Breaking of resonantly excited electron plasma waves

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Simulations of high-amplitude electron plasma waves have been performed by solving the Vlasov equation numerically to clarify the mechanism of wave breaking in smooth density profiles. We show that wave breaking exists as a phenomenon which is distinct from incoherent hot-electron generation and which has a clear meaning in a kinetic theory: By trapping of *entire bunches* of electrons in the wave potential the latter is heavily disturbed and becomes irregular. In contrast to hot-electron generation these electron bunches are not accelerated to high velocities. An analytic criterion for wave breaking is derived which is consistent with the numerical simulations.

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I. INTRODUCTION

The excitation of large-amplitude electron plasma waves has attracted great and continuous attention since 1959, when it was shown that cold plasma oscillations break when the oscillatory velocity v_{os} equals their phase speed $v_\varphi=\omega/k$ [1]. Later on, with the help of a water-bag model this criterion was extended to warm-electron plasma waves and it was found that the electron density n_e of a free harmonic wave in a homogeneous plasma of density n_0 is limited by the inequality $n_e/n_0 \le (v_\varphi/s_e)^{1/2}$, s_e indicating the electron sound speed in the water-bag model. This is an equivalent formulation of the so-called Coffey criterion [2]. It is a straightforward procedure to extend it to a warm Maxwellian plasma of electron temperature T_e and adiabatic coefficient γ, where it reads [3]

$$\frac{n_e}{n_0} \le \left[\frac{v_\varphi}{s_e}\right]^{2/(\gamma+1)}, \quad s_e=\left[\gamma\frac{kT_e}{m_e}\right]^{1/2}. \tag{1}$$

Neglecting heat conduction, for Langmuir waves $\gamma=3$ holds and the inequality becomes formally identical with Coffey's criterion. Owing to this similarity the inequality for a thermal plasma is usually also given the name Coffey criterion or Coffey limit. When approaching this limit, the electrostatic potential of the wave and its enthalpy are growing so rapidly that a single-electron fluid element begins to be reflected from the periodic potential and "hydrodynamic wave breaking" sets in [4].

The possibility of resonant excitation of intense Langmuir waves by high power lasers in inhomogeneous plasmas (resonance absorption [5]) greatly stimulated the search for nonlinear wave phenomena and concomitant kinetic effects. On one hand, criteria for hydrodynamic wave breaking in the resonance region were derived for the cold plasma at rest [6] and flowing at constant speed v_0 [7]. The latter one reads with $v_d=e\hat{E}_d/m_e\omega$ ($\hat{E}_d$: driving field amplitude) as follows:

$$\frac{v_d}{v_0} \ge \left|\frac{1}{2}+i\frac{\pi}{2}e^{i(\pi/2)^2}\int_{-\infty}^{\pi/2}e^{-i\xi^2}d\xi\right|^{-1}=0.36\,. \tag{2}$$

Breaking in the resonance region has to be distinguished from breaking out of resonance [3]: at resonance breaking is due to overlapping of two originally distinct volume elements owing to their different phase shifts in the neighborhood of the resonance point. Due to such shifts a Langmuir wave in a cold (streaming) plasma may break, in contrast to the criterion of Ref. [1], as soon as $v_{os}>0.75v_\varphi$ holds. Here, in the resonance zone, the phase velocity is defined as $v_\varphi=\Delta/(2\pi/\omega)$ (Δ is the resonance width). Outside the narrow resonance region the Langmuir wave becomes free and, in moderately steep density gradients, the Coffey criterion in the form of Eq. (1) applies again if for the electron wave a WKB approximation holds [8].

On the other hand, valuable contributions to the kinetic aspect of high amplitude Langmuir waves have been given by extensive particle-in-cell (PIC) simulations of resonance absorption [9–12]. They undoubtedly revealed that a fraction of the electrons are trapped and accelerated to high velocities in the wave potential (hot electrons) thereby effectively damping the wave. In more recent PIC simulations the calculations were extended to extremely-high-density gradients and again, fast electron generation was found [13,14]. Meanwhile the scientific community became acquainted with the concept of wave breaking, but its meaning in kinetic theory and its relation to the fluid picture remained completely obscure till now. So far only two authors seem to have been fully aware of this situation [15]. In other works, tentative and vague characterizations of kinetic wave breaking were only given occasionally [9,16]. Thus two questions arise: (i) what is a meaningful definition of wave breaking in the kinetic theory, and (ii) what is the appropriate breaking criterion if kinetic effects are taken into account?

