Coherent structures formed by small particles in traveling-wave flows

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(Received 5 December 2023; accepted 25 April 2024; published 8 July 2024)

We experimentally verify the "phase locking model," which describes the formation of one-dimensional coherent structures by low-Stokes-number particles as proposed by Pushkin *et al.* [Phys. Rev. Lett. **106**, 234501 (2011)] in thermocapillary liquid bridges: When the particles form the coherent structures in time-periodic flows, the synchronous coupling of the toroidal vortex and the azimuthally traveling wave is achieved for a range of azimuthal wave number m. We further propose coherent structures of rational wave numbers in the azimuthal direction and unveil those experimentally.

DOI: 10.1103/PhysRevE.110.015101

I. INTRODUCTION

Particle aggregation and dispersion in closed spaces are widely observed in nature and industrial systems. Such has been examined in various types of convection due to the baroclinic instability [1–3], in thermocapillary hanging droplets [4,5], and in lid-driven cavity [6]. As a representative example of the phenomena, Schwabe *et al.* [7] discovered that small particles as the tracers locally accumulate in a time-dependent traveling-wave-type thermal convection due to the hydrothermal wave (HTW) instability [8] in so-called half-zone liquid bridges. This unique phenomenon was termed "particle accumulation structures" (PASs). These small tracer particles form a closed structure in the shape of ordered helical bands under narrow conditions in thermocapillary effect [9]. Their azimuthal wave number $m_{\rm PAS}$ depends on the liquid-bridge aspect ratio ($\Gamma = \text{height/radius} = H/R$) and the volume ratio to coincide with the azimuthal wave number of the HTW itself, $m_{\rm HTW}$, for $m_{\rm HTW} \geqslant 2$ [9–13]: The azimuthal wave numbers of the PASs and HTW are always the same in positive integer, and the PASs as well as the HTW are organized in closed geometry in m symmetry around the axis. In the case of $m_{\rm HTW} = m_{\rm PAS} \geqslant 2$, the coherent structure azimuthally closes after mth turnover of the particles [9,13]. Pushkin *et al.* [14] proposed a model named the "phase locking model" (PL model, hereafter), to produce a coherent structure in the cylindrical liquid bridge with considering the superposition of steady toroidal vortices and azimuthal propagating waves. They suggested that two flows with different directions "synchronize" as fixing the phase and resulting in the occurrence of PAS, nonetheless the model has not been demonstrated. Further, in the case of $m_{\rm HTW} = 1$, it has been indicated through ground-based and microgravity experiments as well as numerical simulations that PAS and HTW have different wave numbers [15–18]. In this research, we conduct experimental verification of the synchronous behavior of the flows by the PL model [14]. Also, we predict the occurrence of PAS of wave numbers not only in integer but also rational numbers based on this model and demonstrate those experimentally.

II. VERIFICATION OF PL MODEL [14]

To verify the PL model, which is based on the Doppler shift of the particle motion [14], we focus on the HTW and coherent structures of various azimuthal wave numbers realized within the liquid bridges. As an example, we examine the well-developed HTW of $m_{\rm HTW} = 1$ (Fig. 1), which gives rise to (i) coherent structure of $m_{\rm PAS} = 1$ formed by particle clusters in the reference frame rotating with traveling-wave flow and (ii) the surface temperature $T = T(r = r_{\rm s}, z, \theta)$ and its deviation $\widehat{T} = T - \overline{T}$. Small particles in the liquid bridges form a closed structure through ordered cyclic motion in the rotating frame of reference. Such behaviors are also observed for $m \geqslant 2$ [9–13,20,21] [see Fig. 2(a)]. Note that the HTW of $m_{\rm HTW}$ exhibits thermal waves associated with $m_{\rm HTW}$-pairs of relatively hot and cold regions [22–25] [bottom panel in Fig. 1(b) for $m_{\rm HTW} = 1$]. The angular velocity of HTW, $\Omega = 2\pi/\tau$, is measured by the propagation speed of the traveling thermal wave, where τ represents the period for HTW of $m_{\rm HTW}$ to complete one azimuthal revolution. The angular velocity of the coherent structures matches the traveling speed of HTW [9,10,26,27]. Focusing on the motion of an individual particle in the laboratory frame [Fig. 1(c)], the particle shows a regular back-and-forth motion in the radial direction, or a turnover motion, while moving in the opposite direction to the azimuthal travel of HTW [9,28,29]. The averaged angular

