Hybrid reward-punishment in feedback-evolving game for common resource governance

Zhengyuan Lu, Shijia Hua,* Lichen Wang, and Linjie Liu†
College of Science, Northwest A & F University, Yangling 712100, China

(Received 2 July 2024; accepted 6 August 2024; published 3 September 2024)

How to maintain the sustainability of common resources is a persistent challenge, as overexploiters often undermine collective efforts by prioritizing personal gain. To mitigate the overexploitation of resources by violators, previous theoretical studies have revealed that the introduction of additional incentives, whether to reward rule-abiding cooperators or to punish those who overexploit, can be beneficial for the sustainability of common resources when the resource growth rate is not particularly low. However, these studies have typically considered rewarding and punishing in isolation, thus overlooking the role of their combination in common resource governance. Here, we introduce a hybrid incentive strategy based on reward and punishment within a feedback-evolving game, in which there is a complex interaction between human decision making and resource quantity. Our coevolutionary dynamics reveal that resources will be depleted entirely, even with cooperative strategies for prudent exploitation, when resource growth is slow. When the rate of resource growth is not particularly low, we find that the coupled system can generate a state where resource sustainability and cooperation can be maintained. Furthermore, when the rate of resource growth is moderate, we find that achieving this state cannot simply allocate all incentive budgets to reward. In addition, the increase in per capita incentives significantly promotes the stability of this state.

DOI: 10.1103/PhysRevE.110.034301

I. INTRODUCTION

Curbing the overexploitation of common resources is a significant challenge facing contemporary society [1–6]. Currently, humans face the dilemma of behavioral choices when extracting common resources [7,8]. On the one hand, individuals who adhere to resource management regulations and engage in sustainable exploitation are faced with the temptation of immediate but unsustainable gains [9]. On the other hand, those who violate these rules and engage in overexploitation often reap more substantial benefits than their more responsible counterparts [10]. This difference in benefit creates an incentive for individuals to pursue overexploitation as a strategy to maximize their immediate interests, even though it is not good for everyone in the long run [11–15]. If not controlled, we might end up with nothing left for everyone, which is known as the tragedy of the commons [16,17]. To avoid this situation, we need to design reasonable regulatory mechanisms to regulate individual behavior and ensure that everyone uses resources responsibly, so that there are sufficient resources for today and the future [18–20].

The most commonly employed incentive strategies to regulate individual behavior are punishment and reward [21–26], with the former being a form of negative incentive aimed at reducing the benefits of individuals who overexploit resources [27–30], and the latter a form of positive incentive designed to increase the gains of those who exploit resources responsibly [31–33]. Previous studies, based on evolutionary game theory, have explored various types of reward and punishment strategies [34–37]. For instance, Szolnoki and Perc [38] explored the evolutionary advantage of combined reward and punishment mechanisms in spatial public goods games. Chen and Szolnoki [39] have demonstrated how the application of punishment and inspection affects behavior decision-making within feedback-evolving games when there is a feedback relationship between individual strategy selection and resource status. Building on this, Wang *et al.* [40] introduced tax-reward and tax-punishment strategies into the feedback-evolving game, revealing the significant role of resource growth rates and per capita incentives in shaping the dynamics of coupled systems.

While previous theoretical research has investigated the roles of various forms of reward and punishment strategies in feedback-evolving games, these studies have typically considered that reward and punishment are examined as if they operate independently within the population. This naturally overlooks scenarios where both reward and punishment are concurrently at play. Which type of incentive strategy, reward, punishment, or a hybrid reward-punishment, can more effectively regulate individual behavior? How should the incentive budget be allocated to better sustain cooperation and ensure the sustainability of resources? To the best of our knowledge, these questions have not yet been investigated.

Here, we introduce a hybrid reward-punishment strategy into feedback-evolving games, where individual behavioral decision making affects resource status, and changes in natural resource status in turn also affect individual decision making. We consider the tax-based incentive [41], where a certain proportion of the total tax revenue is evenly distributed among individuals who exploit resources responsibly

*Contact author: sjhua@nwafu.edu.cn
†Contact author: linjie140@126.com

(cooperators) as a reward, and the remaining portion is evenly allocated to those who overexploit resources (defectors) as the fine. Rewarding cooperators and punishing defectors are implemented by third-party institution. The reward fund is evenly distributed to all cooperators in the group to increase their benefits, while the punishment fund is used to implement punitive actions (for individuals who overexploit resources) to reduce their benefits. Here, the punishment fund is used as a cost for entrusting law enforcement to punish individuals who engage in excessive exploitation. Through analysis, we find that coevolutionary dynamics are mainly influenced by per capita incentive, resource growth rate, and incentive budget allocation rate. Specifically, when the natural growth rate of resources is low, the coupled system will evolve to a state where all individuals reasonably exploit resources, but the resources are depleted. At this point, the implementation of incentives is ineffective, and breaking this deadlock depends on the improvement of resource growth rate. When the growth rate of resources is moderate, we find that the coupled system can evolve to a state where all individuals choose to cooperate and resources are sustainable. A larger per capita incentive, coupled with a tilt of the incentive budget towards punishment, favors the emergence of this outcome. When the resource growth rate is high, the coupled system can produce an outcome where resources remain sustainable even if all individuals choose to defect. To achieve full individuals cooperation and sustainable resource statu, a significant per capita incentive is required, accompanied by a hybrid reward and punishment incentive or pure punishment being adopted.

II. MODEL AND METHODS

We consider an infinitely large, well-mixed population where N individuals are randomly selected to participate in the game. These individuals share a common resource pool with the current resource level denoted by y, which is finite and renewable. Its natural growth rate is r, and we can describe its dynamic change using the logistic growth model [42]

$$\dot{y} = ry\left(1 - \frac{y}{R_m}\right),$$

where $\dot{y}$ represents the derivative of the resource quantity y with respect to time t, and R_m is the maximum carrying capacity of the resource pool.

To simplify the model, we assume that individuals choose between two strategies: cooperation (C) or defection (D). Cooperators comply with the allocation rules and utilize resources in a manner that is considered reasonable and sustainable. The amount of resources they obtain from the common resource pool is represented as $b_L = \frac{b_m y}{R_m}$, where b_m is the maximum amount of resources an individual is allowed to use when the resource level reaches R_m. Clearly, $b_m \leqslant \frac{R_m}{N}$. Defectors do not follow the rules of resource allocation, and thus obtain more resources from the common pool. The benefit they receive is represented as $b_V = (1+\alpha)b_L$, where $\alpha > 0$ represents the severity of the defection [39]. Taking into account that individual behavior impacts the state of the resource, the control equation for the abundance of the common resource can be rewritten as

$$\dot{y} = ry\left(1 - \frac{y}{R_m}\right) - N\frac{b_m y}{R_m}[1 + (1-x)\alpha]. \tag{1}$$

To prevent defectors from unrestrainedly pursuing private gains, undermining the efforts of cooperators, and causing the problem of common resource depletion, we introduce a hybrid reward-punishment strategy based on taxation. Specifically, all individuals in the game group need to pay a tax δ, and the total tax revenue will be controlled by third-party institution. A proportion w of the total tax revenue is evenly distributed among cooperators who adhere to the rules as a reward, while the remaining proportion $1 - w$ is equally distributed among defectors who do not follow the rules as a fine. Consequently, each cooperator receives a reward $\frac{Nw\delta}{N_C}$, and each defector is fined $\frac{N(1-w)\delta}{N_D}$, where N_C is the number of cooperators and N_D is the number of defectors in the game group. Here, we consider that the cost of punishing each defector is the same as the fine imposed on each defector, with a punishment intensity of 1.

