + n_{d0} Q_{d1}). \quad (17)$$

Equations (2), (3), (6), (7), (14), (15), and (17) form a complete set for the study of electrostatic oscillations and instabilities in a dusty plasma with dust charge perturbations. In order to illuminate the importance of the latter, we consider some interesting limiting cases.

First, we focus on $\omega \gg kv_{ij}$, $\mathbf{k}\cdot\mathbf{v}_{e0, i0, d0}$ and obtain from (2), (7), (15), and (17)

$$\omega = -\frac{1}{2} i v_c \pm \frac{1}{2} (4\omega_p^2 - v_c^2)^{1/2}, \quad (18)$$

where $v_c = 4\pi n_{d0} \beta / k^2$ and $\omega_p = (\omega_{pe}^2 + \omega_{pi}^2 + \omega_{pd}^2)^{1/2} \approx \omega_{pe}$. We have denoted $\omega_{pe} = (4\pi n_{e0} e^2 / m_e)^{1/2}$, $\omega_{pi} = (4\pi n_{i0} e^2 / m_i)^{1/2}$, and $\omega_{pd} = (4\pi n_{d0} Q_{d0}^2 / m_d)^{1/2}$. Equation (18) reveals that for $\omega_p^2 \ll v_c^2$, which can be satisfied for long-wavelength ($k \rightarrow 0$) modes in a tenuous plasma, we have a purely damped mode

$$\omega \approx -i v_c. \quad (19)$$

Since the damping rate is inversely proportional to k^2 , purely damped long-wavelength dust modes would have a very short lifetime. The mode arises due to the balance of the dust-charge-number perturbation and the gradient of the space-charge potential. The latter is created because of the oscillations of electrons around the massive ions and the ion trapping around the negatively charged dust grains. Furthermore, we note that the time-dependent dust charge perturbation produces an effective dust-number-density perturbation that gets 90° out of phase with the potential oscillation. The charge fluctuations would thus damp out because of an inadequate phase lag between the dust charge density and the potential disturbance.

Furthermore, for $\omega \approx \omega_p \gg v_c$, we obtain from (18)

$$\omega \approx \omega_{pe} - i v_c, \quad (20)$$

which exhibits the damping of the well-known electron plasma waves. The collisionless damping arises because the effective dust-charge-density perturbation associated with the dust charge perturbation cannot keep in phase with the electron-number-density perturbation. Physically, the dust charge perturbation inhibits the space-charge field between the background electrons and ions. The retardation of the electron velocity vector in the electric field is a kind of drag, which is responsible for the damping of the plasma oscillations that are supported by the background electron motion around the massive ions.

Second, we consider static dust grains ($n_{d1} = 0$) and the frequency regimes $|\omega - \mathbf{k}\cdot\mathbf{v}_{e0}| \ll kv_{te}$ and $|\omega - \mathbf{k}\cdot\mathbf{v}_{i0}| \ll kv_{ti}$. Here, both the electrons and ions are electrostatically shielded and the corresponding number-density perturbations are Boltzmann type. Thus, combining (3), (6), (15), and (17), we obtain the dispersion relation

$$1 + k^2 \lambda_{De}^2 + b + i\nu/\omega = 0, \quad (21)$$

where $b = n_{i0} T_e / n_{e0} T_i$ and $\nu = 4\pi n_{d0} \beta \lambda_{De}^2$. The solution of (21) is

$$\omega = -i\nu / (1 + k^2 \lambda_{De}^2 + b), \quad (22)$$

which is another purely damped electrostatic mode in dusty plasmas. We again note that the damping rate strongly depends on β , which comes from the dust charge perturbation. It is interesting to note that for $k^2 \lambda_{De}^2 \ll 1$, the damping rate is independent of the wavelength, contrary to the first case presented above. For extremely long-wavelength modes, the sum of the electron-number-density perturbation and the dust-charge-density variation involving the dust charge perturbation is exactly balanced by the ion-number-density perturbation in order to preserve the overall quasineutrality of the plasma.

Third, we consider static grains but focus on $|\omega - \mathbf{k}\cdot\mathbf{v}_{e0}| \ll kv_{te}$ and $|\omega - \mathbf{k}\cdot\mathbf{v}_{i0}| \gg kv_{ti}$. This regime corresponds to the presence of immobile charged dust grains in a conventional electron-ion plasma with Boltzmann-distributed electrons and inertial ions. Here, for $k^2 \lambda_{De}^2 \ll 1$, the wave frequency is deduced from (3),

(7), (15), and (17), taking $n_{d1}=0$. We have

$$\omega = -i\frac{1}{2}\nu \pm \frac{1}{2}(-\nu^2 + 4\omega_{ss}^2)^{1/2}, \quad (23)$$

where $\omega_{ss} = kc_{ss} \equiv k(n_{i0}T_e/n_{e0}m_i)^{1/2}$ is the ion sound frequency [11] in dusty plasmas. For $\omega_{ss} \gg \nu$, we observe a collisionless damping of the dust acoustic wave [11] involving immobile grains in dusty plasmas. The physical mechanism of the damping is similar to the one described in the second case.

