Fast-ion interaction in dense plasmas with two-ion correlation effects

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The analytical and numerical investigations of the slowing-down process of a two-ion system in an electron plasma are developed on the basis of the Vlasov-Poisson equations, with particular emphasis on two-ion correlation effects, and the main results are presented. The analysis is based on the assumption that the velocities of the two ions are the same. The forces acting on the two particles are calculated and their resulting effects are (a) the appearance of an angular momentum aligning or misaligning the interionic vector with respect to the velocity vector and (b) the appearance of a net transverse momentum acting on the ion pair and proportional to the angle between the interionic vector and the beam direction.

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I. INTRODUCTION

The energy loss of fast charged particles in an electron gas has been a topic of great interest since the 1950s [1–9] because of its considerable importance for the study of the basic interactions of charged particles in real media; moreover, recently, it has also become a great concern in connection with heavy-ion driven inertial fusion research.

This topic has always been treated in the single-projectile approximation, i.e., under the assumption that the particles of the incident beam interact separately and independently with the medium. This approximation is acceptable in most cases when the beam-to-plasma density ratio n_b/n_0 is very small. On the other hand, for sufficiently high beam densities, the correlated motion between beam particles becomes important and the single-projectile approximation does not describe correctly the slowing-down process [10].

Experimental investigations concerning the interaction of ion clusters with condensed matter have yielded evidence of the presence of correlation effects [11,12] and in some previous theoretical works [13–15] such effects, due principally to the collective response of the plasma to the incoming projectiles, have been further studied.

In a recent paper [10] we calculated, within a dielectric theory, the stopping power of a fast ion moving in the potential wake induced in an electron-collisionless plasma by an aligned leading ion (that is, with the interionic vector and the projectile velocity in the same direction), showing the importance of correlation effects. This work is an extension of that analysis to the more general case of noncollinear motion of two projectile ions, with the explicit computation of the mutual forces acting between the two ions.

In Sec. II, by means of a Fourier analysis, the linearized Vlasov-Poisson equations are solved for two test particles in order to obtain a general form for the linearized potential generated in a Maxwellian plasma [16]. Then the forces acting on the two ions are calculated, resulting in the theoretical evidence of an aligning angular momentum acting on the system when the trailing ion is in the first potential well of the wake created by the leading ion. Furthermore, a net transverse force appears when the ions are not aligned, its intensity being proportional to the angle between the interionic vector and the beam direction.

In Sec III the slowing-down process is analyzed and the stopping power is evaluated in terms of the χ parameter, which measures the strength of correlated motion. In Sec. IV we present a qualitative discussion of the results. In the Appendix a detailed analysis of the plasma dispersion approximations which allow a correct description of the Čerenkov wake, excited by a fast charged particle, is presented and the consequences on the results previously reported in the literature are discussed.

II. SOLUTION OF THE LINEARIZED VLASOV-POISSON EQUATIONS FOR A TWO-ION SYSTEM. FORCES BETWEEN THE TWO IONS

Let us analyze the interaction process of a two-ion system moving with velocity much larger then the thermal electron velocity in a fully ionized electron-collisionless plasma on the basis of a dielectric theory by solving the linearized Vlasov-Poisson equations. Such a description is valid when the plasma parameter is small, i.e., $g \equiv 1/N_D \ll 1$, where N_D is the number of electrons in the Debye sphere ($N_D = n_0 \lambda_D^3$).

In the following we shall use the dimensionless variables, defined as

$$\mathbf{r} \Longrightarrow \frac{\mathbf{r}}{\lambda_D}, \quad t \Longrightarrow \omega_p t, \quad \mathbf{v} \Longrightarrow \frac{\mathbf{v}}{v_{\rm th}}, \qquad f \Longrightarrow \frac{v_{\rm th}^3}{n_0} f, \quad \phi \Longrightarrow \frac{e}{T}\phi, \tag{1}$$

where $\lambda_D=(T/4\pi n_0 e^2)^{1/2}$ is the electron Debye length, $\omega_p=(4\pi n_0 e^2/m)^{1/2}$ is the electron plasma frequency, m is the electron mass, n_0 is the unperturbed electron density, $v_{\rm th}=\sqrt{T/m}$ is the electron thermal speed, and $f(\mathbf{r},\mathbf{v},t)$ and $\phi(\mathbf{r},t)$ are the electron distribution function and the self-consistent electrostatic potential, respectively.

The dimensionless linearized Vlasov-Poisson equations for an electron plasma with a static background ion component read

$$\frac{\partial f_1}{\partial t}+\mathbf{v}\cdot\frac{\partial f_1}{\partial \mathbf{r}}+\frac{\partial \phi_1}{\partial \mathbf{r}}\cdot\frac{\partial f_0}{\partial \mathbf{v}}=0, \tag{2}$$

$$\nabla^2\phi_1=-Z_1\delta(\mathbf{r}-\mathbf{v}_p t)-Z_2\delta(\mathbf{r}-(\mathbf{v}_p t+\mathbf{l}_b))+\int d^3v\, f_1(\mathbf{r},\mathbf{v},t), \tag{3}$$

where $\mathbf{v}_p$ is the velocity of the two ions and $\mathbf{l}_b$ is their interionic distance.

Here, $Z_{{\rm eff},1}$ and $Z_{{\rm eff},2}$, the effective charges of the two ions moving in the plasma, are supposed to be constant throughout the slowing-down process. Such a hypothesis is quite realistic for heavy ions in the high- velocity limit ($v_p \gg 1$), electronic recombination effects being strongly reduced in a fully ionized plasma [17]. All the atomic processes involved in the projectile-plasma interaction (ionization and recombination) are more effective, as a matter of fact, at small velocities ($v_p \lesssim 1$). Even recombination effects caused by the electrons trapped in the potential wake excited by the projectiles are not expected to be effective, the number of resonant electrons being exponentially small at large velocities.

Equations (2) and (3) are obtained by introducing in the Vlasov-Poisson system the expansions $f=f_0+f_1+f_2+\cdots$, $\phi=\phi_1+\phi_2+\cdots$ in the parameter $Z_i/(1+v_p^3)$, where $Z_i=Z_{{\rm eff},i}/N_D$ ($i=1,2$) is the coupling parameter between the incoming projectiles and the plasma, and measures the strength of the perturbation due to the ion beam. The unperturbed state is assumed to be Maxwellian. It must be noticed that for large projectile velocities ($v_p \gg 1$) the linear theory well describes also the situations where $Z_i>1$ because only those few electrons moving with a given projection of their speed along $\mathbf{v}_p$ (*resonant* electrons) will interact strongly with the projectile. Moreover, the successive nonlinear correction to the stopping power turns out to be of relative order Z/v_p^3 (Barkas effect).

By solving Eqs. (2) and (3) in space-time Fourier components, we obtain the following expression for the electrostatic potential in the reference system where the two ions are at rest and the leading projectile is in the origin:

$$\phi_1(\mathbf{r})=\frac{1}{(2\pi)^3}\int d^3k \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{k^2\epsilon(\mathbf{k},\mathbf{k}\cdot\mathbf{v}_p)}(Z_1+Z_2 e^{-i\mathbf{k}\cdot\mathbf{l}_b}), \tag{4}$$

where $\epsilon(\mathbf{k},\omega)=1+W(\omega/k)/k^2$ is the longitudinal dielectric function for a Maxwellian plasma and $W(\xi)=X(\xi)+iY(\xi)$ is the plasma dispersion function [18].