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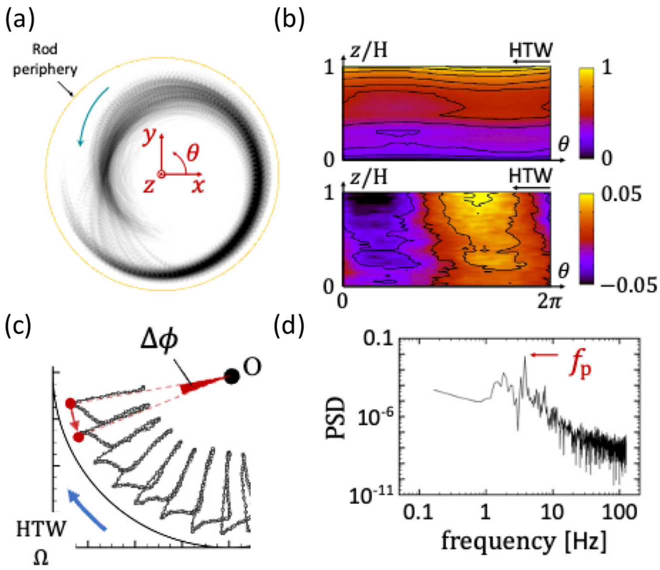

FIG. 1. Example of coherent structure (PAS) in traveling-wave convection and particle behavior forming PAS [19]: (a) Top view of accumulated particle images in reference frame rotating with HTW of $m_{\mathrm{HTW}} = 1$ when PAS of $m_{\mathrm{PAS}} = 1$ occurs. Dots: the particle positions in the particle images; circle in yellow: the outer edge of the rod sustaining the liquid bridge; and arrow: the direction of the particles in the reference frame. Accumulation period of the particle images $\Delta t = 6(2\pi/\Omega)$. (b) Snapshot of temperature $(T - T_{\mathrm{C}})/\Delta T$ (above) and its deviation $\widehat{T}/\Delta T$ (bottom). (c) Time series of positions in (r-ϕ) of a single particle forming PAS [as shown in (a)] in laboratory frame: dot interval corresponds to 4 ms. The particle undergoes azimuthal drift at a rate of $\omega_{\mathrm{p},\phi} = \Delta\phi/\tau_{\mathrm{p}} = \Delta\phi f_{\mathrm{p}}$ while completing one radial oscillation or a single turnover motion within a period $\tau_{\mathrm{p}} = 1/f_{\mathrm{p}}$. (d) Power spectrum density against radial reciprocal motion of a single particle.

velocity of particle azimuthal movement $\omega_{\mathrm{p},\phi}$ is given by $\omega_{\mathrm{p},\phi} = \Delta\phi/\tau_{\mathrm{p}}$, representing "particle azimuthal drift" [14]. Here $\Delta\phi$ is amount of azimuthal movement of particles per turnover, and τ_{p} ($\tau_{\mathrm{p}} = 1/f_{\mathrm{p}}$) is the period for a particle to make a single turnover [Fig. 1(d)]. Extraction of the period of the particle motion in the radial direction, $\omega_{\mathrm{p}} = 2\pi f_{\mathrm{p}} = 2\pi/\tau_{\mathrm{p}}$, from the experimental data for various m_{HTW} allows for the quantitative evaluation of the winding number $W = \widehat{\Omega}/\omega_{\mathrm{p}} = m_{\mathrm{HTW}}(\Omega - \omega_{\mathrm{p},\phi})/\omega_{\mathrm{p}}$, proposed by Pushkin *et al.* [14]. As aforementioned, m_{HTW} is mainly governed by Γ, and m_{PAS} coincides with m_{HTW} for $m \geqslant 2$ [9,10]. It must be noted, however, that the particles exhibit a coherent structure completing two revolutions in the azimuthal direction to form a closed trajectory in the case of HTW of $m_{\mathrm{HTW}} = 1$ [15–17] [top left in Fig. 2(a)]. In previous studies, this type of structure was referred to as "$m = 1$ with a spiral" based on the phenomenological observation [15,16]. In this paper, a fractional wave number is introduced: Suppose the coherent structure closes in p-time revolutions in azimuthal direction after q-time turnover, m_{PAS} is defined as $m_{\mathrm{PAS}} = q/p$, where p and q are the mutually prime positive integers. Thus, the wave number of PAS as "$m = 1$ with a spiral" corresponds to $m_{\mathrm{PAS}} = 1/2$. This definition works perfectly for m_{PAS} of integers in the HTW of $m_{\mathrm{HTW}} = m_{\mathrm{PAS}}$ as well.

