Experimental observation of type-II intermittency in a hydrodynamic system

E. Ringuet, C. Rozé, and G. Gouesbet

Laboratoire d'Energétique des Systèmes et Procédés, Institut National des Sciences Appliquées de Rouen, Complesce de Recherches Interprofessionnel en Aéro-thermochimie, Boîte Postale 08, 76131 Mont Saint Aignan CEDEX, France

(Received 20 July 1992; revised manuscript received 13 October 1992)

Candidates for type-II Pomeau-Manneville intermittency are reported in a hydrodynamic system. Two time series are studied with return maps, power spectra, statistical features of the laminar period lengths, and correlation dimensions. Four degrees of freedom only are seemingly necessary to describe the dynamics.

PACS number(s): 05.45.+b, 47.52.+j

The loss of stability of a limit cycle may theoretically lead to the observation of three kinds of intermittency characterized by laminar phases interrupted by short chaotic bursts [1–3]. In type-I intermittency, a real eigenvalue of the linearized Poincaré map crosses the unit circle at (+1). In type-II intermittency, a pair of conjugate complex eigenvalues simultaneously crosses the unit circle and, in type-III intermittency, a real eigenvalue crosses at (−1). They correspond to saddle-node, Hopf, and subharmonic bifurcations, respectively. Therefore they involve one fundamental frequency F_0, two frequencies F_0 and F_1, and two frequencies F_0 and $F_0/2$, again, respectively. Type-I and type-III intermittencies have already been observed in hydrodynamic systems [4–5]. The only clear-cut experimental observation of type-II intermittency we are aware of was performed in a coupled-oscillator circuit simulating a theoretical model [6]. However, Haucke *et al.* [7] reported measurements on a convecting dilute solution of ^{3}He in superfluid ^{4}He showing a transition from periodic to chaotic behavior via intermittency. They claimed that their observations are consistent with type-II or type-III intermittency but without clearly concluding about the point to know which kind of intermittency was actually observed. In contrast, the present work reports on intermittency behavior in a hydrodynamic system pertaining to the type-II case.

Observations have been carried out in so-called hot-wire experiments in which the free surface of a liquid is heated by a hot wire. For gaining a background in these experiments and associated ones in which heating is performed by using a laser, the reader may consult Refs. [8,9] and references therein. The experimental setup is similar to the one described by Rozé, Darrigo, and Gouesbet [10], but for two modifications: (i) the wire length is 6 cm instead of 3 and (ii) it is current controlled instead of being temperature controlled. The diameter of the wire is 20 μm and the temperature coefficient of the material (platinum) is $\alpha=3.9\times10^{-3}$ K^{-1}. The wire resistance can be written as $R(T)=R_0[1+\alpha(T-T_0)]$ in which R_0 ($\simeq 18\ \Omega$) is the wire resistance at the reference temperature ($T_0=25\,°\mathrm{C}$) and T is the spatially averaged wire temperature. The wire is placed in a silicon oil (Prandtl number of 130) defining an angle α ($\alpha\simeq0.001$ rad) with respect to the free surface. The silicon oil is contained in a tank ($17\times12\times10$ cm^3). Tank walls are kept at a constant temperature (T_0) by using thermostated water. The tank itself is located in a thermostated box which keeps the ambient temperature at T_0 within 0.01 K. The control parameter is the distance d from the wire to the free surface which is adjusted with an accuracy of 20 μm. The wire is supplied with a constant current $I=280$ mA by using a perfect current source monitored by a microcomputer.

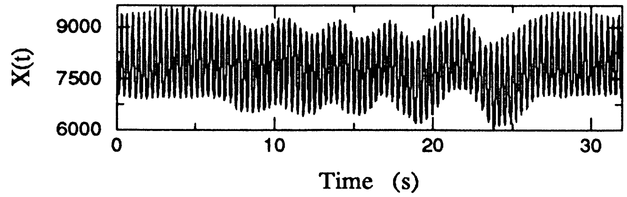

FIG. 1. Short part of the time series for $d=0.66$ mm.