In an infinite well-mixed population, we use the replicator equation to describe the temporal evolution of competing strategies [43,44], as follows:

$$\dot{x} = x(1-x)(P_C - P_D), \tag{2}$$

where P_C and P_D are the average payoffs of cooperators and defectors, determined by the interaction between individuals and the state of the common resource pool, which can be calculated by

$$P_C = \sum_{k=0}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-1-k} \left(b_L - \delta + w\frac{N\delta}{k+1}\right), \tag{3}$$

$$P_D = \sum_{k=0}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-1-k} \left[b_L(1+\alpha) - \delta - (1-w)\frac{N\delta}{N-k}\right]. \tag{4}$$

Combining Eqs. (1)–(4), we can obtain the following feedback-evolving game system

$$\dot{x} = x(1-x)\left(\delta w\frac{1-(1-x)^N}{x} + \delta(1-w)\frac{1-x^N}{1-x} - \frac{b_m y}{R_m}\alpha\right),$$
$$\dot{y} = ry\left(1 - \frac{y}{R_m}\right) - N\frac{b_m y}{R_m}[1 + (1-x)\alpha]. \tag{5}$$

In the following, we will conduct a detailed analysis of the coevolutionary dynamics of the aforementioned system.

III. RESULTS

The system (5) has at most four boundary equilibrium points, which are $(0, 0)$, $(1, 0)$, $(0, R_m - \frac{Nb_m(1+\alpha)}{r})$, and $(1, R_m - \frac{Nb_m}{r})$. There may exist multiple interior equilibrium points. Without loss of generality, we denote them uniformly as (x^*, y^*), where $0 < x^* < 1$ and $0 < y^* < R_m$ are solutions

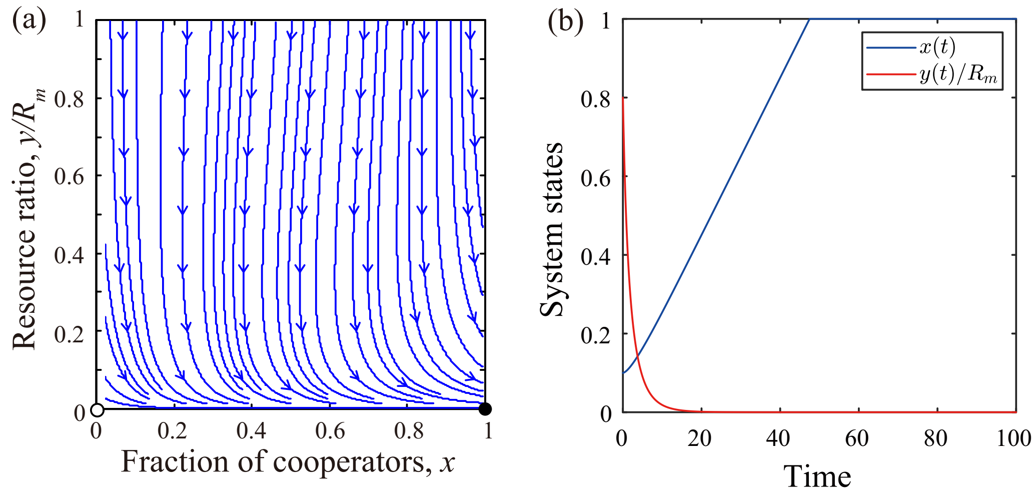

FIG. 1. Coevolutionary dynamics of the coupled system for slowly growing resource. (a) presents the phase diagrams of $x - y/R_m$. (b) displays the temporal evolution of the system's states. When the resource growth rate is low, the resource will inevitably be depleted even if all individuals choose to exploit it rationally. Parameters are $N = 1000$, $R_m = 1000$, $b_m = 0.5$, $w = 0.5$, $\alpha = 0.5$, $r = 0.4$, and $\delta = 0.04$.

to the following equations:

$$\delta w \frac{1-(1-x)^N}{x} + \delta(1-w)\frac{1-x^N}{1-x} - \frac{b_m y}{R_m}\alpha = 0,$$
$$ry\left(1 - \frac{y}{R_m}\right) - N\frac{b_m y}{R_m}[1 + (1-x)\alpha] = 0. \quad (6)$$

Based on the Jacobian matrix for various equilibrium points presented in the Appendix, we analyze the system's evolutionary dynamics in three parameter intervals, primarily using the resource growth rate as the criterion.

A. Slow resource growth rate

When $r < \frac{Nb_m}{R_m}$, the coupled system (5) only has two boundary equilibrium points, namely $(0, 0)$ and $(1, 0)$. By analyzing the eigenvalues of the Jacobian matrices corresponding to these equilibrium points, we find that the equilibrium point $(1, 0)$ is stable, while $(0, 0)$ is unstable. In Fig. 1, we present a specific numerical example to verify our theoretical analysis. In Fig. 1(a), we observe that all trajectories in the phase plane ultimately converge to the point $(1, 0)$. This implies that even if all individuals choose to cooperate, the resource eventually becomes depleted due to an excessively low rate of resource growth. In Fig. 1(b), we show the evolution of the system's state over time under specific initial conditions. We find that the frequency of cooperators gradually increases over time and eventually stabilizes at 1, indicating full cooperation among individuals. Meanwhile, the resource quantity rapidly decreases until it reaches zero.