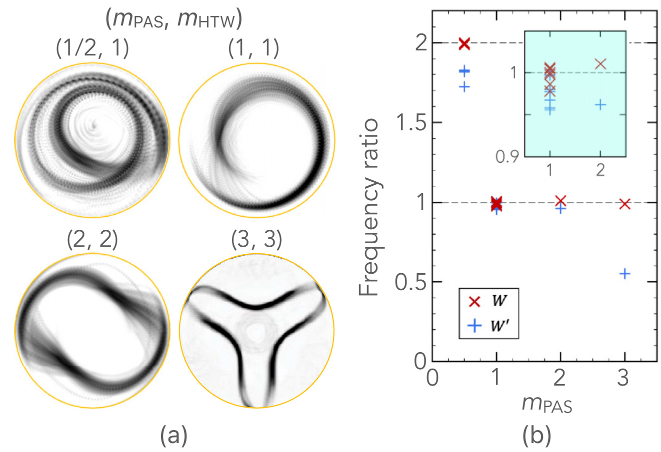

FIG. 2. (a) Top views of integrated particle images of PAS of $m_{\mathrm{PAS}} = 1/2$, 1, 2, and 3 in reference frame rotating with HTW. PASs of $m_{\mathrm{PAS}} = 1/2$ and 1 emerge in the HTW of $m_{\mathrm{HTW}} = 1$, whereas those of $m_{\mathrm{PAS}} \geqslant 2$ emerge in the HTW of $m_{\mathrm{HTW}} = m_{\mathrm{PAS}}$. PAS of $m_{\mathrm{PAS}} = 3$, observed in [13], was realized in the liquid bridge of $R = 2.5$ mm, and the rest of PASs were realized with $R = 0.75$ mm. The particle images are prepared by integrating for $\Delta t = 6(2\pi/\Omega)$. (b) Verification of PL model for various m_{PAS}: $W = \widehat{\Omega}/\omega_{\mathrm{p}} = m_{\mathrm{HTW}}(\Omega - \omega_{\mathrm{p},\phi})/\omega_{\mathrm{p}}$ is the indicator with considering the Doppler shift [14], and W' is the one without the Doppler shift as $W' = m_{\mathrm{HTW}}\Omega/\omega_{\mathrm{p}}$.