II. VLASOV SIMULATIONS

We recognized that both answers cannot be given by PIC calculations, owing to large noise and poor statistics, and decided therefore to solve the Vlasov-Poisson system of equations,

$$\frac{\partial f}{\partial t}+v\frac{\partial f}{\partial x}-\frac{e}{m_e}E(x,t)\frac{\partial f}{\partial v}=0 ,$$
$$\frac{\partial E}{\partial x}=\frac{e}{\varepsilon_0}\left[n_0(x)-\int_{-\infty}^{\infty}f(x,v,t)dv\right] , \tag{3}$$

with the help of a splitting scheme [8,17]. To answer both questions it is sufficient to treat the ions (density n_0/Z) as a fixed neutralizing background of inhomogeneity length $L=n_0/\partial_x n_0$. L is chosen as a free parameter. Our presentation is organized as follows. First we present numerical solutions of system (3) of equations in dependence on the driving laser field and discuss how they relate to the Coffey criterion. Then a definition of wave breaking is given and its occurrence is proved. Finally an analytic breaking criterion is derived, and the meaning of the Coffey limit in presence of electron trapping in a homogeneous warm plasma is shortly discussed.

We use the capacitor model of resonance absorption [8,18,19], which conists of representing the light wave by the oscillating field $E_a=\hat{E}_d\sin\omega t$ parallel to the density gradient of a layered plasma. This model allow both for an analytic treatment of hydrodynamic equations and for reducing the computational demand of the Vlasov simulations to a practicable amount, and it captures the essential aspects of resonance absorption [18,19]. In the following the normalized driver strength η is used:

$$\eta=\frac{E_d}{E_{\rm th}},\quad E_{\rm th}=\frac{m_e v_{\rm th}^2}{e\lambda_D},\quad \lambda_D=\left[\frac{kT_e}{m_e\omega^2}\right]^{1/2} .$$

Initially a Maxwellian distribution

$$f_0=(2\pi)^{-1/2}\exp(-x/L)\exp(-v^2/2v_{\rm th}^2)$$

is assumed ($v_{\rm th}=s_e/\sqrt{\gamma}$). Figure 1 shows the evolution of f in terms of contour lines $f=$const after 20, 25, and 40 periods for $L=300\lambda_D$ and $\eta=0.04$ with the thermal field $E_{\rm th}=kT_e/e\lambda_D$ and Debye length at resonance $\lambda_D=v_{\rm th}/\omega$. After a much higher number of cycles f looks very similar and does not show any new aspects, thus indicating that a quasistationary state is reached at the times considered here. Short reflection may convince the reader that closed loops in the contour plots refer to trapped particles. The contour lines extending to high velocities and leaning to the right are to be assigned to accelerated and detrapped particles. They are modulated by the periodic wave potential and their inclination increases as is characteristic for free streaming particles. The electron density is plotted in Fig. 2. The dashed smooth curve is the Coffey limit. Owing to strong nonlinear damping the Langmuir wave never reaches this limit, except for the first maximum, even at much higher driver strengths η. This behavior is confirmed for three other L values also in Fig. 2. To the first (resonant) density maximum the Coffey criterion does not apply. For instance, in picture (a) it is clearly higher than the Coffey limit, but there is no indication of breaking [see also the first smooth and regular maximum in Figs. 3(a) and 3(b)]. For $\eta\geq 0.04$ all maxima of n_e except the first one do not increase further, but the wave remains periodic and smooth. As a first result, we can clearly see that although fast electrons are generated by trapping, the wave is still regular and periodic in the sense mentioned above. In conclusion, fast electron generation is not indicating wave breaking.