Equation (4) has been discussed by a number of authors for a single test particle (see, for example, the paper by Peter [16] and references therein). In the large-velocity limit ($v_p \gg 1$), the main result is that a test particle couples to the plasma electrostatic modes (collective degrees of freedom), exciting a conical wake behind the particle itself (the Čerenkov cone). A second ion that moves in such a tail will be *correlated* to the leading ion by means of the excited oscillations. The phase factor in Eq. (4) arises from the presence of the second ion and is responsible for the two-ion correlated motion. In the Appendix we shall discuss in detail the Čerenkov cone and the different plasma dispersion approximations to correctly describe such a wake.

Within a Fourier analysis, we look for a stationary or time-asymptotic solution of the problem. Let us calculate the force acting on particle 2 due to the electrostatic field generated in the plasma by particle 1. It reads

$$\mathbf{F}_{12}=-ZN_D\left.\frac{\partial\phi_1^{(1)}}{\partial\mathbf{r}}\right|_{\mathbf{r}=\mathbf{l}_b}=-\frac{Z^2N_D}{(2\pi)^3}\int d^3k\frac{i\mathbf{k}e^{i\mathbf{k}\cdot\mathbf{l}_b}}{k^2\epsilon(\mathbf{k},\mathbf{k}\cdot\mathbf{v}_p)}, \tag{5}$$

where $\phi_1^{(1)}(\mathbf{r})$ is the linear electrostatic potential generated in the plasma by particle 1. Here we have assumed for simplicity $Z_1=Z_2=Z$. If $\mathbf{v}_p=v_p\mathbf{e}_x$ and $\mathbf{l}_b=l_{bx}\mathbf{e}_x+l_{by}\mathbf{e}_y$, Eq. (5) becomes

$$\mathbf{F}_{12}=(F_{12,\parallel}^R+F_{12,\parallel}^{\rm NR})\mathbf{e}_x+(F_{12,\perp}^R+F_{12,\perp}^{\rm NR})\mathbf{e}_y, \tag{6}$$

where

$$F_{12,\parallel}^R=-\frac{Z^2N_D}{2\pi^2}\int_0^{k_{\max}}dk\,k^3\int_0^1 d\mu\frac{\mu Y(\mu v_p)}{(k^2+X)^2+Y^2}\cos(k\mu l_{bx})J_0[kl_{by}(1-\mu^2)^{1/2}], \tag{7a}$$

$$F_{12,\perp}^R=\frac{Z^2N_D}{2\pi^2}l_{by}\int_0^{k_{\max}}dk\,k^4\int_0^1 d\mu\frac{(1-\mu^2)Y(\mu v_p)}{(k^2+X)^2+Y^2}\sin(k\mu l_{bx})J_1[kl_{by}(1-\mu^2)^{1/2}], \tag{7b}$$

$$F_{12,\parallel}^{\rm NR}=\frac{Z^2N_D}{2\pi^2}\int_0^{k_{\max}}dk\,k^3\int_0^1 d\mu\frac{\mu[k^2+X(\mu v_p)]}{(k^2+X)^2+Y^2}\sin(k\mu l_{bx})J_0[kl_{by}(1-\mu^2)^{1/2}], \tag{7c}$$

$$F_{12,\perp}^{\rm NR}=\frac{Z^2N_D}{2\pi^2}l_{by}\int_0^{k_{\max}}dk\,k^4\int_0^1 d\mu\frac{(1-\mu^2)[k^2+X(\mu v_p)]}{(k^2+X)^2+Y^2}\cos(k\mu l_{bx})J_1[kl_{by}(1-\mu^2)^{1/2}], \tag{7d}$$

with $\mu = \mathbf{k}\cdot\mathbf{v}_p/kv_p$ and $k_{\max} = 1/b_{\min} \simeq 4\pi v_p^2/Z$, where $b_{\min}$ is the effective minimum impact parameter. Here $k_{\max}$ has been introduced to avoid the divergence of the integrals caused by the incorrect treatment of the short-range interaction between the projectile and the plasma electrons within the linearized Vlasov theory. We remind the reader that $\mathrm{Im}(1/\epsilon) = -k^2Y/[(k^2+X)^2+Y^2]$ and $\mathrm{Re}(1/\epsilon) = k^2(k^2+X)/[(k^2+X)^2+Y^2]$.

By inspection of Eqs. (7) we can observe the presence of two different kinds of contributions to $\mathbf{F}_{12}$. The components $F^R_{12,\parallel}$ and $F^R_{12,\perp}$ are two *resonant* contributions which depend on $\mathrm{Im}(1/\epsilon)$, that is, they are of dissipative nature. These contributions describe the coupling between the two ions by means of the resonant interaction with the excited plasma oscillations. We shall see in Sec. III that $F^R_{12,\parallel}$ is the correlated-particle contribution to the stopping power and coincides with the contribution calculated by other authors [13,14].

Besides $F^R_{12,\parallel}$, the transverse contribution $F^R_{12,\perp}$ also gives rise to an irreversible exchange of energy between the incoming particles and the plasma, and, as far as we know, it has never been taken into account before. This term represents a net force acting on the system and accelerating it in the transverse direction. As we can see from Eq. (7b), $F^R_{12,\perp}$ is proportional to the angle ϑ between the interionic vector $\boldsymbol{l}_b$ and the velocity $\mathbf{v}_p$ through l_{by}, so that such a term does not appear when the ions are aligned. This contribution depends on the plasma dispersive properties through the Čerenkov cone.

Computing the force acting on particle 1 due to the electrostatic field created by particle 2, $\mathbf{F}_{21}$, we obtain the following relations with $\mathbf{F}_{12}$:

$$\begin{aligned} F^R_{21,\parallel} &= F^R_{12,\parallel} , \\ F^{NR}_{21,\parallel} &= -F^{NR}_{12,\parallel} , \\ F^R_{21,\perp} &= F^R_{12,\perp} , \\ F^{NR}_{21,\perp} &= -F^{NR}_{12,\perp} , \end{aligned} \tag{8}$$

which can be easily derived by exchanging $\boldsymbol{l}_b$ with $-\boldsymbol{l}_b$ in Eqs. (7).

From Eqs. (8) we see that if we sum up the mutual forces acting on the whole system, those related to the term $\mathrm{Re}(1/\epsilon)$, i.e., $F^{NR}_{12,\parallel}, F^{NR}_{21,\parallel}$ and $F^{NR}_{12,\perp}, F^{NR}_{21,\perp}$, cancel out, resulting only in an angular momentum acting on the system. In particular, as we shall see below, for small angles and when $l_b < 2\pi v_p$, where $l_b = |\boldsymbol{l}_b|$, there is an aligning angular momentum of the interionic vector $\boldsymbol{l}_b$ with $\mathbf{v}_p$; this is the theoretical evidence of what has been found in some experimental investigations [15]. The forces acting through the term in $\mathrm{Im}(1/\epsilon)$, on the contrary, sum up, giving a nonzero contribution.

We can obtain simpler formulas, suitable for numerical computation, when $v_p \gg 1$. The wave-vector integrations in Eqs. (7) include contributions due to modes either with $k>1$ or with $k<1$, related to the individual particle and collective aspects of the wave-particle interactions, respectively. To simplify the mathematics underlying Eqs. (7), we neglect the individual-particle contributions to the mutual forces *(collective approximation)*. This working hypothesis is justified because correlations are essentially due to long-wavelength collective modes which are slowly damped, while oscillations characterized by $k>1$ turn out to be strongly Landau damped, at least for $l_b > 2\pi v_p$.

