Pile density is a control parameter of sand avalanches

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Centrifuge experimentation ($10-1000$ m/s^2) on sand avalanches has shown that (i) grain cohesion and elasticity are negligible; (ii) the avalanche size $\delta\theta$ and the maximum angle of repose θ_m depend on the initial pile density d; and (iii) the pile surface evolves spontaneously towards an asymptotic state after a few avalanches, this state being characterized by a mean surface angle equal to the macroscopic friction angle ϕ and a mean avalanche size equal to 1.7°. This demonstrates that the density d is a controlling parameter of the avalanche process. These results are compatible with a unique internal friction angle ϕ with $\phi=34°$; ϕ corresponds to the value measured with other soil-mechanics devices; however, ϕ fluctuates within 2° from pile to pile or test to test. These data are also consistent with a dilatancy effect which depends on density, but the existence of at least one other parameter controlling the avalanche size has been also proved.

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I. INTRODUCTION

Recently much work has been done to understand the mechanism of sandpile avalanches [1–7]; this is mainly due to the pioneering model of Bak, Tang, and Wiesenfeld [1,2] (BTW model). This series of papers has introduced the notion of self-organized criticality to describe the behavior of spatially extended, driven dynamical systems in steady state. The BTW model predicts large fluctuations of avalanche sizes and $1/f$ noise. On the other hand, most experimental investigations on real pile do not get such large fluctuations [4–6], except when very small piles are concerned [7]; furthermore, it has been found experimentally that the typical avalanche size scales as the pile volume and not as the pile surface [4–6], since it concerns a variation $\delta\theta$ of the angle of the free surface before (θ_m) and after (θ_r) the avalanche.

However, one of us has proposed [6] to make a parallel between (i) sandpile-avalanche behavior, (ii) typical results of a triaxial test in soil mechanics, and (iii) behavior of first- and second-order transitions. The interest of this approach is to fill up the gap between the different theories and results; in particular, it makes a link between the theoretical BTW approach, the observed data [4–6] at macroscopic scales, and finite-size effects [7]. In particular, it has been demonstrated in [6] that the dilatancy effect is equivalent to a latent heat of fusion which makes the slope instability subcritical instead of critical. It is then important to control this dilatancy effect if critical fluctuations are to be observed on a large length scale and large finite-size effect [6]. Reference [6] recalls some soil-mechanics results and emphasizes that this dilatancy effect (discovered by Reynolds) is controlled by the pile density; it may be negative for very loose pile, equal to 0 for a given pile density d_c (called by soil-mechanics specialists the "critical density" [8]), or positive for dense enough pile. However, it is also recalled in [6] that some other perturbation may also exist: for instance, cohesion [9] may make the slope stability subcritical too. In order to avoid this last phenomenon, it has been decided to conduct the experiment in a centrifuge at $100G$ (where $G=9.81$ m/s^2), where its relative effect strongly decreases (the cohesion force being kept constant and the weight and stress tensor being multiplied by 100): in order to illustrate this point let us remind the reader that a sand castle on a beach may be as high as 50 cm, but it could not surpass 5 mm at $100G$.

So, this work is devoted to study the effect of pile density on the physics of sand avalanches under high gravity, using a centrifuge, and to determine if the maximum angle of stability θ_m and the avalanche size $\delta\theta=\theta_m-\theta_r$ are governed by sandpile density, by grain cohesion, or by grain elasticity and to demonstrate that sandpile avalanches may be understood within the same concepts as the other results of soil mechanics.

This paper is written as follows: we give first the experimental results and techniques on avalanches. We recall then some classical results on soil mechanics (triaxial tests, dilatancy, etc.). We will interpret our avalanche results within this framework and demonstrate the consistency of classical soil-mechanics results and avalanche experiments. The main result of this paper is to prove that density (and hence dilatancy) is a controlling parameter of the avalanche size and of the maximum angle of stability of the pile slope, but it also proves that it is not the only one.

II. EXPERIMENTS ON AVALANCHES

A. Experimental technique

We have then prepared different homogeneous parallelepipedic samples ($0.4\times0.4\times0.15$ m^3) of the same "normalized" Hostun sand [10] at different initial densities using the pluviation technique [11] and have performed on each of them a series of avalanches at high and constant gravity field (i.e., $100G$ with $G=9.81$ m/s^2), using the 5.5-m-radius centrifuge of the Laboratoire Central des Ponts et Chaussées (LCPC) in Nantes; this centrifuge allows us to supply to a 2000-kg mass a gravity field varying from $1G$ to $100G$. Due to sample and centrifuge sizes, the acceleration homogeneity is not better than 3%; vibrations inside the centrifuge may be neglected (less than $10^{-2}G$), but a special careenage has been designed to prevent any wind perturbation. Hostun sand is a sieved ground silicate sand [10]. Its density range is 1.4–1.7 [10,11], depending on the packing procedure.