B. Moderate resource growth rate

When $\frac{Nb_m}{R_m} < r < \frac{Nb_m(1+\alpha)}{R_m}$, the coupled system (5) has three boundary equilibrium points, namely $(0, 0)$, $(1, 0)$, and $(1, R_m - \frac{Nb_m}{r})$, among which the first two are unstable. The last one is stable when $r < \frac{Nb_m}{R_m}\frac{b_m\alpha}{b_m\alpha - \delta w - \delta(1-w)N}$ and $\delta < \frac{b_m\alpha}{w+(1-w)N}$ or $\delta > \frac{b_m\alpha}{w+(1-w)N}$. According to the theoretical analysis presented in the Appendix, the coupled system (5) may also have one or two interior equilibrium points. When the conditions for the existence of interior equilibrium points are not met (see Appendix), the coupled system only has three boundary equilibrium points. We provide a numerical example based on the stability conditions of the aforementioned equilibrium points. Specifically, when the model parameter satisfies $\delta > \frac{b_m\alpha}{w+(1-w)N}$, the right boundary equilibrium point $(1, R_m - \frac{Nb_m}{r})$ is stable. As shown in Fig. 2(a), all interior trajectories converge to this equilibrium point, indicating that all individuals adopt cooperative strategies and that resources are maintained simultaneously. We further present a new numerical example of a monostable state, where the right boundary equilibrium point is unstable and the coupled system has a stable interior equilibrium point. As shown in Fig. 2(b), the coupled system has four equilibrium points, and all interior trajectories ultimately converge to the stable interior equilibrium point. This implies coexistence of cooperators and defectors within the population while the resource is maintained.

We further provide two representative numerical examples when the coupled system has two interior equilibrium points. As shown in Fig. 2(c), the coupled system has all five equilibrium points, of which the right boundary point and one interior equilibrium point are stable. Depending on the initial conditions, a few trajectories will converge to the right boundary point, while the vast majority of trajectories converge to the stable interior equilibrium point. Regardless of the outcome, cooperation and resources can both be sustained. When we reduce the per capita incentive δ, we find that the stable interior equilibrium point approaches the horizontal axis. As depicted in Fig. 2(d), the level of cooperation and the quantity of resources at this interior stable equilibrium point are both reduced.

Now we are interested in the impact of two important parameters on the coevolutionary outcomes: the allocation weight of the incentive budget w and the per capita incentive δ. As illustrated in the Fig. 3(a), when employing a punishment strategy exclusively, the right boundary point

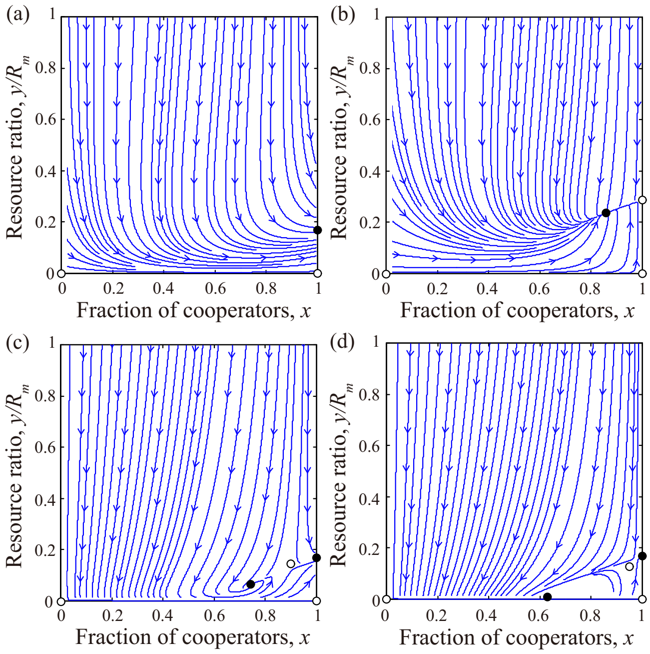

FIG. 2. Coevolutionary dynamics of the coupled system for moderately growing resource. The top row illustrates the phase diagrams where the coupled system exhibits a monostable state. The bottom row presents phase diagrams where the coupled system exhibits a bistable state. Moderate resource growth rate accompanied by reward and punishment strategies can effectively maintain resource sustainability and a high level of cooperation. Parameters are $N = 1000$, $R_m = 1000$, $b_m = 0.5$, $w = 0.5$, $\alpha = 0.5$, $r = 0.6$, and $\delta = 0.04$ in (a); $N = 1000$, $R_m = 1000$, $b_m = 0.5$, $w = 0.999$, $\alpha = 0.5$, $r = 0.7$, and $\delta = 0.05$ in (b); $N = 1000$, $R_m = 1000$, $b_m = 0.5$, $w = 0.1$, $\alpha = 0.5$, $r = 0.6$, and $\delta = 0.004$ in (c); $N = 1000$, $R_m = 1000$, $b_m = 0.5$, $w = 0.1$, $\alpha = 0.5$, $r = 0.6$, and $\delta = 0.001$ in (d).

of the coupled system remains stable. However, when w is significantly large ($w = 0.95$), meaning that the majority of the incentive budget is allocated to reward, in addition to a stable right boundary equilibrium point, the system also has a stable interior equilibrium point [see Fig. 3(b)]. When all incentive budgets are allocated to reward, the right boundary equilibrium point is no longer stable, and the system only has one stable interior equilibrium point [see Fig. 3(c)]. In Fig. 3(d), we present the variation of the attraction basin of the right boundary equilibrium point as the incentive budget allocation rate w changes. It is observed that when w is not particularly large, the attraction basin of this equilibrium point remains at 1. However, as w becomes very large, indicating a significant portion of the incentive budget is allocated to reward, the attraction basin decreases gradually with the increase of w, until it reaches 0. Furthermore, when the incentive budget is evenly allocated ($w = 0.5$), and the per capita incentive is very small, the coupled system exhibits a stable interior equilibrium point close to the horizontal axis [see Fig. 3(e)]. As δ increases, we find that the coupled system can generate bistability, with one stable equilibrium point located on the right boundary of the phase plane, and another located inside the phase plane [see Fig. 3(f)]. With a further increase in per capita incentives, the coupled system only has one stable point located at the right boundary [see Fig. 3(g)]. This indicates that the increase in per capita incentives promotes the stability of this state. In Fig. 3(h), we present the variation of the attraction basin of this right boundary point with the per capita incentive δ, and find that its attraction basin increases as δ increases, eventually stabilizing at 1.

C. Rapid resource growth rate

When $r > \frac{Nb_m(1+\alpha)}{R_m}$, the resource growth rate surpasses the gain rates of both cooperators and defectors. Then the coupled system has four boundary equilibrium points, which are $(0, 0)$, $(1, 0)$, $(0, R_m - \frac{Nb_m(1+\alpha)}{r})$, and $(1, R_m - \frac{Nb_m}{r})$. Provided the conditions outlined in the Appendix are met, the coupled system may also have up to two interior equilibrium points. Due to the high-order complexity involved, we present several representative evolutionary outcomes here.