Individual-particle contributions are negligible if the electrons move along straight lines during the time in which they respond collectively to the induced fields. If τ is the time interval during which the energy exchanged in electronic collisions becomes of the same order of magnitude as the initial energy of the electron, we can formulate the subsequent condition

$$\tau \gg 1 . \tag{9}$$

Here τ can be evaluated as in Ref. [2] and condition (9) coincides with $N_D \gg 1$, i.e.,

$$n_0 \ll 2\times 10^{19} T^3 , \tag{10}$$

where n_0 is measured in cm^{-3} and T in eV. Condition (10) is always true for the range of parameters in which we are interested ($n_0 \sim 10^{17}-10^{22}\ \mathrm{cm}^{-3}$, $T \gtrsim 10$ eV).

In the case of aligned projectiles, it has been verified by the numerical computation of Eqs. (7) and (8) of Ref. [10] that this is an acceptable hypothesis. This is shown in Fig. 1, where the collective (solid line) and the individual (dash-dotted line) contributions to the mutual forces are

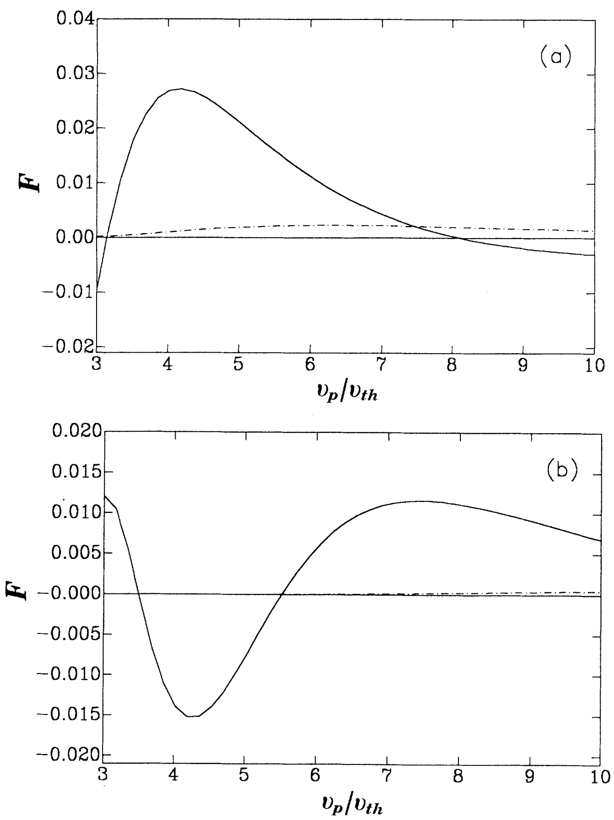

FIG. 1. Collective (solid line) and individual (dashed-dotted line) contributions to the mutual forces for $\vartheta = 0°$ given by Eqs. (7a) and (7c) as a function of v_p (in units of v_{th}) for different values of l_b (in units of λ_D): (a) $l_b = 10$, (b) $l_b = 20$. The plasma parameters are $T = 20$ eV and $n_0 = 10^{18}\ \mathrm{cm}^{-3}$, while $Z_{\mathrm{eff}} = 10$ for the projectiles.

plotted as a function of v_p for two aligned ions.

Considering only the collective contributions in the large-velocity limit and using for $X(\xi)$ the expansion for large arguments up to the second term in order to treat correctly the plasma-wave dispersion properties, i.e., $X(\xi)\simeq -1/\xi^2-3/\xi^4$ (a full discussion of the different plasma dispersion approximations is presented in the Appendix), we have

$$F^{R}_{12,\parallel} = -\frac{Z^2 N_D}{4\pi v_p^2}\int_\alpha^1 dk \frac{1}{k}\cos\left[\frac{l_{bx}}{v_p}(1+3k^2)^{1/2}\right] J_0\left[\frac{l_{by}}{\alpha v_p}(k^2-\alpha^2)^{1/2}\right] , \tag{11a}$$

$$F^{R}_{12,\perp} = \frac{Z^2 N_D}{4\pi v_p} l_{by}\int_\alpha^1 dk \frac{k}{(1+3k^2)^{1/2}}\sin\left[\frac{l_{bx}}{v_p}(1+3k^2)^{1/2}\right] J_1\left[\frac{l_{by}}{\alpha v_p}(k^2-\alpha^2)^{1/2}\right] , \tag{11b}$$

$$F^{NR}_{12,\parallel} = \frac{Z^2 N_D}{4\pi^2}\int_0^1 dk\, k \,⨍_{-1}^{1} d\mu \frac{\mu^3}{\mu^2-\mu_0^2}\sin(k\mu l_{bx}) J_0[k l_{by}(1-\mu^2)^{1/2}] , \tag{11c}$$

$$F^{NR}_{12,\perp} = \frac{Z^2 N_D}{4\pi^2} l_{by}\int_0^1 dk\, k^2 ⨍_{-1}^{1} d\mu \frac{\mu^2(1-\mu^2)}{\mu^2-\mu_0^2}\cos(k\mu l_{bx}) J_1[k l_{by}(1-\mu^2)^{1/2}] , \tag{11d}$$

where $⨍$ is the Cauchy principal-value integral, $\alpha=(1+3/v_p^2)^{1/2}/v_p$, and $\mu_0=(1+3k^2)/kv_p$.

Due to the resonant nature of the integrals (7a) and (7b), we obtained Eqs. (11a) and (11b) for the collective contribution by using the following property of the Dirac δ function:

$$\lim_{Y\to 0}\frac{Y}{f^2(\xi)+Y^2} = \pi\frac{\delta(\xi-\xi_0)}{|f'(\xi_0)|} , \tag{12}$$

where $f(\xi)=X(\xi)+k^2$ and ξ_0 is the zero of $f(\xi)$. Equations (11c) and (11d) instead are to be evaluated as the Cauchy principal value.

The sign of the forces given in Eqs. (11) depends in general on the sign of the integrals and the components of $\mathbf{l}_b$ (l_{bx}, l_{by}). Because they depend on the projections along x and y of the gradient of the electrostatic potential, and the latter has an oscillating character as a function of the interionic vector $\mathbf{l}_b$ when $v_p \gg 1$, we expect that this behavior is found in general in the force components too. Figures 2 and 3 show this behavior. An interesting point that emerges from these figures is that the modulus of $F^{NR}_{12/21,\perp}$ (dash-dotted line in the figures) is always greater than those of the other components, so that its dynamical effect will dominate over the other ones.

In particular, Fig. 2 shows the force components given by Eqs. (11) as functions of v_p for different values of the interionic distance l_b and of the angle ϑ. The regions where $F^{NR}_{12,\perp}<0$ are those governed by an aligning angular momentum.

In Fig. 3 the force components are plotted as functions of l_b for $v_p=7$ and for two different values of ϑ: (a) $\vartheta=10°$ and (b) $\vartheta=60°$; that is, respectively, inside and outside the Čerenkov cone excited by the leading ion [for $v_p=7$ the Čerenkov semivertex angle is $\varphi_C \simeq \arctan(\sqrt{3}/v_p)\simeq 15°$]. In Fig. 3(a) we observe that when the trailing ion is inside the cone and for $l_b<2\pi v_p$ (i.e., in the first potential well created by the leading ion), there is a net alignment of the ion pair, due also to $F^{NR}_{12,\parallel}$. In Fig. 3(b), on the other hand, the trailing ion is outside the Čerenkov cone, and for small interionic distances the system experiences a misaligning angular momentum. It is also evident that outside the Čerenkov cone the force disappears in a few Debye lengths.

We can thus conclude that a two-ion aligned system is stable when $l_b<2\pi v_p$, as indicated in the previous figures. If any perturbation appears which breaks alignment, the plasma electrons act on the perturbing system and push it to the aligned situation.