The experimental avalanche device is sketched on Fig. 1: it is located in the centrifuge basket. It consists in a parallelepipedic container filled with Hostun sand, which can rotate slowly around a horizontal axis in order to create a set of successive avalanches; the flowing sand is collected by a box, where it is weighted. A videocamera records the flow from the container so that we analyze the avalanche dynamics and measure the remaining sand height before and after each avalanche. We measure also the angle of rotation of the container. So, we can determine the angles of the free surface just before (θ_m) and just after (θ_r) any avalanche, the difference $\delta\theta=\theta_m-\theta_r$, and the avalanche weight as a function of the gravity, of the number of the avalanche in a given series, and of the sample initial density.

The container has been kept horizontal during the sample preparation by pluviation [11]. We have recorded

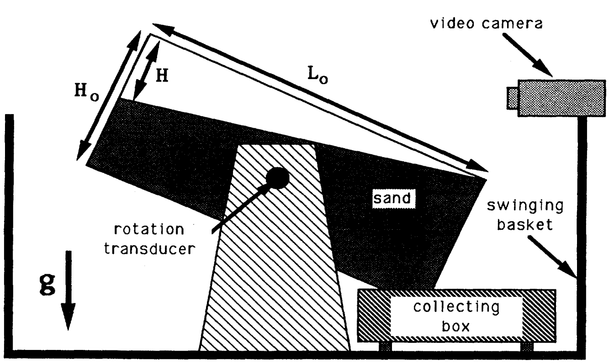

FIG. 1. Sketch of the experiment which is loaded in the centrifuge.

a whole series of avalanches for each sample preparation. Obviously, the real pile density is only known for the first avalanche of each sample since after this first avalanche we may expect the surface pile density to have evolved spontaneously. Thus, for each whole series of avalanches corresponding to a given initial density, we have measured θ_m and $\delta\theta$ as a function of the number n of the avalanche in the series (i.e., for each sample, a series of eight avalanches has been recorded, so $n=1$ to 8); we report these values in Figs. 2(a) and 2(b) for different initial densities, at $100G$.

B. Experimental results on avalanches (Fig. 2)

Let us start our data analysis with θ_m data [Fig. 2(a)]. If we compare the different curves θ_m vs n, one may see first that the evolution after a few avalanches tends to a unique (but fluctuating within 2°) asymptotic limit which is independent of the initial pile density. Furthermore, repeating twice the same experiment with two equivalent samples (i.e., at the same initial density) leads to twice the same evolution. (So our experimental results are stable.)

Now, Fig. 2(a) demonstrates that the maximum angle of stability θ_m exhibits a transient behavior inside each series of avalanches. This transient behavior depends on the initial sandpile density: indeed, we observe that (i) θ_m is continuously increasing for the loose packings ($d=1.45, 1.47$), (ii) it is continuously decreasing for the largest initial densities ($d=1.59, 1.67, 1.68$), (iii) that the θ_m angle tends eventually to a common asymptotic (but

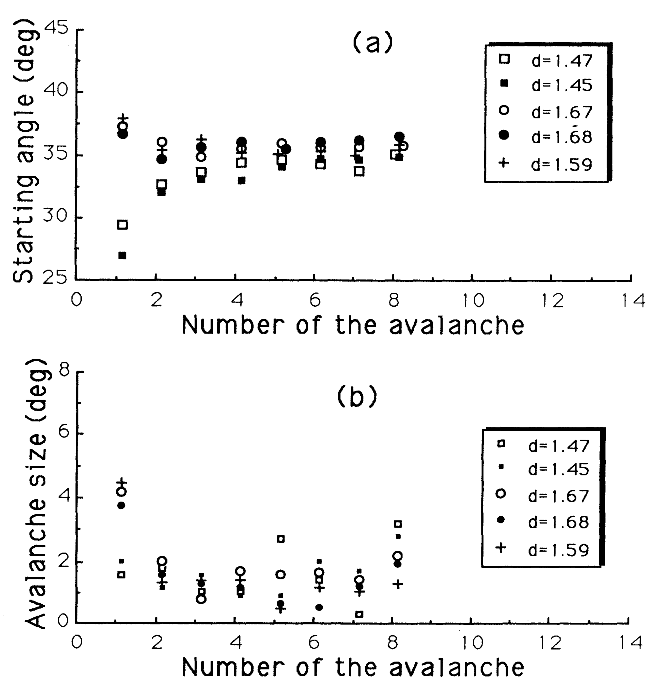

FIG. 2. Evolution of avalanche characteristics, for Hostun sand packed at different initial densities, as a function of the avalanche number in the same series. The gravity was $100G$. (a) The maximum angle of stability: θ_m. (b) The avalanche size: $\delta\theta=\theta_m-\theta_r$.

