Erratum: Contribution of fictitious forces to polarization drag in rotating media [Phys. Rev. E 108, 045201 (2023)]

Julien Langlois and Renaud Gueroult

(Received 18 June 2024; published 8 July 2024)

DOI: 10.1103/PhysRevE.110.019901

There was an error in the general expression of the susceptibility tensor, Eqs. (10) and (15). This error, however, does not affect any of the findings contained within the paper since those were derived from the particular forms (11), (12) (16), and (17) that had been obtained by inverting directly Eq. (9) when written for the configuration $\mathbf{\Omega} \parallel \mathbf{B}_0$ more particularly studied in the paper.

More specifically, the Sherman-Morrison formula which we used to obtain in Appendix B, and from there in Eqs. (10) and (15), the susceptibility tensor $\underline{\boldsymbol{\chi}}'(\omega')$ given its inverse

$$\underline{\boldsymbol{\chi}}'^{-1}(\omega') = -\frac{1}{\omega_{pe}^2}\left[(\omega'^2 + \Omega^2 - \omega_0^2)\underline{\mathbf{I}} + i\omega'\mathbf{\Omega}^* \times \underline{\mathbf{I}} - \mathbf{\Omega}\mathbf{\Omega}\right] \tag{1}$$

is inapplicable since the tensor

$$-\frac{1}{\omega_{pe}^2}\left[(\Omega^2 - \omega_0^2)\underline{\mathbf{I}} - \mathbf{\Omega}\mathbf{\Omega}\right], \tag{2}$$

which we would like to write as a dyadic product $\mathbf{ab}$ with $\mathbf{a}$ and $\mathbf{b}$ in $\mathbb{R}^3$ is of rank 3. The Sherman-Morrison formula can, however, be used by isolating the tensor $\omega_{pe}^{-2}\mathbf{\Omega}\mathbf{\Omega}$ in $\underline{\boldsymbol{\chi}}'^{-1}(\omega')$. Specifically, we introduce

$$\underline{\boldsymbol{\chi}}_{\dagger}'^{-1}(\omega') = -\frac{1}{\omega_{pe}^2}\left[(\omega'^2 + \Omega^2 - \omega_0^2)\underline{\mathbf{I}} + i\omega'\mathbf{\Omega}^* \times \underline{\mathbf{I}}\right] \tag{3}$$

so that

$$\underline{\boldsymbol{\chi}}'^{-1}(\omega') = \underline{\boldsymbol{\chi}}_{\dagger}'^{-1}(\omega') + \frac{1}{\omega_{pe}^2}\mathbf{\Omega}\mathbf{\Omega}, \tag{4}$$

and find from $\underline{\boldsymbol{\chi}}_{\dagger}'^{-1}(\omega')\underline{\boldsymbol{\chi}}_{\dagger}'(\omega') = \underline{\mathbf{I}}$ that

$$\underline{\boldsymbol{\chi}}_{\dagger}'(\omega') = -\frac{\omega_{pe}^2}{\left(\omega'^2 + \Omega^2 - \omega_0^2\right)^2 - \omega'^2\Omega^{*2}}\left[(\omega'^2 + \Omega^2 - \omega_0^2)\underline{\mathbf{I}} - \frac{\omega'^2}{\omega'^2 + \Omega^2 - \omega_0^2}\mathbf{\Omega}^*\mathbf{\Omega}^* - i\omega'\mathbf{\Omega}^* \times \underline{\mathbf{I}}\right]. \tag{5}$$

We then apply the Sherman-Morrison formula with $\mathbf{a} = \mathbf{b} = \omega_{pe}^{-1}\mathbf{\Omega}$ to now obtain

$$\underline{\boldsymbol{\chi}}'(\omega') = \underline{\boldsymbol{\chi}}_{\dagger}'(\omega') - \frac{\underline{\boldsymbol{\chi}}_{\dagger}'(\omega')\cdot\mathbf{\Omega}\mathbf{\Omega}\cdot\underline{\boldsymbol{\chi}}_{\dagger}'(\omega')}{\omega_{pe}^2 + {}^T\mathbf{\Omega}\cdot\underline{\boldsymbol{\chi}}_{\dagger}'\cdot\mathbf{\Omega}}. \tag{6}$$

Putting these pieces together, we have

$$\underline{\boldsymbol{\chi}}'(\omega') = \underline{\boldsymbol{\chi}}_{\dagger}'(\omega') - \frac{\underline{\boldsymbol{\chi}}_{\dagger}'(\omega')\cdot\boldsymbol{\varpi}\boldsymbol{\varpi}\cdot\underline{\boldsymbol{\chi}}_{\dagger}'(\omega')}{1 + {}^T\boldsymbol{\varpi}\cdot\underline{\boldsymbol{\chi}}_{\dagger}'\cdot\boldsymbol{\varpi}}, \tag{7a}$$

where

$$\underline{\boldsymbol{\chi}}_{\dagger}'(\omega') = -\frac{\omega_{pe}^2}{\left(\omega'^2 + \Omega^2 - \omega_0^2\right)^2 - \omega'^2\Omega^{*2}}\left[(\omega'^2 + \Omega^2 - \omega_0^2)\underline{\mathbf{I}} - \frac{\omega'^2}{\omega'^2 + \Omega^2 - \omega_0^2}\mathbf{\Omega}^*\mathbf{\Omega}^* - i\omega'\mathbf{\Omega}^* \times \underline{\mathbf{I}}\right] \tag{7b}$$

and

$$\boldsymbol{\varpi} = \omega_{pe}^{-1}\mathbf{\Omega}. \tag{7c}$$

Equation (7) here should hence replace Eq. (10) in the original paper as the general expression for the susceptibility tensor modified by rotation. One verifies that this expression reduces as it should to Eqs. (11) and (12) when $\mathbf{\Omega} \parallel \mathbf{B}_0$. Similarly summing contributions from the various species and taking $\omega_0 = 0$ should replace the formula for the plasma, Eq. (15).

We thank Tie-Huai Zhang for calling our attention to these unfortunate errors and for suggesting this remedy.