Gemmell *et al.* [12] have observed experimentally this aligning effect in the case of ion clusters injected in solids. They have explained it theoretically within the cold-plasma approximation by using the Neufeld form of the electrostatic potential, i.e., Eq. (A8) in the Appendix. This approximation can be obtained by taking $X\simeq -1/\xi^2$ in the dielectric function $\epsilon(\mathbf{k},\omega)$ and does not take into account correctly the plasma dispersion properties. In conclusion, that model does not describe all the aspects of correlation effects and in particular cannot describe correctly the action of the angular momentum when the trailing ion is in the Čerenkov wake excited by the leading ion.

III. SLOWING-DOWN PROCESS

Now we can calculate the stopping power for the system considered as the force that the ions experience due to the field induced by themselves along their motion direction. Within the linear theory we have

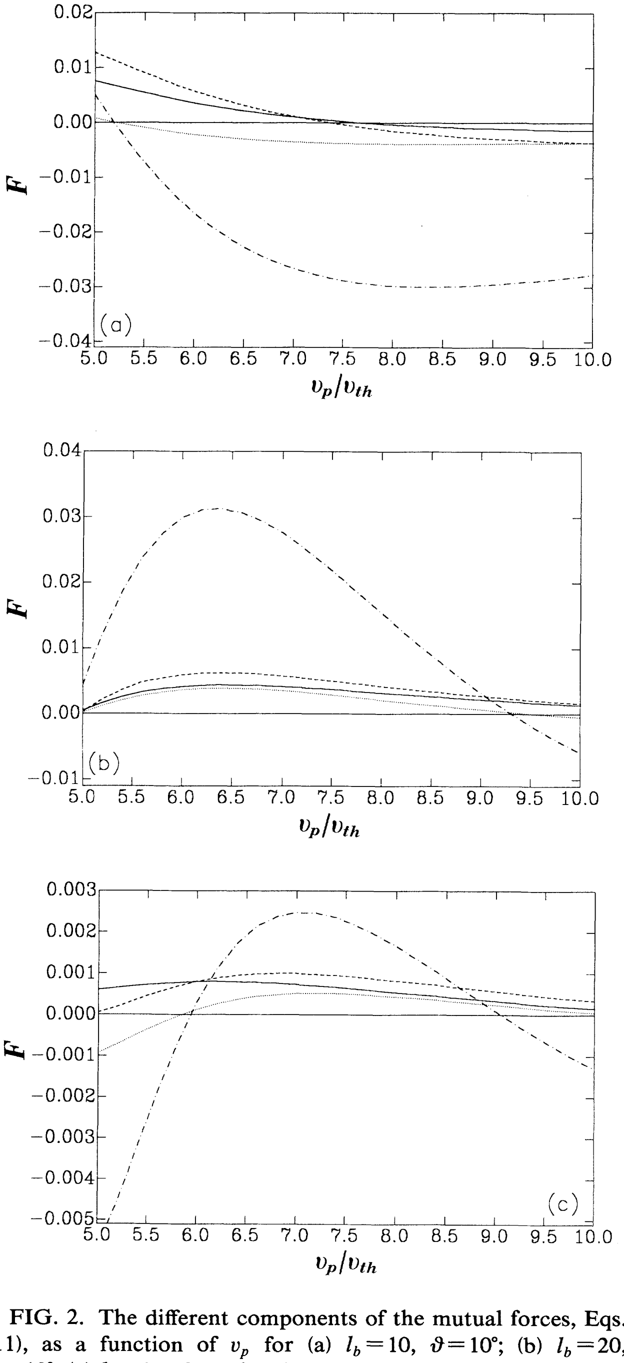

FIG. 2. The different components of the mutual forces, Eqs. (11), as a function of v_p for (a) $l_b=10$, $\vartheta=10°$; (b) $l_b=20$, $\vartheta=10°$; (c) $l_b=20$, $\vartheta=30°$. The plasma parameters are $T=100$ eV and $n_0=10^{20}$ cm^{-3}, and $Z_{\rm eff}=10$. Solid line $F^R_{12,\parallel}$; dotted line, $F^R_{12,\perp}$; dashed line, $F^{\rm NR}_{12,\parallel}$; dash-dotted line, $F^{\rm NR}_{12,\perp}$. It can be noted that $F^{\rm NR}_{12,\perp}$ is, in modulus, always greater than the other components.

$$-\left[\frac{dE}{dx}\right]=-\mathbf{F}\cdot\mathbf{e}_x$$
$$=Z_1N_D\frac{\partial\phi_{\rm ind}}{\partial x}\bigg|_{\mathbf{r}=\mathbf{r}_1(t)}+Z_2N_D\frac{\partial\phi_{\rm ind}}{\partial x}\bigg|_{\mathbf{r}=\mathbf{r}_2(t)}, \tag{13}$$

where $\mathbf{r}_1(t)=\mathbf{v}_pt$ and $\mathbf{r}_2(t)=\mathbf{v}_pt+\boldsymbol{l}_b$ are the projectile trajectories in the laboratory system. The sign has been chosen so that $-(dE/dx)$ is positive if the energy is lost. After differentiation of Eq. (4) we obtain

$$-\left[\frac{dE}{dx}\right]=\frac{N_D}{(2\pi)^3}\int d^3k\frac{i\mathbf{k}\cdot\mathbf{v}_p}{v_p}\frac{1}{k^2}\left[\frac{1}{\epsilon(\mathbf{k},\mathbf{k}\cdot\mathbf{v}_p)}-1\right]$$
$$\times[Z_1^2+Z_2^2+2Z_1Z_2\cos(\mathbf{k}\cdot\boldsymbol{l}_b)]\ . \tag{14}$$

Proceeding as in Sec. II and assuming $Z_1=Z_2=Z$, Eq. (14) becomes

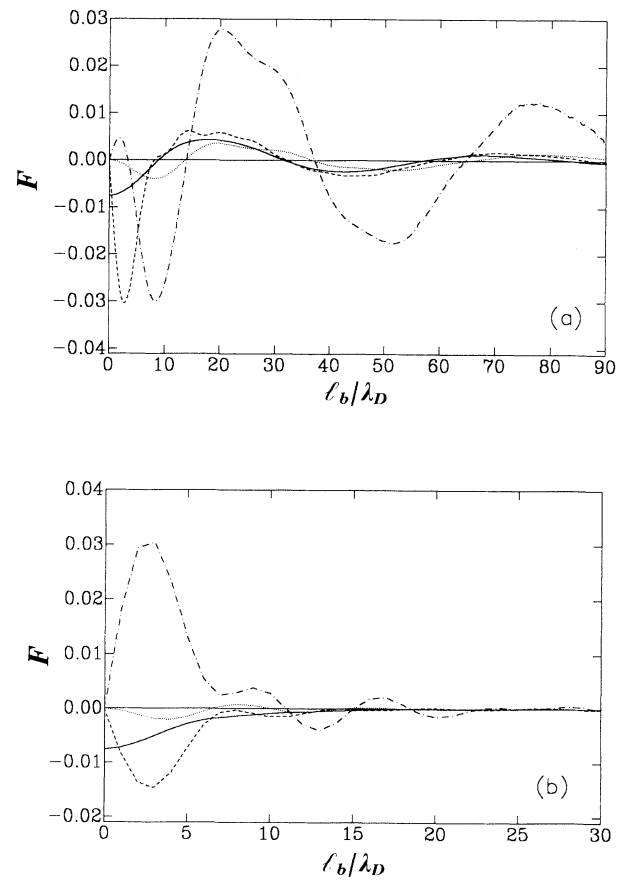

FIG. 3. The different components of the mutual forces, as in Fig. 2, as a function of l_b for $v_p=7$ and for (a) $\vartheta=10°$, (b) $\vartheta=60°$.

$$-\left[\frac{dE}{dx}\right]=\frac{Z^2N_D}{\pi^2}\int_0^{k_{\max}}dk\,k^3\int_0^1 d\mu\frac{\mu Y(\mu v_p)}{(k^2+X)^2+Y^2}\{1+\cos(k\mu l_{b\parallel})J_0[kl_{b\perp}(1-\mu^2)^{1/2}]\}\ , \tag{15}$$

where $l_{b\parallel}=l_{bx}$ and $l_{b\perp}=|l_{by}|$.