fluctuating) value of about 34°–35°. Another point worth noticing concerns the data on initially low-density piles: in this case the first two avalanches correspond in fact to a macroscopic compaction process since no flow of sand out of the container has been recorded and weighted. This detail will be discussed later.

Figure 2(b) reports the evolution of the avalanche size $\delta\theta$ as a function of the number in a given series for different pile densities. This size is measured using the variation of slope. In the case of initially dense piles, we observe that the first avalanches are larger than the forecoming. We observe also that whatever the pile density is the avalanche size tends to an asymptotic behavior, which is characterized by a mean avalanche size of 1.7° and fluctuations of about 1° around this value. This asymptotic behavior is quite similar to that of other results at $1G$. Let us now introduce some classical results of soil mechanics and demonstrate their consistency with these data.

III. CLASSICAL TRIAXIAL TEST RESULTS IN SOIL MECHANICS

A. Triaxial test method

Soil-mechanics specialists are commonly using triaxial tests [8,12–16] in order to characterize the mechanical behaviors of granular materials, of clays, or even of rocks. As sketched on Fig. 3, a triaxial apparatus is composed of a cylindrical flexible membrane of radius R and height h closed at its bottom and top by two pistons (volume $v = \Pi R^2 h$); this membrane contains the material to be studied (i.e., the granular medium here); it is immersed in a liquid at a given pressure p and it is submitted to a vertical overload q, called the deviatoric stress (see Fig. 3).

The basic principle of the experiment is to submit the sample to a given set of successive stress-strain transformations and to record the material response. For instance, one may submit the sample to cyclic deviatoric loadings or to cyclic compressions, . . . , but the simplest tests consist in applying a monotonous one-way path such as a single compression (p =const, q increases), a single oedometric compression (R =const, h decreases continuously by varying continuously the overload q, which implies in turn that p increases), and so on. The mechanical state at a given stage of transformations is defined by the set of the four experimental data: the liquid pressure p, the deviatoric stress q, the volume change $\delta v = v_0 - v$, and the height decrease $\delta h = h_0 - h$. (We note that volume and height decreases are positive as used in soil mechanics.)

We will be concerned exclusively in this paper by tests which consist of a single continuous decrease of the sample height [$d(\delta h)/dt > 0$ where t is time]. Furthermore, the rate $d(\delta h)/dt$ will be sufficiently small to keep the sample in a quasistatic equilibrium, at the limit of plasticity, so that time will not enter directly in the test response. Furthermore, and for sake of simplicity, we will exclusively consider here tests which keep constant p and vary q.

When one wants to investigate the mechanical properties of a consolidated soil, such as drained clay or rocks, one puts directly in the triaxial cell a boring of the natural soil so that the initial conditions of the sample are the natural *in situ* conditions. But when studying an unconsolidated soil, such as sand or any noncohesive granular material, this cannot be done and the way of settling the sample in the cell defines the initial conditions. We then have to discuss briefly the set of initial conditions which may be studied and the way of making a sample obey such initial conditions.