However, above a certain threshold η^* the distribution function undergoes a *qualitative* change: the Langmuir wave becomes irregular; it breaks, as illustrated by Fig. 3 for n_e and E at $\eta=0.2$ in a density profile of scale length $L=300\lambda_D$. Inspection of the corresponding evolution of

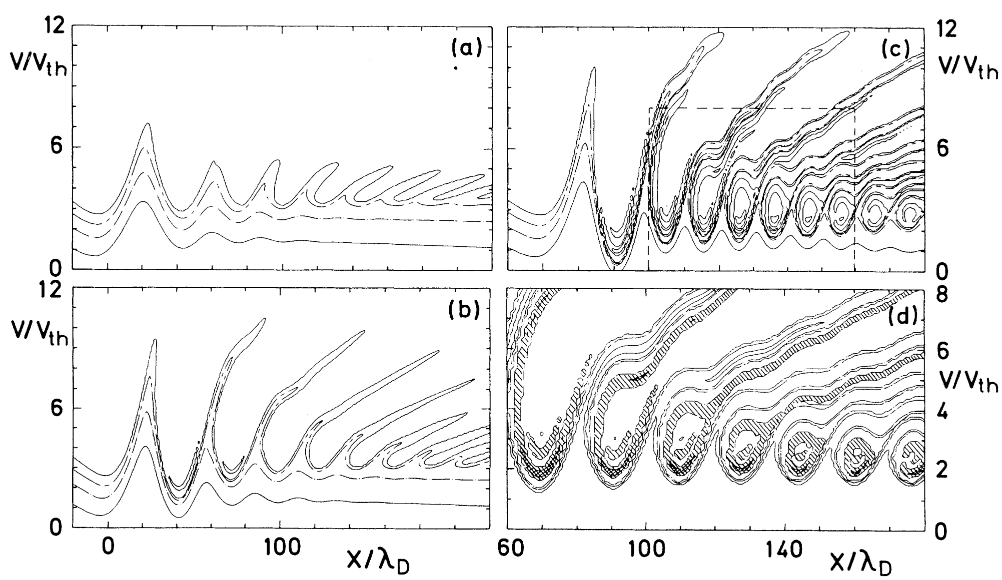

FIG. 1. Distribution function evolving from the initial distribution $f_0(x,v)=(2\pi)^{-1/2}\exp(-x/L)\exp(-v^2/2v_{\rm th}^2)$ with $L=300\lambda_D$ and $\eta=0.04$ after (a) 20, (b) 25, and (c) 40 periods; (d) enlarged plot of area indicated in (c) after 50 periods. The contour lines refer to $f=10^{-1}$, 10^{-2}, 10^{-3}, and 10^{-4}; the critical density with $\omega_p=\omega$ is located at $x=0$. Trapped electrons form closed loops (d); the velocity modulation of the detrapped electrons is due to the periodic electric field of the Langmuir wave.

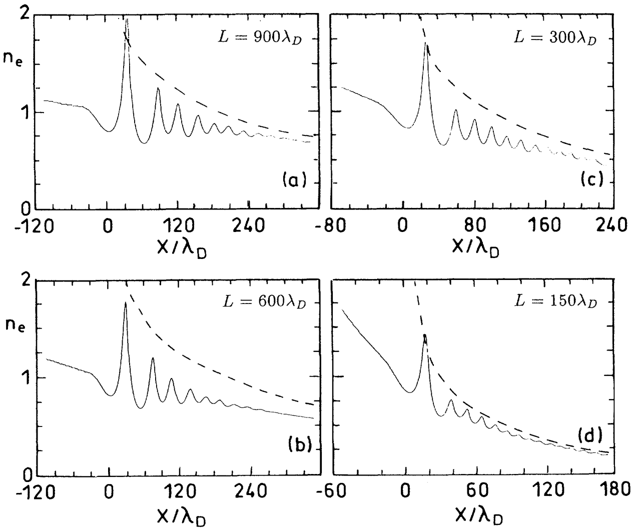

FIG. 2. Resonantly excited electron plasma waves in ion density profiles with different scale lengths L. Electron density n_e and breaking limit after Coffey (dashed curves); out of resonance this limit is never reached owing to nonlinear Landau damping. (a) $\eta = 0.06$, $\omega t = 120\pi$; (b) $\eta = 0.06$, $\omega t = 80\pi$; (c) $\eta = 0.08$, $\omega t = 100\pi$; (d) $\eta = 0.1$, $\omega t = 70\pi$.

the distribution function (cf. Fig. 4) reveals the reason for such behavior. The mean oscillatory velocity in the resonance region becomes so large that trapping of whole *bunches* of rather slow electrons occurs. These bunches of coherently moving and partly coalescing electrons remain trapped at least for several wavelengths, thus creating an additional aperiodic macroscopic electric field. From a fluid point of view the phenomenon is similar to what is called intense mixing of volume elements exhibiting different oscillation phases. (A Grassberger-Procaccia analysis of the electric field and the electron density at fixed x reveals the transition from a quasiperiodic attractor to a chaotic one [8].) The electrons in these bunches contribute to the heat conduction, so that the heat flux q increases suddenly with wave breaking. For example, at $x = 50\lambda_D$ we obtain