There are two contributions to the stopping power of the ion pair. The first one is the uncorrelated particle contribution and represents the energy loss of the two projectiles by emission of Langmuir waves. Such a mechanism can be interpreted as a local competition between spontaneous emission of plasma waves and its corresponding Landau damping. For $v_p \gg 1$ the latter mechanism is overcome by the first one. The second contribution, on the contrary, is responsible for the correlated motion of the two ions by means of the resonant interaction with the excited plasma oscillations. This contribution arises for the phase factor in Eq. (4).

These two terms are responsible for the irreversible transfer of the two-ion energy to the plasma by means of resonant electrons. The Bessel function J_0 takes into account the dependence on the angle ϑ between the interionic vector $\mathbf{l}_b$ and the velocity $\mathbf{v}_p$. Correlation effects are expected to be maximum when the two test particles are aligned, while they go to zero (at least for $l_b \gtrsim 10$) when $\vartheta \rightarrow 90°$. The explanation is connected with the behavior of the potential: from 0° to 90° the oscillation amplitude decreases (Čerenkov cone) and the two ions tend to an uncorrelated motion (except when $l_b \ll v_p$, in which case each ion is affected by the unscreened field of the other ion).

Since correlation effects are strong for large projectile velocities ($v_p \gg 1$), we have computed Eq. (15) in that limit. The first integral gives the classical Bohr formula for the single-projectile stopping power [8]. The second integral can be split into two contributions: the collective one ($k<1$) and the individual one ($k>1$). Equation (15) takes the form

$$-\left[\frac{dE}{dx}\right]=\frac{Z^2N_D}{2\pi v_p^2}[\ln(k_{\max}v_p)+I_c]+\frac{Z^2N_D}{\pi\sqrt{2\pi}v_p^2}I_i\ , \tag{16}$$

where

$$I_c=\int_\alpha^1 dk\frac{1}{k}\cos\left[\frac{l_{b\parallel}}{v_p}(1+3k^2)^{1/2}\right]J_0\left[\frac{l_{b\perp}}{\alpha v_p}(k^2-\alpha^2)^{1/2}\right]\ , \tag{17a}$$

$$I_i=\int_0^{v_p}d\xi\,\xi^2e^{-\xi^2/2}\int_1^{k_{\max}}dk\frac{1}{k}\cos\left[k\frac{l_{b\parallel}}{v_p}\xi\right]\times J_0\left[k\frac{l_{b\perp}}{v_p}(v_p^2-\xi^2)^{1/2}\right]\ , \tag{17b}$$

with $\alpha=(1+3/v_p^2)^{1/2}/v_p$. Equations (17) have been obtained by taking into account the plasma-wave spatial dispersion.

In the collective approximation we shall neglect the contribution of I_i to the correlated motion. In Fig. 4, $-(dE/dx)$, given by Eq. (16), has been plotted versus v_p in this approximation, for different angles. It can be noticed that when ϑ increases correlation effects decrease, as it is expected.

We can compute analytically Eq. (17a) in the cold-plasma limit with $X(\xi)\simeq -1/\xi^2$; however, in doing so, we do not take into account the plasma-wave spatial dispersion (see the Appendix). In this case the latter equation becomes

$$I_c\simeq\cos\left[\frac{l_{b\parallel}}{v_p}\right]\int_1^{v_p}d\xi\frac{1}{\xi}J_0\left[\frac{l_{b\perp}}{v_p}(\xi^2-1)^{1/2}\right]=\cos\left[\frac{l_{b\parallel}}{v_p}\right]\int_0^{(v_p^2-1)^{1/2}\sim v_p}d\eta\frac{\eta}{1+\eta^2}J_0\left[\frac{l_{b\perp}}{v_p}\eta\right]\ . \tag{18}$$

For $l_{b\perp}\neq 0$, in the limit of $v_p \gg 1$, the upper extreme can be sent to infinity because of the convergence of the integrand function (for $\eta\rightarrow\infty$ it decreases as $1/\eta^{3/2}$). With this approximation the integral can be simply evaluated:

$$I_c\simeq\cos\left[\frac{l_{b\parallel}}{v_p}\right]K_0\left[\frac{l_{b\perp}}{v_p}\right]\ , \tag{19}$$

where K_0 is the modified Bessel function of zeroth order. Assuming $I_i\simeq 0$, Eq. (16) becomes

$$-\left[\frac{dE}{dx}\right]=\frac{Z^2N_D}{2\pi v_p^2}\left[\ln(k_{\max}v_p)+\cos\left[\frac{l_{b\parallel}}{v_p}\right]K_0\left[\frac{l_{b\perp}}{v_p}\right]\right]\ . \tag{20}$$

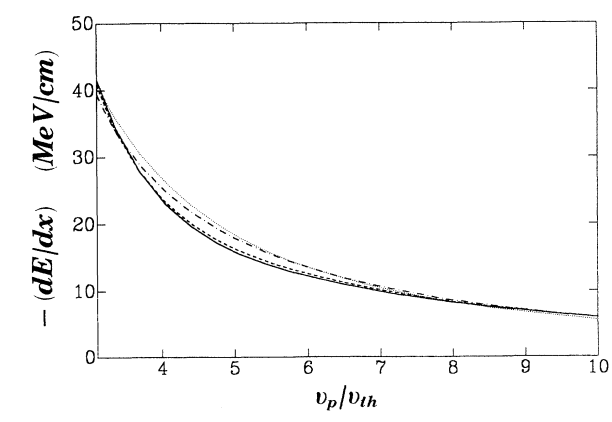

FIG. 4. Stopping power (in MeV/cm) of a two-ion correlated system with $l_b=10$ as a function of v_p for different values of ϑ. Solid line, $\vartheta=0°$; dashed line, $\vartheta=10°$; dash-dotted line, $\vartheta=25°$; dotted line, single-particle approximation. Parameters as in Fig. 2.

Although this is a very crude calculation, it agrees with the result of Basbas and Ritchie [14] when $l_{b\perp}q_{\max} \gg 1$, where $q_{\max}=(k_{\max}^2-1/v_p^2)^{1/2}$ is the maximum transverse momentum transferred in a "close" collision. From the analysis of the Čerenkov cone, this solution is in good agreement with the numerical evaluation of Eq. (16), with Eqs. (17), outside the cone. In this sense, $q_{\max}$ depends on the semivertex angle of the cone.

We must observe that Eqs. (19) and (20) diverge logarithmically in the limit of $l_{b_\perp}\to 0$ due to the inconsistency of the two simultaneous assumptions $l_{b\perp}\to 0$ and $v_p\to\infty$ (which is equivalent to considering a cold-plasma limit).

This can be explained with the following arguments. By putting $T=0$ or $v_p\to\infty$, the Čerenkov cone becomes infinitely narrow and "collapses" in a straight line behind the fast ion which has created it. This has unphysical consequences on the description of the wake and on the evaluation of the stopping power for $l_{b\perp}\to 0$.