Monostable state. We initially present three monostable outcomes. When the model parameters satisfy the condition $\delta > \max\{\frac{b_m\alpha}{w+N-Nw}, \frac{b_m\alpha}{1+Nw-w}\}$, the right boundary equilibrium point $(1, R_m - \frac{Nb_m}{r})$ is stable, while the left boundary equilibrium point $(0, R_m - \frac{Nb_m(1+\alpha)}{r})$ is unstable. As depicted in Fig. 4(a), the phase plane contains four equilibrium points. All interior trajectories converge to the right boundary equilibrium point of the phase plane, which suggests that all individuals in the population choose the cooperative strategy and the resource can be maintained. When $\delta < \min\{\frac{b_m\alpha}{1+Nw-w}, \frac{b_m\alpha}{w+N-Nw}\}$ and $r > \max\{\frac{Nb_m^2(1+\alpha)\alpha}{R_m(b_m\alpha+\delta w-\delta-\delta wN)}, \frac{Nb_m^2\alpha}{R_m(b_m\alpha-\delta w-\delta N+wN\delta)}\}$, we know that the left boundary equilibrium point $(0, R_m - \frac{Nb_m(1+\alpha)}{r})$ is stable, while the right boundary equilibrium point $(1, R_m - \frac{Nb_m}{r})$ is unstable. A representative diagram is presented in Fig. 4(b), where the phase plane includes four boundary equilibrium points. The left boundary equilibrium point is stable, and all trajectories converge to this stable equilibrium point, which implies that even if all individuals choose to overexploit the resources, the resource sustainability is still achievable. When $\{\frac{b_m\alpha}{1+Nw-w} < \delta < \frac{b_m\alpha}{w+N-Nw}$ and $r > \frac{Nb_m^2\alpha}{R_m(b_m\alpha-\delta w-\delta N+\delta wN)}\}$ or $\{\delta < \min\{\frac{b_m\alpha}{1+Nw-w}, \frac{b_m\alpha}{w+N-Nw}\}$ and $\frac{Nb_m^2\alpha}{R_m(b_m\alpha-\delta w-\delta N+wN\delta)} < r < \frac{Nb_m^2(1+\alpha)\alpha}{R_m(b_m\alpha+\delta w-\delta-\delta wN)}\}$ are satisfied, both boundary equilibrium points become unstable. We provide a numerical example that includes a stable interior equilibrium point, as shown in Fig. 4(c). We show that here are five equilibrium points in the phase plane, consisting of four unstable boundary equilibrium points and one stable interior equilibrium point. All trajectories converge to this stable point, suggesting that cooperators and defectors stably coexist within the population, and resources can also be sustained.

Bistable state. Here, due to the complexity of theoretically analyzing the stability of the interior equilibrium points, we provide two numerical examples to illustrate the outcomes of bistability. As illustrated in Fig. 4(d), the phase plane exhibits six equilibrium points, comprising four boundary equilibrium points and two interior equilibrium points. Among these, the right boundary equilibrium point and the lower-left interior equilibrium point are stable. The former represents an ideal state where all individuals opt for cooperation, ensuring the

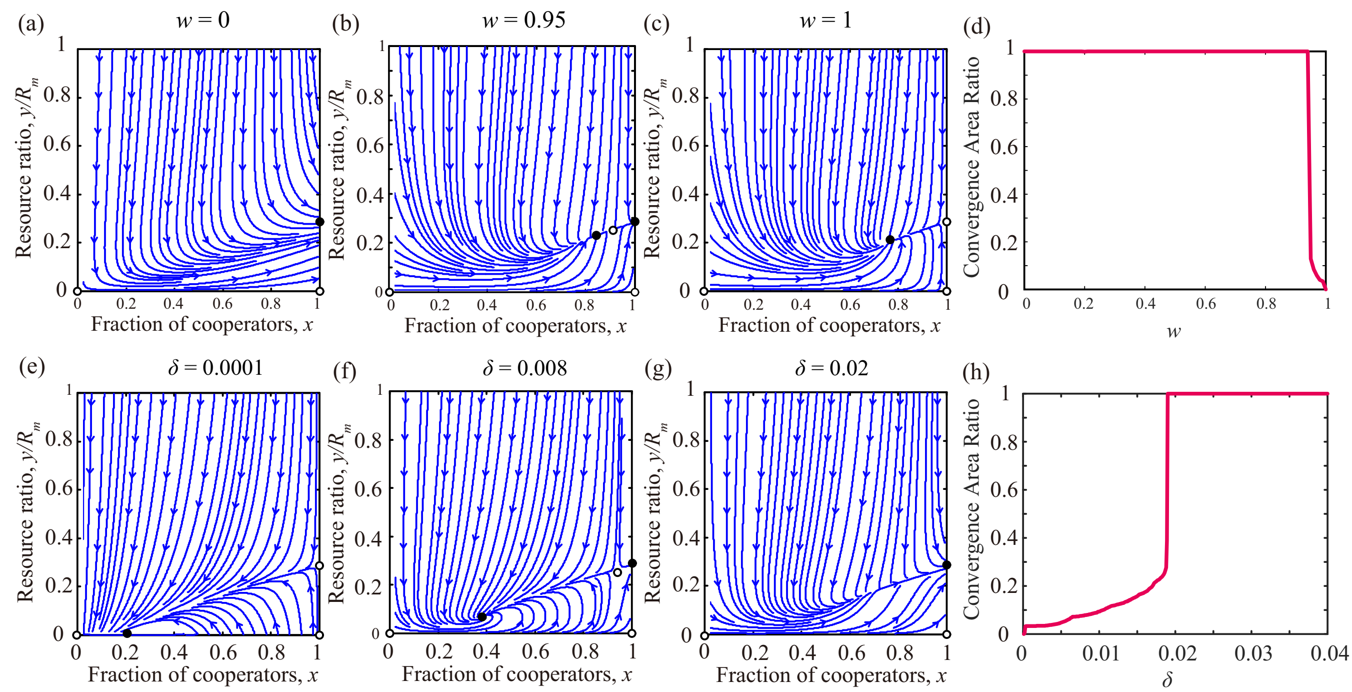

FIG. 3. Coevolutionary dynamics with different values of w and δ in the coupled system. (a)–(c) present representative phase diagrams of $x - y/R_m$ for three different values of w. (d) shows the variation of attraction basin of the right boundary equilibrium point $(1, R_m - \frac{Nb_m}{r})$ with the change in the allocation rate w. (e)–(g) present representative phase diagrams of $x - y/R_m$ for three different values of δ. (h) shows the variation of attraction basin of the right boundary equilibrium point $(1, R_m - \frac{Nb_m}{r})$ with the change in the per capita incentive δ. Punishment or a combination of rewards and punishments is superior to pure reward in maintaining full cooperation and a constant level of resources. The improvement of per capita incentives is conducive to the realization of this state. Parameters are $N = 1000$, $R_m = 1000$, $b_m = 0.5$, $\delta = 0.04$, $\alpha = 0.5$, and $r = 0.7$ in (a)–(d); $N = 1000$, $R_m = 1000$, $b_m = 0.5$, $w = 0.5$, $\alpha = 0.5$, and $r = 0.7$ in (e)–(h).

sustainability of resources. The latter indicates a stable coexistence of cooperators and defectors within the population, with the resource can also be sustained. Another bistable outcome is presented in Fig. 4(e), where the phase plane contains five equilibrium points, including four boundary equilibrium points and one interior equilibrium point. Notably, both the left boundary equilibrium point $(0, R_m - \frac{Nb_m(1+\alpha)}{r})$ and the right boundary equilibrium point $(1, R_m - \frac{Nb_m}{r})$ are stable. The majority of trajectories converge towards the right boundary equilibrium point.