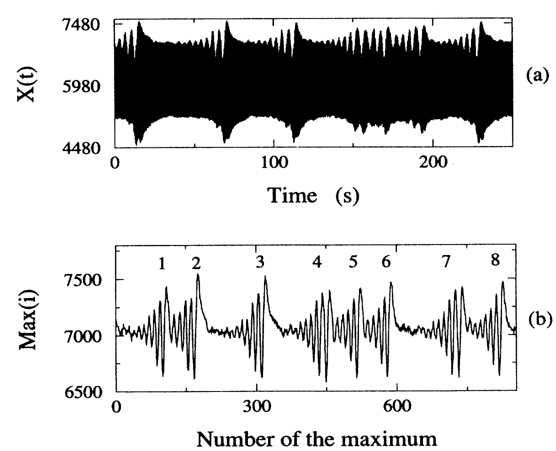

FIG. 2. (a) Long part of the time series for $d=0.64$ mm. (b) The time evolution of the maximum is used to detect the laminar periods (the end is numbered).

The studied signal $X(t)$ is the wire voltage which is related to the spatially averaged temperature of the wire, simply called "temperature" in this report. When the free surface of the liquid is motionless, the temperature is constant defining a basic steady state. When oscillatory instability occurs, a temperature gradient appears moving along the wire. Then the time signal exhibits oscillatory variations which are studied by using a discrete fast Fourier transform (FFT) routine on a sequence of 8192 sampled values, and also by other tools to be described. The cutoff frequency of the time signal is about 100 Hz.

When the distance is decreased, one observes a supercritical Hopf bifurcation from the basic steady state to a limit cycle at $d \simeq 4$ mm. At $d = 0.7$ mm, the signal is still periodic with a fundamental frequency equal to 2.8 Hz. At $d = 0.68$ mm, another bifurcation has occurred in which a second frequency appears at about 0.2 Hz in the power spectrum. The obtained signals then correspond to good candidates for type-II intermittency.

An example of a laminar period is displayed in Fig. 1, for $d = 0.66$ mm, in which the growing influence of the second frequency is clearly evidenced. A long-time series at $d = 0.64$ mm is displayed in Fig. 2(a) revealing the intermittent appearance and development of the second frequency. To clarify the meaning of the words laminar and turbulent in the context of the present experiments, the visual characterization of the time series is, however, better exemplified by only plotting the time evolution of the maxima Max(i) of the signal, as displayed in Fig. 2(b), showing eight laminar periods.

From the discrete times signal Max(i), it is possible to define a third return map Max($i+3$) vs Max(i) as displayed in Fig. 3. for a long laminar period. In this map, one observes an unstable fixed point from which the orbit departs by spiraling outward, and near which it is eventually reinjected, missing a homoclinic connection. It is, however, to be noted that the displayed map is actually a projection of a three-dimensional map (see, for instance, Ref. [11]) corresponding to a flow in $\mathbb{R}^4$, in contrast with a Shil'nikov behavior which may be described in $\mathbb{R}^3$. This point will be later confirmed by fractal dimension evaluations. In connection also with the fact that the flow evolves in $\mathbb{R}^4$, it must be pointed out that the secondary Hopf bifurcation is subcritical (a supercritical secondary Hopf bifurcation would lead to a quasiperiodic motion on a two-torus in $\mathbb{R}^3$).

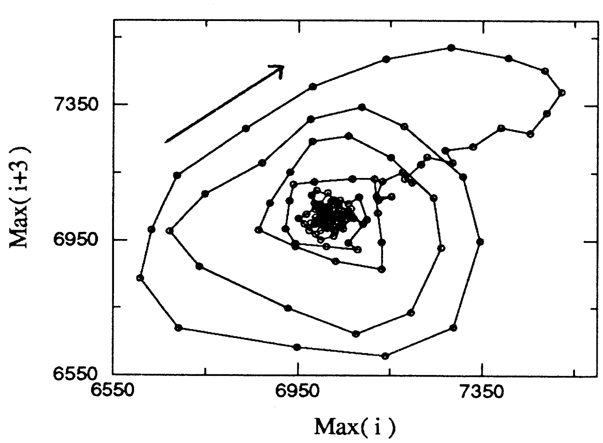

FIG. 3. Illustration of the growing influence of the second frequency by using a third return map.