When the two ions are aligned ($l_{b\perp}=0$) the stopping power should be computed by considering the thermal corrections to the plasma dispersion. The relevant computations and results have been published in Ref. [10]. There, the most important consequences of considering the spatial dispersion of the excited waves consist of the finite value of the stopping power for $l_{b\perp}=0$ and the decreasing importance of the individual particle contributions to the correlated motion for increasing l_b, thus supporting our hypothesis of a collective approximation.

It is useful to write the stopping power in terms of two contributions,

$$-\left[\frac{dE}{dx}\right]=-\left[\frac{dE}{dx}\right]_{\rm sp}-\left[\frac{dE}{dx}\right]_c , \tag{21}$$

where $-(dE/dx)_{\rm sp}$ is the single-particle contribution to the stopping power and $-(dE/dx)_c$ is the correlated motion contribution. In this way we can introduce the χ parameter, which describes the intensity of correlation effects with respect to the completely uncorrelated motion:

$$\chi\equiv\frac{-\left[\dfrac{dE}{dx}\right]_c}{-\left[\dfrac{dE}{dx}\right]_{\rm sp}} , \tag{22}$$

and Eq. (21) can be written as

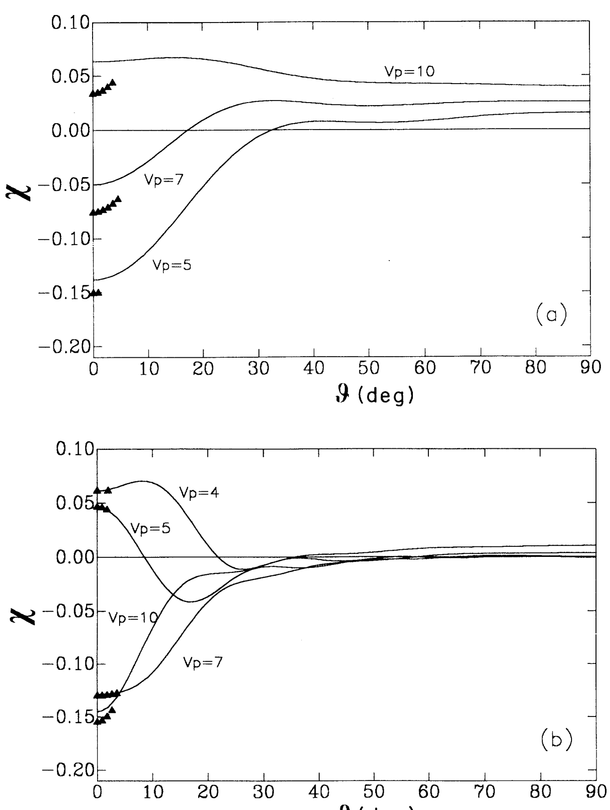

FIG. 5. The χ parameter as a function of ϑ for (a) $l_b=10$, (b) $l_b=20$, with the parameters as in Fig. 2. Solid line, $\chi=\chi_{\rm coll}$; ▲, $\chi=\chi_{\rm coll}+\chi_{\rm ind}$ computed for $\vartheta<10°$.

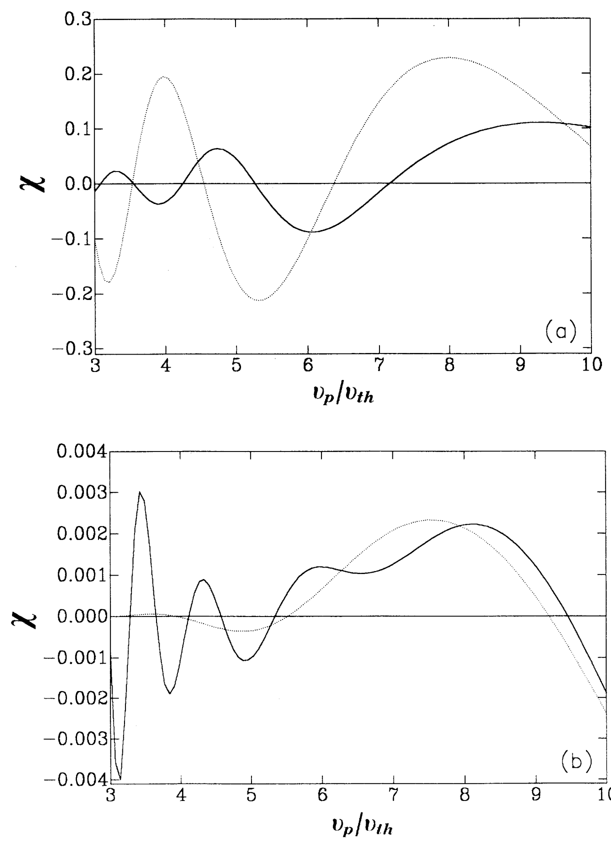

FIG. 6. Comparison between the χ parameter within the two approximations $X\simeq-1/\xi^2-3/\xi^4$ (solid line) and $X\simeq-1/\xi^2$ (dotted line) as a function of v_p with $l_b=50$ and with (a) $\vartheta=0°$, (b) $\vartheta=30°$. The plasma parameters are $T=10$ eV and $n_0=10^{18}$ cm^{-3}, with $Z_{\rm eff}=10$ for the projectiles.

$$-\left[\frac{dE}{dx}\right]=-\left[\frac{dE}{dx}\right]_{\rm sp}[1+\chi] . \tag{23}$$

In Ref. [12] the χ parameter is called the *vicinage function.* In that case the χ parameter has been calculated with a statistical average on the orientations of the projectiles. Even the χ parameter can be split into a collective contribution and an individual one, and following the same arguments as above we shall assume $\chi \simeq \chi_{\rm coll}$.

In Fig. 5 the values of the parameter $\chi_{\rm coll}(\vartheta)$ are shown for different projectile velocities for (a) $l_b=10$ and (b) $l_b=20$. In particular, $\chi_{\rm coll}$ (solid line) is compared with $\chi=\chi_{\rm coll}+\chi_{\rm ind}$ (▲) for small angles; the approximation adopted improves with increasing l_b.

Figures 6 and 7 show the comparison between the collective contribution to the χ parameter evaluated in the two different approximations, i.e., $X \simeq -1/\xi^2-3/\xi^4$ (solid line) and $X \simeq -1/\xi^2$ (dotted line) for different angles.

In Fig. 6, $\chi_{\rm coll}$ vs v_p is shown for $l_b=50$, while Fig. 7 shows $\chi_{\rm coll}$ vs l_b for $v_p=7$. Here the difference between the two approximations is remarkable for small angles, that is for $\vartheta < \varphi_C \simeq \arctan(\sqrt{3}/v_p)$, when the trailing ion is inside the Čerenkov cone excited by the leading ion ($\varphi_C \simeq 15°$ for $v_p=7$).

On the contrary, the cold-plasma picture well describes the slowing down of the ion pair when $\vartheta > \varphi_C$ (outside the cone). The reason for the disagreement when inside the cone is connected to the lack of the spatial dispersion effects in the description of the plasma, caused by the use of the cold-plasma limit. The formation of the Čerenkov cone is not described at all.

Due to the tendency of close projectile ions to align with each other (see the discussion at the end of Sec. II), a good evaluation of the stopping power can rely on the results of Ref. [10], dedicated to the aligned particles. The results obtained here justify the main results presented in Ref. [10] for the collinear case.