Next, we analyze the impact of incentive budget allocation rate w and per capita incentive δ on coevolutionary dynamics in rapid growth rate scenario. When all incentive budgets are used to punish defectors ($w = 0$), we find that bistable outcome can appear, that is, depending on the initial conditions, the system trajectory either converges to the right boundary point $(1, R_m - \frac{Nb_m}{r})$, implying that all individuals cooperate and resources are sustainable, or converges to the left boundary point $(0, R_m - \frac{Nb_m(1+\alpha)}{r})$, implying that all individuals defect but resources are sustainable [see Fig. 5(a)]. As w increases but does not exceed the median value, we find that the equilibrium point on the left boundary becomes unstable, and all interior trajectories converge to the right boundary point [see Fig. 5(b)]. When only a small amount of incentives are allocated to punishment, we find a new bistable outcome, where the trajectory either converges to the right boundary point or converges to the stable interior point, depending on the initial conditions [see Fig. 5(c)]. When all incentives are used to reward cooperators, we find that the right boundary equilibrium point becomes unstable, and all system trajectories converge to the interior equilibrium point [see Fig. 5(d)]. In Fig. 5(e), we investigate the variation of the attraction domain of the right boundary equilibrium point with the incentive budget allocation rate w. We find that as w increases, the attraction domain first increases, then reaches its maximum value of 1, and then gradually decreases to 0. This emphasizes that relying solely on reward or punishment may not be optimal and requires both reward and punishment to work together. For the effect of per capita incentives, we find that as per capita incentives increase, the system is more likely to stabilize at the right boundary equilibrium point [see Figs. 5(f)–5(i)].

Finally, we provide numerical simulations to study the evolutionary outcomes of coupled systems for different model parameters. As shown in Fig. 6, when the resource growth rate is low, we find that no matter how the incentive budget is allocated, we cannot escape the tragedy of resource depletion. As the growth rate of resources increases, the system can achieve an ideal state where all individuals choose cooperative behavior and resources are sustainable [orange area in Fig. 6(a)]. In addition, we can observe that as the incentive budget shifts from punishment to reward, the attractive domain of this ideal state gradually narrows, indicating that shifting the incentive budget towards punishment is beneficial for cooperation and sustainable resources. In addition, when the per capita incentive is small, the attraction domain of the

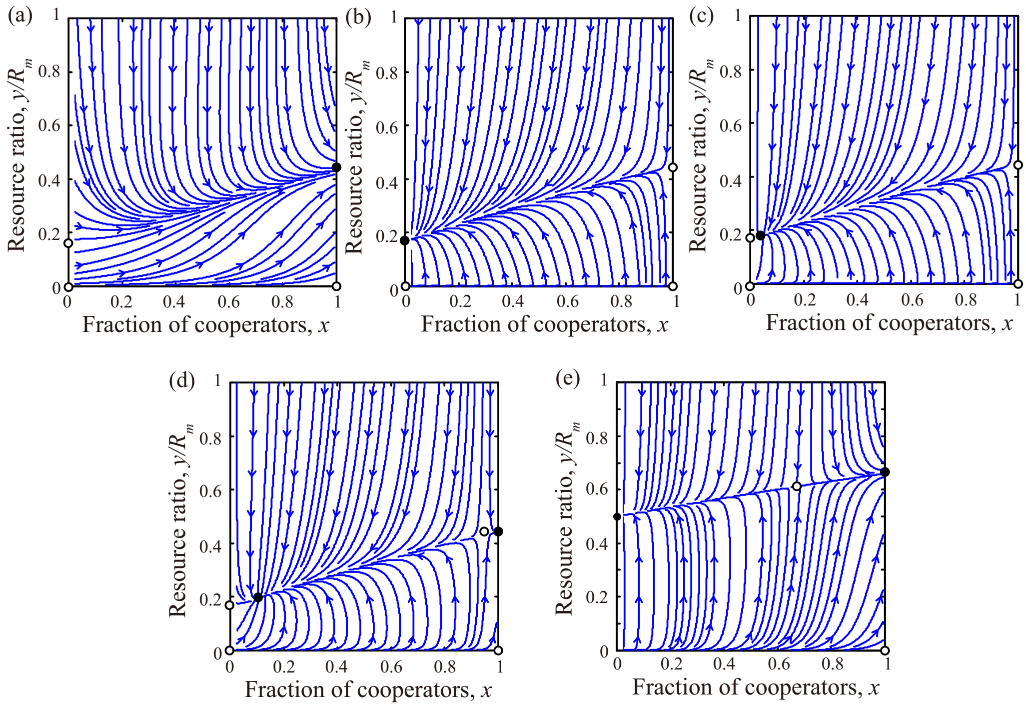

FIG. 4. Coevolutionary dynamics of the coupled system for rapidly growing resource. (a)–(e) present representative phase diagrams of $x - y/R_m$. In the scenario of rapid growth rate, coupled system can generate monostable (top row) and bistable results (bottom row), and resources can be maintained regardless of which result appears. Parameters are $N = 1000$, $R_m = 1000$, $b_m = 0.5$, $w = 0.5$, $\alpha = 0.5$, $\delta = 0.08$, and $r = 0.9$ in (a); $N = 1000$, $R_m = 1000$, $b_m = 0.5$, $w = 0.5$, $\alpha = 0.5$, $\delta = 0.00001$, and $r = 0.9$ in (b); $N = 1000$, $R_m = 1000$, $b_m = 0.5$, $w = 0.9$, $\alpha = 0.5$, $\delta = 0.002$, and $r = 0.9$ in (c); $N = 1000$, $R_m = 1000$, $b_m = 0.5$, $w = 0.5$, $\alpha = 0.5$, $\delta = 0.01$, and $r = 0.9$ in (d); $N = 1000$, $R_m = 1000$, $b_m = 0.5$, $w = 0.001$, $\alpha = 0.5$, $\delta = 0.05$, and $r = 1.5$ in (e).