Utilizing these experimental findings, we extract the quantities Ω, $\omega_{\mathrm{p},\phi}$, and ω_{p} as shown in Fig. 1 to verify of the PL model [Fig. 2(b)] . We evaluate the azimuthal wave number with considering the Doppler shift as $\widehat{\Omega} = m_{\mathrm{HTW}}(\Omega - \omega_{\mathrm{p},\phi})$. The quantity $\widehat{\Omega}$ represents the wave number experienced by the particles in the azimuthal direction when they undergo a single turnover [Fig. 1(c)]. Pushkin *et al.* [14] proposed that the phase of the particles with respect to the traveling wave is fixed by the "synchronization" between the particles and the azimuthal waves, which leads to the formation of PAS: They numerically demonstrated that PAS occurs when W is equal to unity. This criterion is evinced to rigidly work for PASs of different wave numbers for PASs of m_{PAS} in integers, whereas W becomes $1/m_{\mathrm{PAS}}$ for $m_{\mathrm{PAS}} < 1$ ($m_{\mathrm{PAS}} \neq m_{\mathrm{HTW}}$): The fractional wave number of PAS satisfies $m_{\mathrm{PAS}} = m_{\mathrm{HTW}}/W$ [cross marks in Fig. 2(b)]. On the other hand, the frequency ratio $W' = m_{\mathrm{HTW}}\Omega/\omega_{\mathrm{p}}$, which arbitrarily neglects the Doppler shift, does not converge to unity nor remain constant with respect to m_{PAS} (plus marks). These experimental results suggest that the Doppler shift is a crucial factor, and the azimuthal wave number $\widehat{\Omega}$ experienced by the particles governs the formation of PAS in a system with a time-periodic flow with an azimuthal wave [14]. Note that it has not been demonstrated whether a nonzero azimuthal drift ($\omega_{\mathrm{p},\phi} \neq 0$) is a necessary condition for the formation of coherent structures, notwithstanding. We then examine particle trajectory when the azimuthal wave number of the traveling wave per one turnover of the particle is intentionally varied in the rotating frame of reference. We extract the particle motion within a steady toroidal vortex flow with "minimal" azimuthal drift ($\omega_{\mathrm{p},\phi} \approx 0$) as the fundamental particle motion [Figs. 3(i)(a)].