IV. DISCUSSION OF THE RESULTS AND CONCLUSIONS

The purpose of this work is to extend the analysis of the slowing-down process of a two-ion system in collinear motion in a Maxwellian plasma performed in a recent paper [10] to the more general situation of arbitrary relative positions of the two projectiles. Indeed, we have studied the ion-plasma interaction process, allowing the trailing ion to be disaligned with respect to the leading ion. The disalignment is described by the angle between the projectile velocity $\mathbf{v}_p$ and the interionic vector $\boldsymbol{l}_b$.

The first result that emerges from our study is that the characteristics of the correlated motion of a two-ion system are direct consequences of the plasma dispersion properties, which are necessary to correctly describe the Čerenkov cone. The cold-plasma approximation matches the correct solution only outside the Čerenkov wake, and our solution for the stopping power coincides in this case with the result presented by Basbas and Ritchie [14].

The analysis of the forces acting on the projectiles has

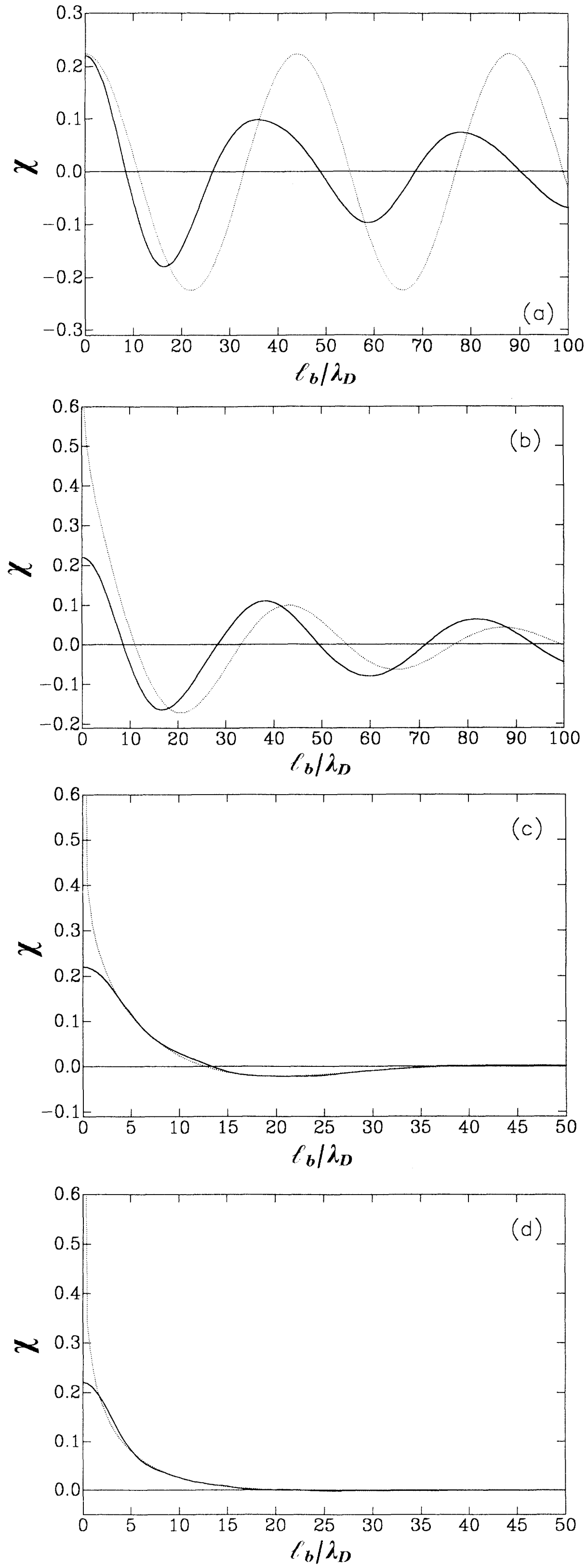

FIG. 7. Comparison between the χ parameter within the two approximations, as in Fig. 6, as a function of l_b for $v_p=7$. (a) $\vartheta=0°$, (b) $\vartheta=5°$, (c) $\vartheta=30°$, (d) $\vartheta=60°$. The parameters are the same as in Fig. 6.

been done by taking into account the dispersion effects. The main result shows that, when l_b and $\mathbf{v}_p$ are not aligned, the two-ion system gains a net momentum in the direction perpendicular to $\mathbf{v}_p$, proportional to the angle between l_b and $\mathbf{v}_p$; as far as we know, this effect has never been discussed before. This transverse momentum has obviously great importance in the dynamical evolution of the whole plasma-projectile system and is expected to influence either the dynamics of the particles or the process of dissipation of the ion energy into the plasma (plasma heating).

Furthermore, the mutual forces acting between the two ions give rise to an angular momentum which tends to rotate the system. In particular, our results show that there is a net aligning angular momentum when the trailing ion is in the first potential well of the oscillations excited by the leading ion, inside the Čerenkov cone. On the other hand, when the trailing ion is on one of the peaks of the oscillations or outside the cone, we expect that the angular momentum is misaligning. Besides this effect, we must add the motion created by the transverse momentum which shifts the motion axis. Obviously, the effect of this transverse momentum decreases in the case of an aligning action of the angular momentum.

The global effects of the forces acting on the two-ion system could be fully described by means of a dynamical particle code for the projectile-plasma system, which is out of the context of this work.

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APPENDIX: DISCUSSION OF THE ČERENKOV CONE

Let us recall the linearized potential created by an ion in a plasma

$$\phi_1(\mathbf{r})=\frac{Z}{(2\pi)^3}\int d^3k\frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{k^2\epsilon(\mathbf{k},\mathbf{k}\cdot\mathbf{v}_p)}\ , \tag{A1}$$

where $\epsilon(\mathbf{k},\omega)$ has been defined in Sec. II. Using spherical coordinates for $\mathbf{k}$ and cylindrical ones for $\mathbf{r}(x,\rho,\varphi)$, with $\mathbf{v}_p=v_p\mathbf{e}_x$, Eq. (A1) becomes

$$\phi_1(\mathbf{r})=\frac{Z}{4\pi^2}\int_0^\infty dk\,k^2\int_{-1}^{+1}d\mu\frac{e^{ik\mu x}}{k^2+X+iY}\times J_0[k\rho(1-\mu^2)^{1/2}]\ , \tag{A2}$$

where $\mu=\mathbf{k}\cdot\mathbf{v}_p/kv_p$.

The k integration can be split into two contributions, $k<1$ and $k>1$, relevant to plasma collective and individual-particle aspects, respectively. Since here we are mainly interested in analyzing the Čerenkov cone excited in the plasma by a fast projectile, in evaluating Eq. (A2) we shall retain only the collective oscillating (or resonant) contribution to the potential, $\phi_{\rm res}(\mathbf{r})$ (cf. Peter [16]). By taking the expansion of X for large arguments, $X(\xi)\simeq-1/\xi^2-3/\xi^4$, we get

$$\phi_{\rm res}(\mathbf{r})\simeq\frac{Z}{2\pi v_p}\Theta(-x)\int_{(1+3/v_p^2)^{1/2}}^{2}d\eta\frac{\eta^2}{\eta^2-1}\sin\left[\frac{x}{v_p}\eta\right]J_0\left[\frac{\rho}{v_p}\left[\eta^2-\left[1+\frac{3}{v_p^2}\right]\right]^{1/2}\right]. \tag{A3}$$