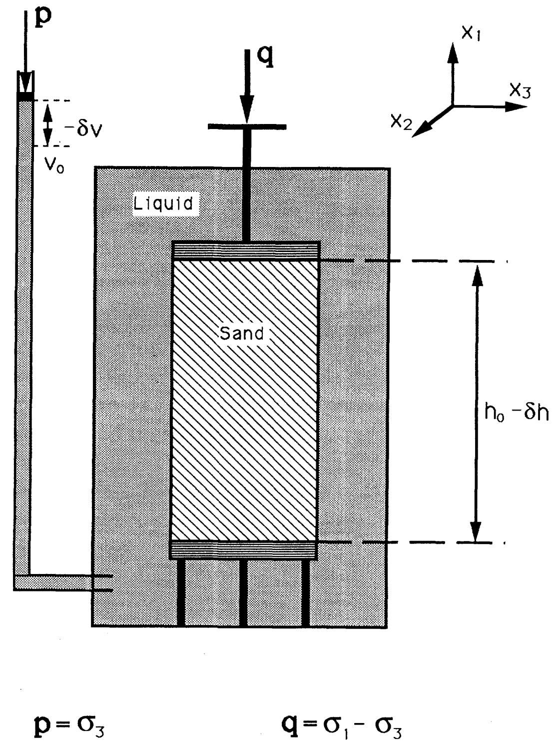

FIG. 3. Triaxial test: a granular material is contained in a flexible cylindrical membrane closed at its bottom and top by two pistons; the membrane is immersed in a liquid at pressure p and a vertical additional stress q is applied to the top piston. q is called the deviatoric stress. The test consists in measuring simultaneously the variations of height $\delta h = h_0 - h$, of volume $\delta v = v_0 - v$, of q, and of p for a given series of mechanical transformation.

In the case of noncohesive materials, piles may be built at different densities by pluviation [11]; furthermore, a good sample homogeneity is obtained when spraying the sand all over the surface and building the pile in successive horizontal slices. One may also submit the sample to vibrations in order to get very large densities. In sum, there are many different procedures of sample preparation which depend on the soil-mechanics specialist. Nevertheless, it is well accepted by these specialists that homogeneous and isotropic piles may be built at different densities within a certain range. One may then submit this homogeneous sample to a first uniform compression and reach the pressure p; this will be the set of initial conditions ($p, q = 0$) we will consider in the following.

B. Typical experimental results on sand

We consider then a triaxial test on noncohesive granular media made up of rigid grains, such as sand; the initial pile is assumed to be isotropic, homogeneous, and at a given density d_0. So, the initial conditions of the triaxial test are the pressure p, the deviatoric stress $q_0=0$, the initial volume v_0, the initial height h_0, and the initial density d_0.

The granular sample is then submitted to an imposed decrease dh of the vertical height h obtained by varying the overload stress q applied to the top piston and by keeping p constant, as explained formerly; δh (and then q) are varied slowly enough to keep the material in a quasistatic equilibrium so that the system is kept in its limit of plasticity. The test consists in keeping the sample as homogeneous as possible and in recording at the same time the volume change δv and the deviatoric stress q as functions of the height variations dh. The results are commonly summed up by giving the two plots of q/p and $\delta v/v_0$ as functions of $\delta h/h_0$. Typical results are sketched in Fig. 4; the three sets of two curves of Figs. 4(a) and 4(b) correspond to the same sand at different initial densities, as mentioned in the caption. We recall that positive δh and δv correspond to the height and volume decrease according to soil-mechanics uses so that sample volumes larger than the initial volume are located above the $\delta h/h_0$ axis as usual.