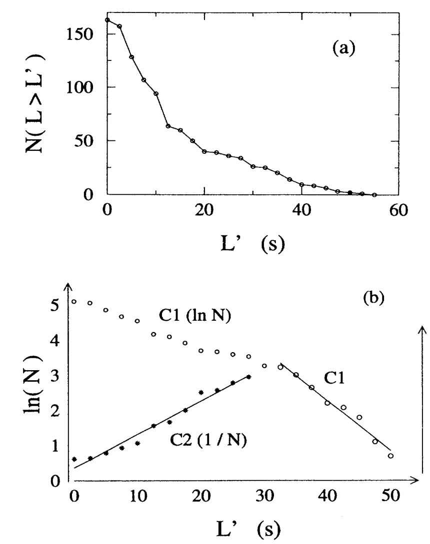

FIG. 4. (a) Statistic study of the reinjection process. (b) Comparison of our experimental results with theoretical predictions.

Statistical features of the reinjection process are illustrated in Fig. 4(a) displaying the number N $(L \geq L')$ of laminar period having a length L larger than or equal to L'. Very long laminar periods may be observed. Indeed, an orbit which would exactly establish a homoclinic connection would take an infinite time. Due to the presence of a small slow drift during the experiment and of external noise, the characterization of the shortest laminar periods ($L \lesssim 7$ s) may be sometimes difficult to accurately define. However, this fact would not spoil the overall information conveyed by Fig. 4(a). These results in Fig. 4(a) are qualitatively similar to the ones obtained in a type-II intermittency model by Richetti, Argoul, and Arneodo [2] in which the distribution N exhibits short laminar periods and shows the presence of long ones with, however, the difference that long ones are privileged. Figure 4(a) is also similar to Fig. 5 in Haucke *et al.* [7]

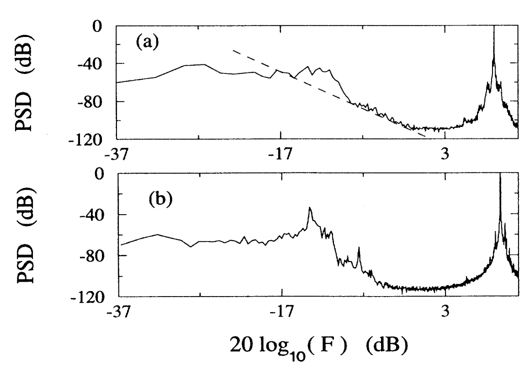

FIG. 5. Averaged power spectral density obtained by using (a) 12 power spectra, $d = 0.66$ mm and (b) 14 power spectra, $d = 0.64$ mm.

exemplifying the fact that such a distribution is not by itself an unambiguous signature of type-II intermittency. It simply rules out the possibility of type-I intermittency which does not exhibit a large number of short laminar periods.

A complementary presentation of these results is given in Fig. 4(b) showing $\ln(N)$ and $1/N$ vs L' in order to point out power-law behavior. For points labeled C1 [$\ln(N)$ vs L'], a clear-cut power-law behavior is exhibited for L' bigger than about 30 s. This is in agreement with the Manneville prediction for type-II intermittency in the limit of big values of L' [12]. Conversely, when L' is small, the Manneville prediction implies that $1/N$ is proportional to L', as shown in Fig. 4(b) (points labeled C2).

The display of power spectra is also of interest. Averaged power spectra for $d=0.66$ and 0.64 mm are displayed in Figs. 5(a) and 5(b), respectively. Figure 5(a) shows the existence of a power-law behavior with a slope equal to (-4) specified by a dashed line. Such a behavior is also in agreement with predictions of a theoretical model [2]. It is, however, not observed in Fig. 5(b). This is attributed to the fact that the distance from the bifurcation threshold has increased from Fig. 5(a) to 5(b). Indeed, in the theoretical model by Richetti, Argoul, and Arneodo, a power-law behavior is observed in the spectrum [Fig. 5(b), in Ref. [2]] when the control parameter is immediately above the intermittency threshold. Considering in our experiments that the modification between Figs. 5(a) and 5(b) corresponds to a change of d equal to 20 μm only, it is clear that approaching nearer to the threshold is certainly a difficult task indeed. Also, in any case, the power-law behavior does not extend to arbitrary small frequencies corresponding to arbitrary long laminar periods.