Far away from the projectile trajectory ($\rho\gg0$) Eq. (A3) becomes

$$\phi_{\rm res}(\mathbf{r})\simeq\sqrt{3}\frac{Z}{2\pi}\Theta\left[-x-\frac{\rho v_p}{\sqrt{3}}\right]\frac{e^{-\Gamma|x|}}{v_p^2}\frac{\cos[\Upsilon(x,\rho)]}{\Upsilon(x,\rho)}\ , \tag{A4}$$

where

$$\Upsilon(x,\rho)=\left[\frac{3x^2}{v_p^4}-\frac{\rho^2}{v_p^2}\right]^{1/2}, \tag{A5}$$

and $e^{-\Gamma|x|}$ takes into account the Landau damping of the excited Langmuir waves. In Eq. (A4), the Heaviside function Θ describes a Čerenkov cone ($x<-\rho v_p/\sqrt{3}$) in the trail of the projectile with semivertex angle $\varphi_C=\arctan(\sqrt{3}/v_p)$. It is important to notice that the structure of the cone depends strongly on the spatial dispersion properties of the medium. In fact, if we consider the cold-plasma limit by taking $X(\xi)\simeq-1/\xi^2$, the wave cone vanishes. In this case Eq. (A2) takes the form

$$\phi_{\rm res}(\mathbf{r})=\frac{Z}{2\pi v_p}\Theta(-x)\sin\left[\frac{x}{v_p}\right]\times\int_0^{(v_p^2-1)^{1/2}}d\xi\frac{\xi}{\xi^2+1}J_0\left[\frac{\rho}{v_p}\xi\right]. \tag{A6}$$

On the projectile trajectory ($\rho=0$) Eq. (A6) becomes

$$\phi_{\rm res}(\mathbf{r})=\frac{Z}{2\pi v_p}\Theta(-x)\sin\left[\frac{x}{v_p}\right]\ln v_p\ . \tag{A7}$$

The gradient of this expression yields the collective contribution of the stopping power of the particle. On the other hand, far away from the ion trajectory ($\rho\gg0$) Eq.

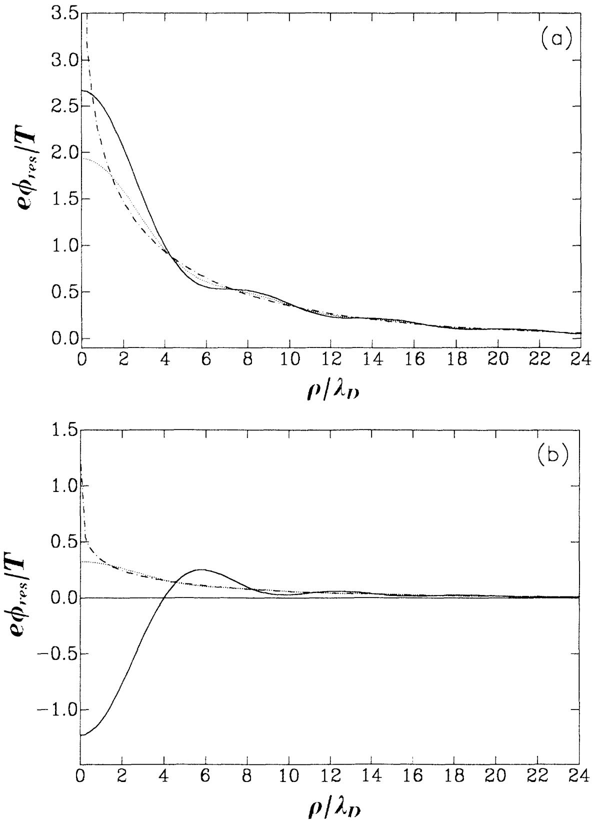

FIG. 8. Resonant contribution to the potential of an ion penetrating in a Maxwellian plasma with $v_p=10$ as a function of ρ for different values of x. (a) $x=10$, (b) $x=30$. Solid line, Eq. (A3); dotted line, Eq. (A6), dashed-dotted line, Eq. (A8).

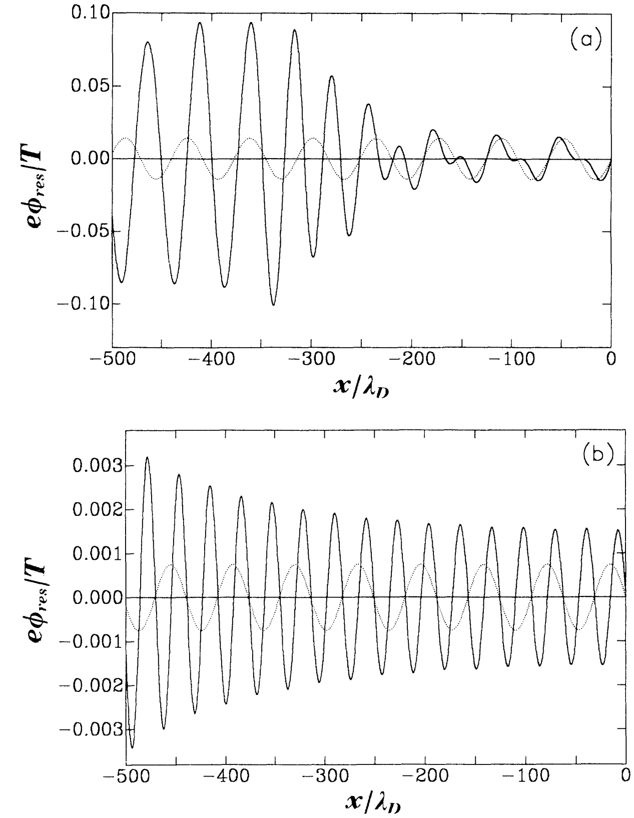

FIG. 9. Resonant contribution to the potential of an ion with $v_p=10$ as a function of x for different values of ρ. (a) $\rho=40$, (b) $\rho=100$. Solid line, Eq. (A3); dotted line, Eq. (A6).

(A6) can be evaluated by sending the upper limit of the integration to infinity:

$$\phi_{\rm res}(\mathbf{r})\simeq\frac{Z}{2\pi v_p}\Theta(-x)\sin\left[\frac{x}{v_p}\right]K_0\left[\frac{\rho}{v_p}\right]. \qquad (A8)$$

In the relevant computations we have neglected the Landau damping term for simplicity. Equation (A8) coincides with the result obtained by Neufeld and Ritchie [2]. The same result can be found by treating the plasma electrons as classical noninteracting oscillators. An important point that emerges from Eqs. (A7) and (A8) is that there is no continuity in passing from $\rho=0$ to $\rho\neq 0$ and vice versa. In particular, there is a logarithmic divergence in Eq. (A8) when $\rho\to 0$, due essentially to the erroneous treatment of the infinite upper limit in the integration. In this case the Čerenkov cone vanishes, degenerating into a line (the particle trajectory). Chen, Langdon, and Lieberman [19] obtained a similar result for $\rho\gg 1$ and small $k_\perp=k(1-\mu^2)^{1/2}$, outside the Čerenkov cone.

Figure 8 shows that the cold-plasma limit Eq. (A6), and in particular Eq. (A8), well describe the electrostatic field outside the Čerenkov cone. In this limit we lose information that depends on the spatial dispersion properties of the medium.

In Figure 9, $\phi_{\rm res}(\mathbf{r})$ corresponding to Eqs. (A3) and (A6) are compared for fixed ρ. The Čerenkov cone is described by retaining the plasma spatial dispersion.

We conclude that the explanation of the aligning effect observed in the cluster-target experiments done by Gemmell *et al.* [12] in terms of the Neufeld form of the electrostatic potential, Eq. (A8), does not take into account all the aspects of correlation effects and in particular cannot describe correctly the acting angular momentum, because the trailing ion is not affected by the Čerenkov wake excited by the leading ion.

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