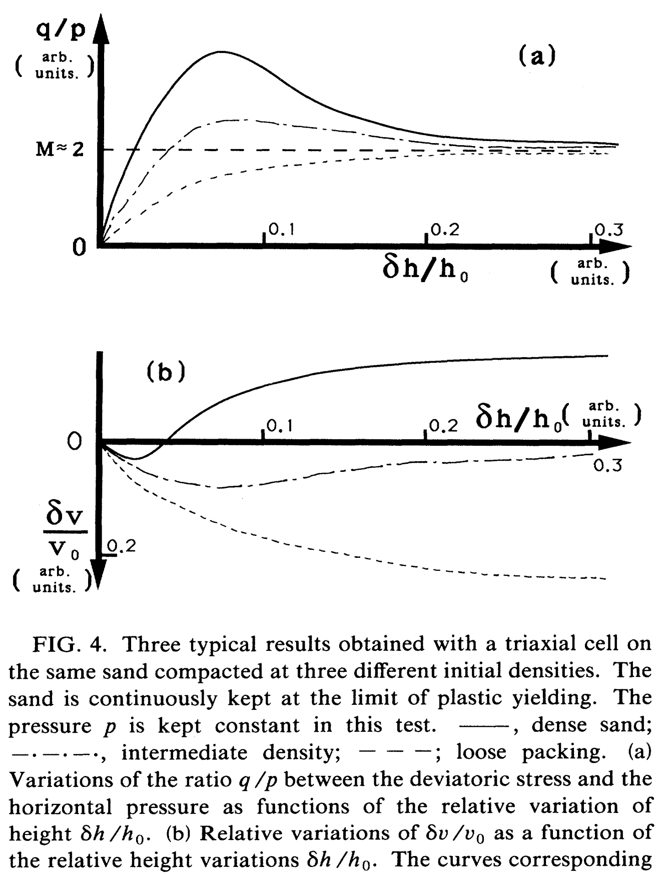

FIG. 4. Three typical results obtained with a triaxial cell on the same sand compacted at three different initial densities. The sand is continuously kept at the limit of plastic yielding. The pressure p is kept constant in this test. ——, dense sand; —·—·—·, intermediate density; — — —; loose packing. (a) Variations of the ratio q/p between the deviatoric stress and the horizontal pressure as functions of the relative variation of height $\delta h/h_0$. (b) Relative variations of $\delta v/v_0$ as a function of the relative height variations $\delta h/h_0$. The curves corresponding to the two dense media both exhibit some dilatancy effect [$K>0$ if we define the dilatancy K by $K=-(dv/v)/(dh/h)$]. The so-called "characteristic state" of Luong and Habib [13,14] is the state for which the volume is minimum ($K=0$) and for which q/p crosses the M horizontal line; the so-called "critical" state is the state obtained at large plastic yielding, which is also characterized by a $K=0$ dilatancy. The maximum value of $\delta h/h_0$ in a contraction test is obviously 1.

The q/p vs $\delta h/h_0$ variations of Fig. 4 indicate that the deviatoric stress ratio q/p tends towards the same limit value M at large $\delta h/h_0$ (i.e., $\delta h/h_0>0.3$), independent of the initial density of the packing. Furthermore, a systematic investigation [12–15] has demonstrated that M is independent of p; it depends only on the nature of the material and may be considered as an intrinsic parameter which is related to the friction angle ϕ [12–15] [it is straightforward to demonstrate that $\sin\phi=q/(2p+q)$ according to a simple Mohr-Coulomb approach without cohesion, so $\sin\phi=M/(2+M)$]. A typical value of the friction angle is 30° so that a typical value of M is 2. (ϕ ranges between 25°–40°, depending on the material.) Soil-mechanics specialists say that the material is in its "critical" state when the material has yielded largely and when its q/p value has reached M. It appears also that the density d_c of the pile in its critical state depends only on the pressure p [8,12–15]; d_c is called the critical state density.

However, the way the material reaches the critical state depends on the initial density. Figure 4 shows that when the initial sample is very loose, the ratio q/p increases monotonically and the material is always contracting. On the contrary, for an initial sufficiently dense packing, the deviatoric stress ratio q/p begins increasing, crosses the $q/p=M$ line, then reaches a maximum q_m/p value which depends on the initial density and then decreases before reaching the M value asymptotically. If one looks at the same time at variations of the volumetric strain $\delta v/v_0$, one sees that the material is first contracting until the q/p ratio crosses the $q/p=M$ line. It is then dilatant till it reaches an asymptotic value. Looking now at the density d_c of the material at large yielding at this final state, one finds experimentally that it is independent of the initial density d_0 [8,12–15] and depends only on p.

Typical experimental values are the following: $M=2$; $\phi=30°$; $M<q_m/p<10$; for Hostun sand: $1.4<d_0<1.7$; typical investigated pressure range: 30 kPa–10 MPa; typical investigated range for $\delta h/h_0$: 0.3; typical investigated range for $\delta v/v_0$: 0.2; $\delta h/h_0=0.01–0.05$ for the characteristic state.

At this stage, we want to emphasize two points. First, we have considered a material with rigid grains, such as sand. The Young modulus of each grain is then infinite so that the strain in Fig. 4 is due to plastic irreversible yielding. It might then occur (and it does) that under some special test procedure a finite stress ratio q_0/p is required to get a small yielding. Thus the curves in Fig. 4 may contain a segment of the vertical axis from 0 to the ordinate q_0/p. For instance, this is observed when the pile is not isotropic initially or when the test is carried out after a first loading-unloading cycle. This is not observed when the pile is isotropic initially.

The second point concerns the instability of a packing which has reached its q_m/p maximum value and has been submitted to a dq/dt = constant triaxial test: a slight increase of q is no longer possible at this maximum of q and a microscopic stable $d(\delta h)/h_0$ response is impossible so that the packing is unstable and a macroscopic motion, which is often a failure, occurs. This is the rather large difference between triaxial tests carried out under strain- and stress-rate control.