η	q (a.u.)
0.06	0.31
0.09	0.32
0.12	0.49

As a consequence, the following definition can be given: Wave breaking is the loss of periodicity in at least one of the macroscopically observable quantities. This definition extends on both hydrodynamic as well as kinetic descriptions. In contrast to a linear wave where an irregularity occurs in all variables simultaneously, breaking may appear to a different degree in the various quantities (e.g., n_e and E in Fig. 3). The first resonant density maximum may either satisfy Coffey's inequality or exceed this limit, even when the wave does not break (cf. Figs. 2 and 3). At this point we wish to point out for clarity that, as stated in the introduction, Coffey's criterion Eq. (1) was originally derived for the water-bag model and only later on has it become customary to use the same criterion for any kind of warm plasma without or with a hot-electron component present.

Starting from the breaking condition for the streaming cold plasma, we replace the streaming velocity by the group velocity of the plasma wave, because the latter is the speed of energy transport now (at least approximately in a nonlinear wave). From the fluid theory of resonance absorption [19] we obtain for the group velocity at the end of the resonance zone $v_s = s_e^2/v_\varphi \approx 2(L/\lambda_D)^{-1/3} v_{\text{th}}$. Inserting this into Eq. (2) at the place of v_0 yields

$$\eta > \eta^* = 0.72(L/\lambda_D)^{-1/3} . \tag{4}$$

This breaking threshold is in very good agreement with the results of the Vlasov simulations, which are displayed in Fig. 5. The reason for the validity of inequality (4) at finite electron temperature is that in the resonance region the electronic oscillatory motion v_e is mainly determined by the total electric field $E = E_d + E_{\text{wave}}$, and is only slightly affected by the much smaller force due to the electron pressure gradient [3].

Next, the threshold intensity for wave breaking is calculated. Making use of the scaling $L/\lambda_D \sim I_L^{-1/2}$ from Ref. [11], where I_L is the vacuum laser intensity, Eq. (3) translates into $I_L^* = 2 \times 10^{15}$ W/cm^2 for the Nd laser if a degree of absorption of 25% is assumed.

Finally we mention that our kinetic analysis of wave breaking of freely running Langmuir waves in a *homogeneous* plasma shows that the same phenomenon occurs here too, that the reason for breaking is again trapping of *entire bunches* of electrons. However, the limit at which the wave breaks is much higher than the Coffey

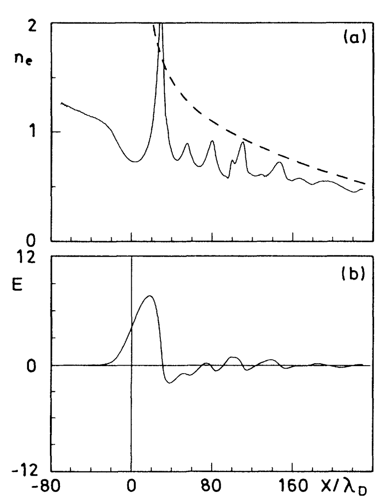

FIG. 3. Langmuir wave excited by driver of strength $\eta = 0.2$ in a plasma of scale length $L = 300\lambda_D$: the irregular shape indicates breaking. (a) Electron density n_e; (b) electric field E as a function of space; dashed curve, Coffey limit.