Fourth, we consider the influence of the dust charge perturbation on the extremely low-frequency ($kv_{id} \ll \omega \ll kv_{te,i}$) dust-acoustic waves, which involve Boltzmann-distributed electrons and ions, as well as mobile inertial dust fluid [9]. For this case, we use (3), (6), (14), (15), and (17) to derive the dispersion relation without the equilibrium ion and dust drifts:

$$1 + k^2\lambda_{De}^2 + b - (k^2c_{rs}^2/\omega^2) + i\nu/\omega = 0, \quad (24)$$

where $c_{rs} = \omega_{pd}\lambda_{De}$. For $k^2\lambda_{De}^2 \ll 1$, the frequency spectra are similar to (23), except that ω_{ss} is replaced by $\omega_{rs} = kc_{rs}/(1+b)^{1/2}$ and ν by $\nu_b = \nu/(1+b)$. Clearly, the dust charge perturbation again causes a collisionless damping of the dust-acoustic waves [9] involving mobile grains in dusty plasmas. The damping arises because the first-order dust-charge-density part involving the dust charge variation is out of phase with that involving the dust-number-density perturbation n_{d1} .

We now turn our attention to some interesting two-stream instabilities in the presence of the dust charge perturbation. For illustrative purposes, we present three limiting cases. First, consider the dust-acoustic instability in the presence of streaming of dust particles against Boltzmann-distributed electrons and ions. Here, instead of (24), we have

$$1 + k^2\lambda_{De}^2 + b - [k^2c_{rs}^2/(\omega - \mathbf{k} \cdot \mathbf{v}_{d0})^2] + i\nu/\omega = 0. \quad (25)$$

For $\nu_b < \omega$ and $k^2\lambda_{De}^2 \ll 1$, the approximate solution of (25) is

$$\omega \approx \mathbf{k} \cdot \mathbf{v}_{d0} \pm \omega_{rs}(1 - i\nu_b/2\omega). \quad (26)$$

The lower sign (corresponding to the negative-energy wave) has a solution with $\text{Im}\omega > 0$. Equation (26) implies that a negative-energy wave causes instability by coupling to a dissipative medium, since the amplitude of the negative-energy wave grows by dissipating its energy [23].

Next, in the presence of an equilibrium ion drift against stationary grains and Boltzmann-distributed electrons, we have for $\nu < \omega$ and $k^2\lambda_{De}^2 \ll 1$:

$$\omega \approx \mathbf{k} \cdot \mathbf{v}_{i0} \pm \omega_{ss}(1 - i\nu/2\omega), \quad (27)$$

which predicts an instability of negative-energy waves.

Finally, for inertial electrons and stationary ion and dust fluids, we have for $\nu < \omega$:

$$\omega \approx \mathbf{k} \cdot \mathbf{v}_{e0} \pm \omega_{pe}(1 - i\nu/2\omega), \quad (28)$$

which also predicts an instability of negative-energy waves in plasmas with fixed ions and charged dust particulates.

In closing, we mention that the instability analysis, in-

cluding the dynamics of ion and dust fluids with equilibrium drifts of both the species, would be based on an equation that is a fifth-order polynomial in ω . Here, one should resort to numerical studies in order to obtain the complex solutions. However, this investigation is beyond the scope of the present paper.

III. SUMMARY

In this paper, we have investigated the effect of the dust charge perturbation on electrostatic oscillations and instabilities in a uniform unmagnetized dusty plasma. The dust charge fluctuations, which arise from the wave-motion-induced oscillating plasma currents that flow to the grains, are found to introduce an additional source of dissipation in plasmas. We have found that the dust charge perturbation can give rise to two interesting purely damped modes. On the other hand, the electron plasma waves as well as the dust-acoustic waves, without and with the dust dynamics, are subjected to a collisionless damping. It appears that a dusty plasma with the dust charge perturbation behaves like a dissipative system and the damping of the modes arises because of the phase difference between the total perturbed dust charge density and the wave potential. Furthermore, accounting for the dust charge perturbation, we have also established the existence of some interesting electrostatic instabilities for three cases, in which (i) the inertial charged dust fluid streams with respect to the Boltzmann-distributed electrons and ions, or (ii) the inertial ion fluid drifts against the stationary dust grains and the Boltzmann-distributed electrons, or (iii) the inertial electron fluid streams with respect to stationary ions and charged dust particulates. The instabilities are attributed to a linear coupling between a negative-energy wave and a resistive medium [23].

Our investigation assumes that the characteristic wave frequency and damping rate of electrostatic oscillations is much larger than the attachment frequency of the electrons to the dust. When this inequality does not hold, then we must include a model loss term $-\nu_{at}(n_j - n_0)$ on the right-hand side of the continuity equation, where ν_{at} is the attachment frequency. It turns out that the attachment process would cause an additional damping of the respective density perturbations. Consequently, the plasma waves would be further damped. For example, in a plasma with $\omega \sim \omega_{pe} \gg kv_{te}, \mathbf{k} \cdot \mathbf{v}_{e0}$, $\nu_{at} \sim \nu_c$, the spectrum (18) remains the same except that ν_c is replaced by $\nu_c + \nu_{at}$. Thus, the attachment effect in dusty plasmas is similar to the recombinational damping in partially ionized plasmas.

It is of interest to note that a high level of electrostatic wave activity has been observed in laboratory [4] and space [24,25] plasmas, which contain charged particulates. Specifically, the experimental data in the vicinity of the Phobos orbit [25] exhibit the substantial increase of the electric-field spectral density within the frequency range from a few hertz to several hundred hertz. Accordingly, the results of the present investigation should be useful for understanding the existing plasma-wave spectra. Besides, it is suggested that more laboratory ex-

periments and numerical-simulation studies on dusty-plasma wave phenomena are needed to help guide and verify theory.

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