Interestingly these behaviors of compaction or dilatancy may be understood within the framework of the dilatancy theory [6] commonly used in soil mechanics to describe the typical sandpile behaviors described previously. In this soil-mechanics theory, it is stated that sandpile mechanics is governed by a unique friction angle ϕ, which depends only on the grain nature, but that the static equilibrium law has to be modified to also take into account the effect of dilatancy [6]. For instance, this theory predicts that if the pile is denser than d_c, its mechanics will be governed by an effective pseudofriction angle θ_m which will be larger than ϕ; conversely, if $d < d_c$ one finds a pseudofriction angle $\theta_m < \phi$ (see Ref. [6] for more information). Within this framework, the pile reaches a unique state which is characterized by its density d_c and the friction angle ϕ at large deformation. This state is called the "critical state" in soil mechanics [6,8]. Furthermore, in the same time as θ_m tends to ϕ, the density evolves to d_c, and the pile exhibits either dilatancy (for $d_0 > d_c$) or contractancy (for $d_0 < d_c$).

We want also to emphasize that in simple models such as the Granta-Gravel model [8], q_m/p cannot overpass M except when d_0 is larger than d_c. This is not true experimentally and one may observe some cases where the initial and final densities are equal but where the pile is first contracting and then dilatant; this effect then combines the existence of a maximum in the curve q/p and of a minimum in δv.

Let us now discuss briefly what would be the effect of adding some cohesion or when the grains are elastic instead of perfectly rigid on typical triaxial test results; let us start with cohesion.

Since Coulomb, it is known that a cohesion c added to a friction ϕ introduces an effective friction coefficient M which depends on the pressure p; the larger the p the smaller the M, according to

$$M = 2[\sin\phi/(1-\sin\phi)][(c/p)+1] .$$

One gets also that the maximum angle of stability θ_m is given by $\tan\theta_m = \tan(\phi) + (c/p)$ which means that the failure likely occurs deep in the pile (where p is large). The fact that M and $\tan\theta_m$ do not depend on the pressure p implies then that there is no cohesion. In the same way, the fact that the asymptotic behavior of the avalanche size $\delta\theta$ does not depend on gravity (and hence on p) proves that there is no effect of cohesion.

Turning now to the case of elastic grains, it is found experimentally that the elasticity reduces the peak of the q_m/p ratio. This implies for avalanches in particular that their size $\delta\theta$ should diminish when decreasing the Young modulus or increasing the gravity. As this is not observed experimentally, we may conclude that the grains are rigid.

IV. RETURN TO THE AVALANCHE PROBLEM; INTERPRETATION USING TRIAXIAL TEST RESULTS AND SOIL-MECHANICS CONCEPTS

We may then attempt to interpret the experimental results on avalanches of Fig. 2 as follows: first of all, the asymptotic steady state which is independent of density proves that the characteristics of the surface of the initial pile evolves to asymptotic characteristics. Furthermore, the transient behavior observed on θ_m displays a density effect at least for small density piles; it is then tempting to attribute the evolution of θ_m as a function of the avalanche number in a given series as a change of the pile density at the free surface during the first avalanches; in this case, the scenario will be the following: successive avalanches modify the structure of the pile just below the free surface due to the action of shearing forces. It results from this that if the pile was initially dense, it becomes looser, but if it was initially loose, it is compacted at the free surface, and the surface density evolves and eventually reaches a constant density d_c, independent of the initial density d.

So, we think that our experimental data of Fig. 2(a) are in agreement with the dilatancy approach [6]: in this case, θ_m and density evolve spontaneously till they reach their limit value ($\theta_m = 34°, d = d_c$). Within this framework, 34° should be equal to the internal friction angle ϕ measured at large deformations by any soil-mechanics apparatus. From the literature [11], we find indeed that $\phi = 33-35°$ for Hostun sand which is the value found here. Furthermore, this theoretical approach agrees also with the facts that θ_m should increase continuously when the initial pile is loose and decrease when the initial pile is dense, which confirms this soil-mechanics analysis. This limit value $\theta_m = 34°$ demonstrates also that cohesion is negligible in this experiment (at least its effect is smaller than the 1°–2° variation of ϕ). It is also in agreement with the compactification process observed during the two first avalanches in the case of low-density packings.