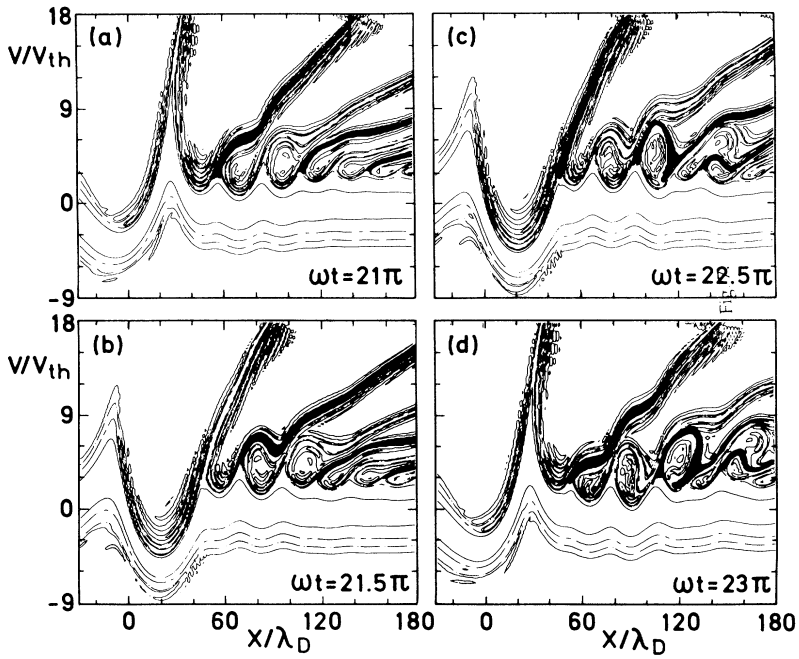

FIG. 4. Route to wave breaking. The distribution function for $L=300\lambda_D$ and $\eta=0.2$ (cf. Fig. 3) after (a) 21, (b) 21.5, (c) 22.5, and (d) 23 pèriods. Contour lines for $f=10^{-1}$, 10^{-2}, 10^{-3}, and 10^{-4}. Trapping and coalescence of entire electron *bunches* is evident (see black colored areas).

criterion would indicate. A detailed analysis of this case is under preparation.

III. CONCLUSION

We conclude that wave breaking is a phenomenon on its own, to be distinguished from trapping of more or less uncorrelated electrons. The definition of wave breaking we have presented here has a clear meaning and is applicable to a kinematic description, too. We have further shown that a criterion for wave breaking in smooth ion density profiles can be deduced from the model of a cold streaming plasma. This suggests that coherence in the resonant wave persists until the hydrodynamic description leads to inconsistent multivalued density and flow distributions in the resonance region. Beyond the breaking limit given by Eqs. (2) and (4) one has to rely on a kinetic analysis which shows that although breaking, i.e., phasing mixing, originates from resonance, its existence becomes manifest only downstream as a lack of wave coherence. Furthermore, it has become clear that Coffey's criterion should be applied with care; in general, it is not applicable to resonance absorption.

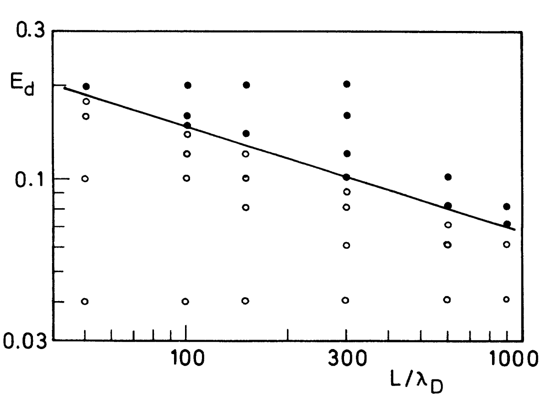

FIG. 5. Driver strength threshold η^* for wave breaking as a function of scale length L. Straight line: Eq. (3). The circles in the $\eta-L$ plane represent Vlasov simulations with wave breaking (solid) or without wave breaking (blank).

Wave breaking will lead to an increased plasma fluctuation level, spectral broadening of reflected laser light, temporal pulsations, and possibly to lower saturation levels of stimulated Raman and Brillouin scattering. As an interesting by-product of the calculations presented here we have observed that the absorption rate can be obtained from the linear theory of resonance absorption for laser intensities up to at least 200 times higher than one would deduce from simple standard criteria. A further refinement of our treatment would be the self-consistent determination of L and the inspection of caviton and soliton formation. However, in view of the present understanding of wave breaking it is perfectly justified not to take ion motion into account. (i) The results in flat density profiles presented here are new and interesting. (ii) The case of a stationary smooth profile (no holes, no radiation trapping) is generally assumed to occur in experiments with clean smooth laser pulses. In fact, with such pulses initial irregularities are washed out soon; ablation has this effect, as hydrosimulations clearly show. (iii) It would be interesting to understand unsteady profiles (fluctuating, or ripples, etc.). Such calculations are planned for the future.

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