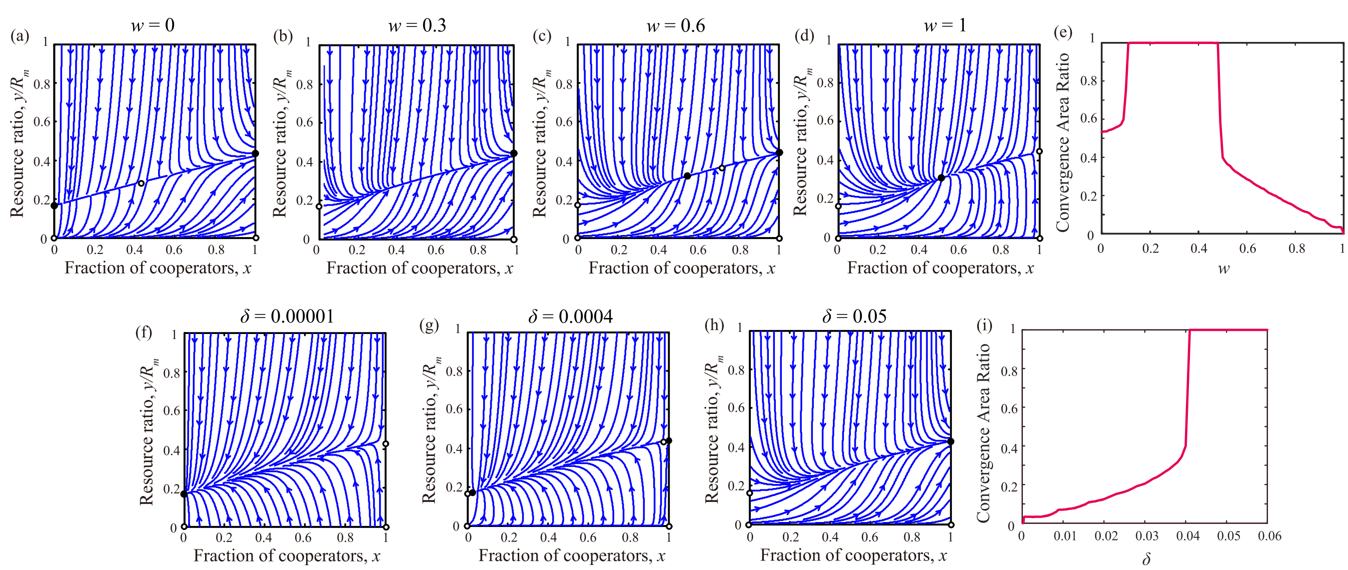

FIG. 5. Coevolutionary dynamics with different values of w and δ in the coupled system for rapidly growing resource. (a)–(d) present representative phase diagrams of $x - y/R_m$ for four different values of w. (e) shows the variation of attraction basin of the right boundary equilibrium point $(1, R_m - \frac{Nb_m}{r})$ with the change in the allocation rate w. The results show that a hybrid reward and punishment mechanism is more conducive to achieving full cooperation and sustainable resource status compared to pure reward and punishment. (f)–(h) present representative phase diagrams of $x - y/R_m$ for three different values of δ. (i) shows the variation of attraction basin of the right boundary equilibrium point $(1, R_m - \frac{Nb_m}{r})$ with the change in the per capita incentive δ. Higher per capita incentives are more conducive to achieving full cooperation and sustainable resource status. Parameters are $N = 1000$, $R_m = 1000$, $b_m = 0.5$, $\delta = 0.04$, $\alpha = 0.5$, and $r = 0.9$ in (a)–(e); $N = 1000$, $R_m = 1000$, $b_m = 0.5$, $w = 0.5$, $\alpha = 0.5$, and $r = 0.9$ in (f)–(i).

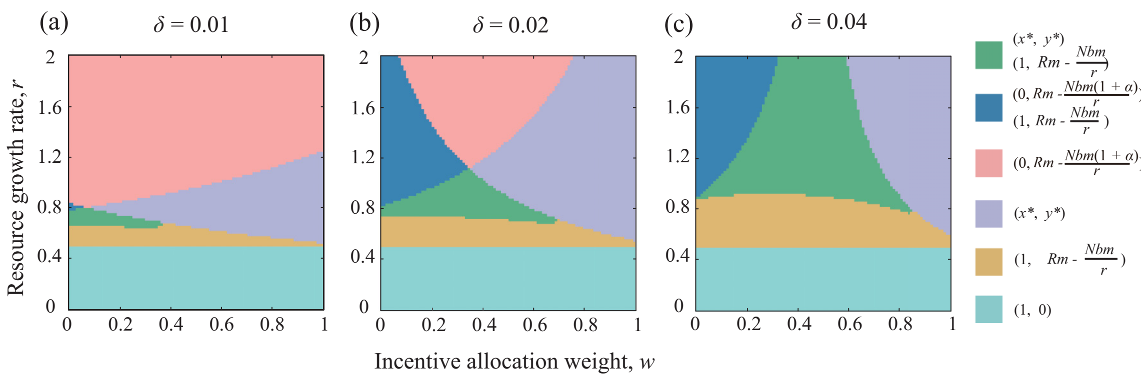

FIG. 6. Evolutionary outcomes vary with incentive allocation weight w and resource growth rate r for three different values of per capita incentive δ. When the growth rate of resources is moderate, pure punishment or a mixed reward and punishment is more conducive to full cooperation and resource sustainability, while higher per capita incentives are more conducive to achieving this state. Parameters are $N = 10$, $R_m = 10$, $b_m = 0.5$, $\alpha = 0.5$, and $\delta = 0.01$ in (a); $\delta = 0.02$ in (b); $\delta = 0.04$ in (c).

left boundary equilibrium point, implying that all individuals choose to defect and resources are sustainable, is very large [see the pink area in Fig. 6(a)]. As per capita incentives increase, we find that the orange area is getting larger [see Figs. 6(b) and 6(c)], while the pink area is getting smaller until it no longer exists [Fig. 6(c)]. Our results emphasize that increasing per capita incentives and allocating more incentive budgets to punishments are beneficial for cooperation and the maintenance of common pool resources. These results are presented more intuitively in Fig. 7. Compared to using all incentive budgets for reward [see Fig. 7(c)], using all incentive budgets for punishment ($w = 0$) can promote the stability of the ideal state in a larger parameter range [see Fig. 7(a)]. Meanwhile, the increase in per capita incentives also promotes full cooperation and resource sustainability.

IV. DISCUSSION

How to effectively use reward and punishment mechanisms to regulate individual behavior and promote sustainable use of resources when dealing with the problem of common resource extraction is a question worthy of in-depth investigation [45,46]. Here, we have established a feedback-evolving game model and introduced a hybrid reward and punishment strategy. We have found that the coevolutionary dynamics of the system are influenced by the growth rate of resources, the allocation rate of incentive budgets, and per capita incentives. Specifically, when the growth rate of resources is relatively low, regardless of the incentives used, even if all individuals choose to exploit resources reasonably, resources will inevitably fall into a state of depletion. When the growth rate of resources is moderate, the coupled system can produce monostable and bistable states, and resources can achieve sustainability. In addition, we have also found that achieving a state of all individuals choosing to cooperate and resource sustainability cannot excessively allocate incentive budgets to reward, and higher per capita incentives are more conducive to achieving this state. When the resource growth rate is high, even if all individuals choose to overexploit, the resources can still be sustainable.