We turn now to the variations of the avalanche size [cf. Fig. 2(b)] and we focus first on the behavior of the first two avalanches of each series (the transient regime). Clearly, it exhibits some correlation between the initial density and the avalanche size $\delta\theta$, since the larger the initial density d_0 the larger the avalanche size $\delta\theta$; for instance, we find first avalanche sizes of about $\delta\theta = 4°$ for the densest packings ($d = 1.67, 1.68$) instead of 1° or 2° for the looser packings ($d = 1.45, 1.47$). This feature is in agreement with the approach based on soil mechanics [6] described previously which predicts an increase of the avalanche size when increasing density. We note also that after this transient regime an asymptotic regime is observed which exhibits size fluctuations, but which leads to a mean avalanche size $\langle \delta\theta \rangle$ of about 1.7°. This 1.7° value compares well with the other data from literature [4–7], obtained on avalanches with other devices and different silicate granular materials (glass spheres of different diameters, sand of different sizes). Another interesting result is that $\langle \delta\theta \rangle$ does not depend on the gravity since our $\langle \delta\theta \rangle$ value at $100G$ [Fig. 2(b)] is identical to those found in the other experiments [4–7] (i.e.,

$\langle \delta\theta \rangle \approx 2°$); we may then conclude that the cohesion effect between grains is negligible from these data.

In the same spirit, we note that the asymptotic value of $\theta_m = \phi$ fluctuates within 2°. This width compares well to the mean limit size of the avalanche $\langle \delta\theta \rangle = 1.7°$ as found from Fig. 2(b) data, so that the physics of the avalanche seems to be governed by an erratic process as if the internal friction angle ϕ could not be defined within an accuracy better than 2°. Coming back to our analogy with a soil-mechanics theoretical approach, we can try and test this hypothesis and look at a triaxial test curve to demonstrate the existence of such fluctuations in the internal friction angle ϕ. Indeed, the scatter of the experimental data from a trial to another one allows us to measure the friction angle within an accuracy of at most 2°; however, if we examine carefully each trial curve, we find that the stress-strain experimental curves are smooth in general (their fluctuations correspond to $\delta\phi = 0.5°$ and not to 2°). So it seems to us that we cannot really conclude whether the erratic size of the avalanche is due to a random distribution of the friction angle ϕ as a function of the strain (or of space), or if these $\langle \delta\theta \rangle$ fluctuations observed during the avalanche experiment are induced directly by a nonlinear relaxation process and are the signature of specific physical and mechanical processes involved in the avalanche mechanism itself.

A point which is worth emphasizing is the following: indeed soil-mechanics theory explains the existence of large avalanches for dense piles, but it cannot predict the existence of avalanches for loose piles. For loose piles, this theoretical approach predicts a continuous densification process with continuous, slow, and infinitely small local yieldings. So, the two first macroscopic events observed in Figs. 2(a) and 2(b) for $d = 1.45$ and 1.47 cannot be understood within this framework [6]. In order to investigate this phenomenon, we have then measured the weight of the sand flowing from the box during the avalanche processes and have found that it is proportional to the measured $\delta\theta$, *except for these four peculiar events where no sand was found to be flowing out.* This makes the "avalanche" phenomenon when $d < d_c$ a little more understandable: it is no longer a flowing motion but only local rearrangements of grains which makes the pile denser in its whole. These four events prove only that densification is a discontinuous process instead of a continuous one, with sudden breaks of stability at a global scale (not only at a local scale) which enhance macroscopic events, so that the whole pile is concerned.

This eventually allows us to reinterpret these results within the framework of the soil-mechanics theory, which predicts that for piles looser than d_c a continuous densification occurs as far as θ increases from θ_m ($\theta_m < \phi$ for $d < d_c$) to ϕ without the flow of sand out from the container. We do observe this densification indeed, except that it is a discontinuous evolution. The existence of this discontinuity makes the mechanics of granular material a little more nonlinear.

V. CONCLUSION

As a conclusion, we think that these experiments demonstrate the existence of a density effect which is more or less in agreement with triaxial data [6] and with a "critical state analysis" [6] as it is called in soil mechanics: in particular, we have shown that the denser the packing the larger the maximum angle of repose and that the first avalanche size $\delta\theta$ depends on the packing density. We have found that an asymptotic regime is reached after a few avalanches. It is characterized by a typical avalanche size $\langle \delta\theta \rangle = 2°$ which is gravity independent. These results seem to us very stimulating since they also demonstrate the effect of density on the avalanche size; the denser the packing the larger the avalanche. It has also been found that a macroscopic discontinuous densification occurs in the case of very loose packings. These effects were not known and not forecast by soil mechanics.