Finally, correlation dimensions are evaluated by using a time-delay method with a Grassberger-Procaccia algorithm. Details in our notations and our implementation of the algorithm may be found in Refs. [8] and [9], with references therein devoted to prior pioneering literature. $N=80\,000$ experimental data points sampled over about $\frac{1}{2}$ hour, corresponding to a total observation time of about 5000 short periods $\mathcal{T}_0$, have been used. Short period here means the period associated with the basic frequency appearing after the first Hopf bifurcation. With $\mathcal{T}_0 \simeq \frac{1}{3}$ s, about 15 points are sampled per $\mathcal{T}_0$. An example of local slopes versus $\log_{10}(r)$ [8,9] with $m=1000$ central vectors and reconstructed phase-space dimensions from $n=10$ to 80 by steps $\Delta n=10$ is shown in Fig. 6. For this figure, correlation dimensions evaluated on the chosen scaling domain [for $\log_{10}(r)$ between 2.3 and a bit more than 2.5] are found to be $D_2=3.1$ and 3.4 for $n=40$ and 80, respectively. For $n=50$, correlation dimensions are found to be 3.3 ± 0.3 and 3.1 ± 0.2 for $d=0.66$ and 0.64 mm, respectively. Although evaluations of such dimensions on experimental signals are somewhat difficult and inaccurate, our results indicate that the description of the dynamics would require four degrees of freedom in agreement with the model proposed by Richetti, Argoul, and Arneodo [2]. Finally, it is worthwhile noting that, although this report focuses on type-II intermittency, hot-wire experiments may also exhibit other behaviors including quasiperiodicity or type-I intermittency [13].

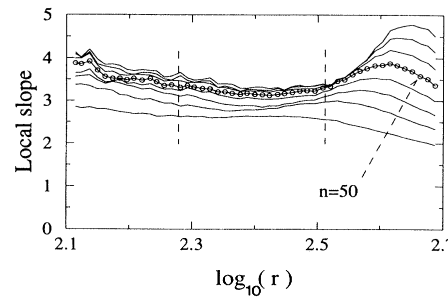

FIG. 6. Local slopes computed from the time series obtained with $d=0.64$ mm.

We thank P. Manneville for helpful critical remarks and Y. Pomeau for his interest in this work. The Laboratoire d'Energétique des Systèmes et Procédés is "Unité de Recherche Associée du Centre National de la Recherche Scientifique No. 230."

[1] Y. Pomeau and P. Manneville, Commun. Math. Phys. **74**, 189 (1980).
[2] P. Richetti, F. Argoul, and A. Arneodo, Phys. Rev. A **34**, 726 (1986).
[3] M. A. Rubio, M. De La Torre, and J. C. Antoranz, Physica D **36**, 92 (1989).
[4] P. Bergé, M. Dubois, P. Manneville, and Y. Pomeau, J. Phys. (Paris) Lett. **41**, L344 (1980).
[5] M. Dubois, M. A. Rubio, and P. Bergé, Phys. Rev. Lett. **51**, 1446 (1983).
[6] J. Y. Huang and J. J. Kim, Phys. Rev. A **36**, 1495 (1987).
[7] H. Haucke, R. E. Ecke, Y. Maens, and J. C. Wheatley, Phys. Rev. Lett. **53**, 2090 (1984).
[8] G. Gouesbet, Phys. Rev. A **42**, 5928 (1990).
[9] S. Meunier-Guttin-Cluzel, B. Maheu, and G. Gouesbet, Physica D **58**, 423 (1992).
[10] C. Rozé, G. Darrigo, and G. Gouesbet (unpublished).
[11] F. Argoul, A. Arneodo, and P. Richetti, J. Phys. (Paris) **49**, 767 (1988).
[12] P. Manneville, J. Phys. (Paris) **41**, 1235 (1980).
[13] E. Ringuet, C. Rozé, and G. Gouesbet (unpublished).