In our coupled social-resource system, the natural growth rate of resources plays a crucial role in affecting the coevolutionary dynamics of the system. The growth rate of resources not only determines to what extent they can self recover and

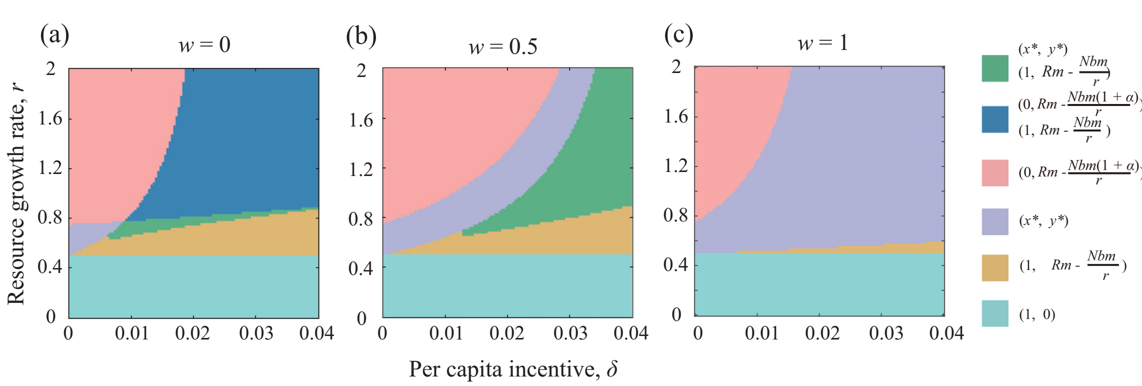

FIG. 7. Evolutionary outcomes vary with per capita incentive δ and resource growth rate r for three different values of incentive allocation weight w. When the resource growth rate is moderate, a strategy that combines both punishment and reward, or even pure punishment, is more conducive to achieving full cooperation and resource sustainability. Additionally, higher per capita incentives further support the realization of this state. Parameters are $N = 10$, $R_m = 10$, $b_m = 0.5$, $\alpha = 0.5$, and $w = 0$ in (a); $w = 0.5$ in (b); $w = 1$ in (c).

regenerate, but also determines individual strategic choices and the design of control measures. When the growth rate of resources is low, any degree of exploitation and utilization will quickly lead to resource depletion, forcing humans to immediately stop exploitation in order to avoid the tragedy of the commons. However, when the growth rate of resources is high, they can be quickly restored and replenished, and even if all individuals overexploit resources within a controllable range, the resources will not be depleted. Previous theoretical studies have already demonstrated these points [39,40], but our work differs from these in that it involves more diverse dynamics, such as different forms of bistability that can be generated. More importantly, we have provided more effective incentive budget allocation schemes for achieving full cooperation and sustainable resource status under different resource growth rate scenarios.

Reward and punishment are commonly used incentive methods to regulate individual behavior [47,48]. Previous theoretical research has introduced tax-reward and tax-punishment strategies into coupled social resource systems, and analyzed the coevolutionary dynamics under different resource growth rate scenarios when these two incentives act separately [40]. However, in the real world, reward and punishment incentives are often used simultaneously. Our model uniquely integrates both punishment and reward mechanisms. This dual approach allows us to explore a broader spectrum of interactions and their impacts on cooperative behaviors, which has not been examined in prior studies. Thus, what kind of incentive can better regulate individuals' behavior in resource extraction? Pure reward? Pure punishment? Or a hybrid incentive? If it is the latter, how to allocate the incentive budget to better promote the realization of the ideal state, where all individuals can reasonably exploit resources while ensuring resource sustainability? These questions are all unclear. Our current research reveals that when the resource growth rate is not too low, achieving this ideal state cannot rely entirely on reward, a mixed use of reward and punishment or pure punishment is required.

In this work, we assume a dynamical feedback relationship between individual behavior and resource quantity. However, we have overlooked an important factor, the resource growth rate, which is also influenced by individual behavior and the quantity of resources. In our current model it is constant. When the quantity of resources is already low and individuals continue to overexploit, the natural growth rate of resources will be severely affected [37]. For example, when the fish population is low, overfishing can affect the reproductive ability of the fish population, thereby reducing the natural growth rate of fish resources. Therefore, if we consider the coupling relationship between individual behavior, resource growth rate, and resource quantity, without relying on external incentives, it is worth studying whether resources can be maintained. On the other hand, when considering the implementation of incentives for individual resource extraction, how the allocation weight of incentive budgets adapts to changes in population and resource states is a path worth studying in the future. Importantly, our work considers an infinite well-mixed population, ignoring the more realistic population structure where individuals only interact with their surrounding neighbors [49,50]. The study of reward and punishment mechanisms in spatial games has a long history [28,51], such as Szolnoki and Perc have investigated the evolutionary advantages of mixed reward and punishment mechanisms in spatial public goods games [38]. Another point to note is that our model does not take into account individual moral preferences, as it plays an important role in guiding individuals to exploit resources. Previous research has outlined the mathematical foundation of moral preferences [52], which drive doing the right thing to promote ideal behavior.

ACKNOWLEDGMENTS

This research was supported by the National Natural Science Foundation of China (Grants No. 62306243 and No. 62406255).

APPENDIX

In the Appendix, we first analyze the conditions for the existence of interior equilibrium points, and then investigate the stability of the equilibrium points by analyzing the signs of the eigenvalues of the Jacobian matrix corresponding to each equilibrium point [53]. According to Eq. (6), we can get

$$r\left[1-\frac{\delta w\frac{1-(1-x)^N}{x}+\delta(1-w)\frac{1-x^N}{1-x}}{b_m\alpha}\right]-\frac{Nb_m}{R_m}[1+(1-x)\alpha]=0.$$

Considering $\frac{1-(1-x)^N}{x}=\sum_{i=0}^{N-1}(1-x)^i$ and $\frac{1-x^N}{1-x}=\sum_{i=0}^{N-1}x^i$, we can obtain the following equation:

$$r\left[1-\frac{\delta w\sum_{i=0}^{N-1}(1-x)^i+\delta(1-w)\sum_{i=0}^{N-1}x^i}{b_m\alpha}\right]-\frac{Nb_m}{R_m}[1+(1-x)\alpha]=0.$$

For convenience, we set $F(x)=r[1-\frac{\delta w\sum_{i=0}^{N-1}(1-x)^i+\delta(1-w)\sum_{i=0}^{N-1}x^i}{b_m\alpha}]-\frac{Nb_m}{R_m}[1+(1-x)\alpha]$. By taking the derivative, we have

$$F'(x)=\frac{r\delta w\sum_{i=1}^{N-1}i(1-x)^{i-1}-r\delta(1-w)\sum_{i=1}^{N-1}ix^{i-1}}{b_m\alpha}+\frac{Nb_m\alpha}{R_m},$$

$$F''(x)=-\frac{r\delta w}{b_m\alpha}\sum_{i=2}^{N-1}i(i-1)(1-x)^{i-2}-\frac{r\delta(1-w)}{b_m\alpha}\sum_{i=2}^{N-1}i(i-1)x^{i-2}.$$

Since $F''(x)<0$, we know that $F'(x)$ monotonically decreases on (0, 1). Considering that

$$F'(0)=\frac{r\delta w\frac{N(N-1)}{2}-r\delta(1-w)}{b_m\alpha}+\frac{Nb_m\alpha}{R_m},$$

$$F'(1)=\frac{r\delta w-r\delta(1-w)\frac{N(N-1)}{2}}{b_m\alpha}+\frac{Nb_m\alpha}{R_m}.$$

We can draw the following conclusions:

(1) If $F'(0) < 0$, we know that $F(x)$ monotonically decreases on (0, 1). At this point, when $F(0) > 0 > F(1)$, $F(x)$ has a single root on (0, 1).