As a final remark, part of this work was dedicated to the search of a parameter controlling the avalanche size and it was expected that density was this one. It has clearly succeeded in demonstrating that density is one of them, but we have not yet been able to make the avalanche size vanish and to get a state of critical bifurcation, obeying the BTW model, as we were hoping [6]. We have to seek for another controlling parameter, so we hope that this work will stimulate further developments of avalanche experiments in centrifuge.

In any case we note that we have not yet succeeded in getting avalanche sizes tending to 0, so that we cannot yet approach the conditions under which critical fluctuations of avalanche size on large samples may be observed [6]. However, we have repeated the experiment reported by Held *et al.* [7] using a similar device (a conic pile and very gentle pluviation on the pile top). We have obtained similar results: we confirm then that one may obtain critical fluctuations of the avalanche size for piles small enough, but there is a crossover to classical fluctuations, with $\delta\theta = 2°$, when the pile slope is long enough, i.e., when it exceeds 30 to 40 bead diameters ($2R$). We have measured the critical exponent governing the probability distribution of avalanche masses $P(m)$ in the critical regime and have found the same behavior [i.e., $P(m) = m^{-1.6}$]. We notice also that the limit ratio $L_M/2R$ between the slope length L_M and the bead size $2R$, which defines the crossover between the critical behavior and the subcritical one, is such as $\langle \delta\theta \rangle L_M = 2R$ (where R is the grain size). As $\langle \delta\theta \rangle$ is also the accuracy $\delta\phi$ with which the friction angle ϕ may be measured with a slope-stability experiment in the macroscopic regime, these results confirm the relationship $\langle \delta\theta \rangle L_M = 2R$ about the finite-size effect which has been stated first in Ref. [6] and restated in Ref. [17] a little later.

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[1] P. Bak, C. Tank, and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987); Phys. Rev. A **38**, 364 (1988); C. Tang and P. Bak, Phys. Rev. Lett. **60**, 2347 (1989).

[2] L. P. Kadanoff, S. R. Nagel, L. Wu, and S.-M. Zhou, Phys. Rev. A **39**, 6524 (1989).

[3] S. B. Savage and Y. Nohguchi, Acta Mech. **75**, 153 (1988).

[4] P. Evesque and J. Rajchenbach, C. R. Acad. Sci. Paris Ser. II **307**, 223 (1988); *Poudres et Grains,* edited by J. Biarez and R. Gourvès (Balkema, Rotterdam, 1989), pp. 217–224.

[5] H. M. Jaëger, C.-H. Liu, and S. Nagel, Phys. Rev. Lett. **62**, 40 (1989).

[6] P. Evesque, Phys. Rev. A **43**, 2720 (1991); J. Phys. (Paris) **51**, 2515 (1990).

[7] G. A. Held, D. H. Solina II, D. T. Keane, W. J. Haag, P. M. Horn, and G. Grinstein, Phys. Rev. Lett. **65**, 1120 (1990).

[8] A. N. Schofield and C. P. Wroth, *Critical State of Soil Mechanics* (McGraw-Hill, London, 1968).

[9] P. Habib, Rev. Fr. Géotech. **31**, 5 (1986).

[10] E. Flavigny, J. Desrues, and B. Palayer, Rev. Fr. Géotech. **53**, 67 (1990).

[11] As a matter of fact, it is well known that one can pack a granular material at different densities by changing building conditions. For instance, one way of getting different packings is to let fall from a hooper at a given height h the granular material under a given flow Q (Q is controlled by the width of the aperture as in the hour glass); the pile density depends on h and Q: the higher the fall from the hooper and/or the smaller the hooper aperture (and then the flow Q), the denser the pile. Furthermore, a good sample homogeneity is obtained when spraying the sand all over the surface and building the pile in successive horizontal slices.

[12] M. P. Luong, C. R. Acad. Sci. Paris Ser. B **237**, 305 (1978).

[13] P. Habib and M. P. Luong (unpublished).

[14] M. P. Luong, in *Powders and Grains,* edited by J. Biarez and R. Gourvès (Balkema, Rotterdam, 1989), pp. 485–492.

[15] P. W. Rowe, in *Stress-Strain Behaviours of Soils,* edited by R. G. H. Parry (Cambridge University, Cambridge, England, 1971), pp. 143–194; Geotechnique **19**, 75 (1969).

[16] E. Michalski and A. Rahma, *Banque de Donnėes Modélisol No. 89SGN117GEG,* edited by Bureau de Recherches Géologiques et Minières (Service Géologique National, Orléans, 1989).

[17] C.-H. Liu, H. M. Jaeger, and S. R. Nagel, Phys. Rev. A **43**, 7091 (1991).