(2) If $F'(1) > 0$, we know that $F(x)$ monotonically increases on (0, 1). At this point, when $F(0) < 0 < F(1)$, $F(x)$ has a single root on (0, 1).

(3) If $F'(1) < 0 < F'(0)$, we know that $F(x)$ first monotonically increases to the maximum value $F(\bar{x})$, and then monotonically decreases on (0, 1).

(1) When $F(0) < 0, F(\bar{x}) > 0$, and $F(1) < 0, F(x)$ has two roots x_1 and x_2 on (0, 1).

(2) When $F(0) > 0$ and $F(1) > 0$, $F(x)$ has single root on (0, 1).

(3) When $F(0) < 0$ and $F(1) > 0$, $F(x)$ has single root on (0, 1).

In addition to the above conditions, the existence of the interior equilibrium point (x^*, y^*) also needs to ensure that $0 < y^* < R_m$.

Next, we will analyze the stability of the equilibrium points of the coupled system. We set that

$$f(x, y) = x(1-x)\left(\delta w \frac{1-(1-x)^N}{x} + \delta(1-w)\frac{1-x^N}{1-x} - \frac{b_m y}{R_m}\alpha\right),$$

$$g(x, y) = ry\left(1 - \frac{y}{R_m}\right) - N\frac{b_m y}{R_m}[1 + (1-x)\alpha].$$

We present the Jacobian matrix of the system as follows:

$$J = \begin{bmatrix} \frac{\partial f(x,y)}{\partial x} & \frac{\partial f(x,y)}{\partial y} \\ \frac{\partial g(x,y)}{\partial x} & \frac{\partial g(x,y)}{\partial y} \end{bmatrix},$$

where

$$\frac{\partial f(x, y)}{\partial x} = \delta w[(1+N)(1-x)^N - 1] + \delta(1-w)(1-(1+N)x^N) + \frac{\alpha b_m y}{R_m}(2x-1),$$

$$\frac{\partial f(x, y)}{\partial y} = -\frac{\alpha b_m}{R_m}x(1-x),$$

$$\frac{\partial g(x, y)}{\partial x} = \frac{\alpha N b_m}{R_m}y,$$

$$\frac{\partial g(x, y)}{\partial y} = r - \frac{N b_m}{R_m} - \frac{2r}{R_m}y - \frac{\alpha N b_m}{R_m}(1-x).$$

Specifically, for $(x, y) = (0, 0)$,

$$J(0, 0) = \begin{bmatrix} \delta[1+(N-1)w] & 0 \\ 0 & r - \frac{(1+\alpha)N b_m}{R_m} \end{bmatrix},$$

since $\delta[1 + (N-1)w] > 0$, it is unstable.

For $(x, y) = (1, 0)$,

$$J(1, 0) = \begin{bmatrix} -\delta[N-(N-1)w] & 0 \\ 0 & r - \frac{N b_m}{R_m} \end{bmatrix},$$

it is stable when $r < \frac{N b_m}{R_m}$, while when $r > \frac{N b_m}{R_m}$, it is unstable.

For $(x, y) = (0, R_m - \frac{N b_m (1+\alpha)}{r})$,

$$J\left(0, R_m - \frac{N b_m(1+\alpha)}{r}\right) = \begin{bmatrix} \delta[1+(N-1)w] - \alpha b_m + \frac{\alpha(1+\alpha)N b_m^2}{R_m r} & 0 \\ \alpha N b_m - \frac{\alpha(1+\alpha)b_m^2 N^2}{R_m r} & \frac{(1+\alpha)N b_m}{R_m} - r \end{bmatrix},$$

since $R_m - \frac{N b_m(1+\alpha)}{r} > 0$, we know $\frac{(1+\alpha)N b_m}{R_m} - r < 0$. Thus it is stable when $\delta[1+(N-1)w] - \alpha b_m + \frac{\alpha(1+\alpha)N b_m^2}{R_m r} < 0$, while it is unstable when $\delta[1 + (N-1)w] - \alpha b_m + \frac{\alpha(1+\alpha)N b_m^2}{R_m r} > 0$.

For $(x, y) = (1, R_m - \frac{N b_m}{r})$,

$$J\left(1, R_m - \frac{N b_m}{r}\right) = \begin{bmatrix} -\delta[N-(N-1)w] + \alpha b_m - \frac{\alpha N b_m^2}{R_m r} & 0 \\ \alpha N b_m - \frac{\alpha N^2 b_m^2}{R_m r} & \frac{N b_m}{R_m} - r \end{bmatrix},$$

since $R_m - \frac{N b_m}{r} > 0$, we know $\frac{N b_m}{R_m} - r < 0$. Thus it is stable when $-\delta[N-(N-1)w] + \alpha b_m - \frac{\alpha N b_m^2}{R_m r} < 0$, while it is unstable when $-\delta[N-(N-1)w] + \alpha b_m - \frac{\alpha N b_m^2}{R_m r} > 0$.

For the interior equilibrium point, denoted as (x^*, y^*), where $0 < x^* < 1$ and $0 < y^* < R_m$, the Jacobian matrix is given by:

$$J(x^*, y^*) = \begin{bmatrix} a_{11} & -\frac{\alpha b_m}{R_m}x^*(1-x^*) \\ \frac{\alpha N b_m}{R_m}y^* & -y^*\frac{r}{R_m} \end{bmatrix},$$

where $a_{11} = x^*(1-x^*)[-\delta w \sum_{i=0}^{N-1} i(1-x^*)^{i-1} + \delta(1-w)\sum_{i=0}^{N-1} i(x^*)^{i-1}]$. Then it is stable when $a_{11} - y^*\frac{r}{R_m} < 0$ and $-a_{11}y^*\frac{r}{R_m} + \frac{\alpha b_m}{R_m}x^*(1-x^*)\frac{\alpha N b_m}{R_m}y^* > 0$.

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