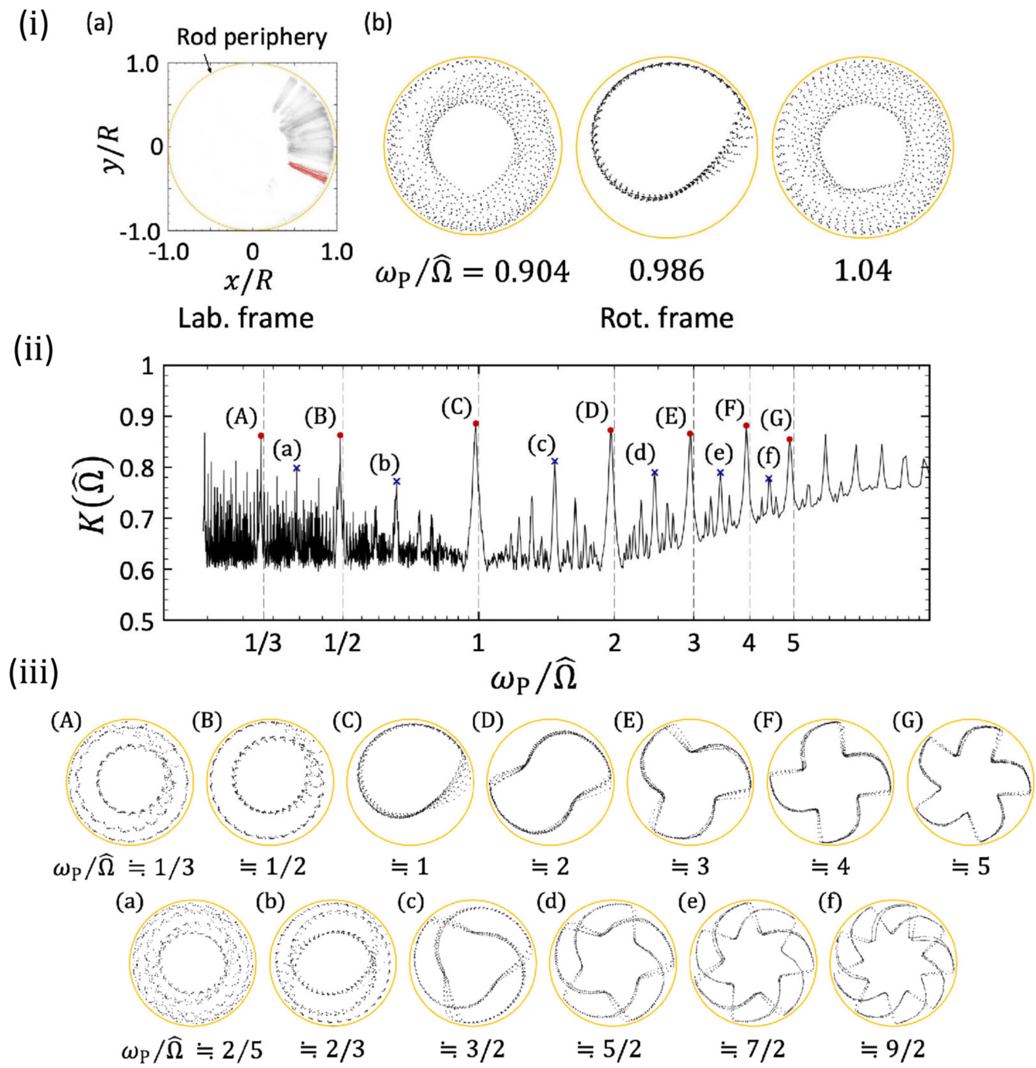

FIG. 3. (i) (a) Top view of path lines of particles in steady flow observed (integrated for approximately 17 turnovers). A single path line in red is the trajectory of a single particle for the reconstruction by the examination in the case without azimuthal draft ($\omega_{\mathrm{p},\phi} \approx 0$). (b) Accumulated position of single particle in imaginary traveling wave with varying ratio of particle's turnover against imaginary azimuthal component. (ii) Variation of accumulation measure $K(\widehat{\Omega})$ against $\omega_\mathrm{p}/\widehat{\Omega} = W^{-1}$. Peaks are found at rational values of $\omega_\mathrm{p}/\widehat{\Omega}$. (iii) Examples of highly accumulated trajectories. The labels for primary peaks (A)–(G) and those for subpeaks (a)–(f) correspond to those in frame (ii).

In this simplified examination, $\widehat{\Omega}$ is approximately equal to Ω: We neglect any azimuthal drift. That is, the particle undergoes simple back-and-forth motion in the radial direction in the laboratory frame. When an imaginary azimuthal flow of angular velocity $\widehat{\Omega}$ is applied to the particle motion with $\omega_{\mathrm{p},\phi} \approx 0$, the particle trajectory exhibits tight accumulation like a coherent structure in the rotating frame of reference under the condition of $\omega_\mathrm{p}/\widehat{\Omega} \approx 1$ [$\omega_\mathrm{p}/\widehat{\Omega} = 0.986$ in Figs. 3(i)(b)]. As the ratio $\omega_\mathrm{p}/\widehat{\Omega}$ slightly deviates from unity, the particle positions disperse in the rotating frame of reference. This demonstrates that the synchronous behavior between the turnover motion of the particles and the azimuthal flow is a direct factor in the formation of coherent structures. A coherent shape of trajectories is successfully reproduced by applying imaginary traveling waves through the particle turnover motions in any steady flows within the liquid bridges of various Γ ($1.0 \leqslant \Gamma \leqslant 1.8$).

III. PREDICTION OF COREHENT STRUCTURES

To quantitatively measure the degree of particle aggregation in the rotating frame of reference when an imaginary azimuthal flow of the angular velocity $\widehat{\Omega}$ is applied to the turnover motion of a single particle in a steady flow, we utilize the so-called "accumulation measure" [30] defined as $K(\widehat{\Omega}) = \{2(N_\mathrm{p} - \overline{N})\}^{-1}\sum_{i=1}^{N_\mathrm{cells}} |N_i(\widehat{\Omega}) - \overline{N}|$, where N_p is the total number of particles, $\overline{N}$ is the average number of particles per inspection cell, and $N_i(\widehat{\Omega})$ is the number of particles in the ith inspection cell [31]. High degree of particle aggregation is realized for $\omega_\mathrm{p}/\widehat{\Omega} \approx 1/\xi$ (ξ: integer) [denoted as (A) and (B) in Fig. 3(ii)], as well as for cases where $\omega_\mathrm{p}/\widehat{\Omega}$ becomes an integer [(C) $\sim$ (G) in Fig. 3(ii)] with varying $\widehat{\Omega}$. Focusing on the cases where high $K(\widehat{\Omega})$ values are achieved with $\omega_\mathrm{p}/\widehat{\Omega}$ being an integer, the distributions of particle positions in the rotating frame of reference [Fig. 3(iii)] correspond well to the

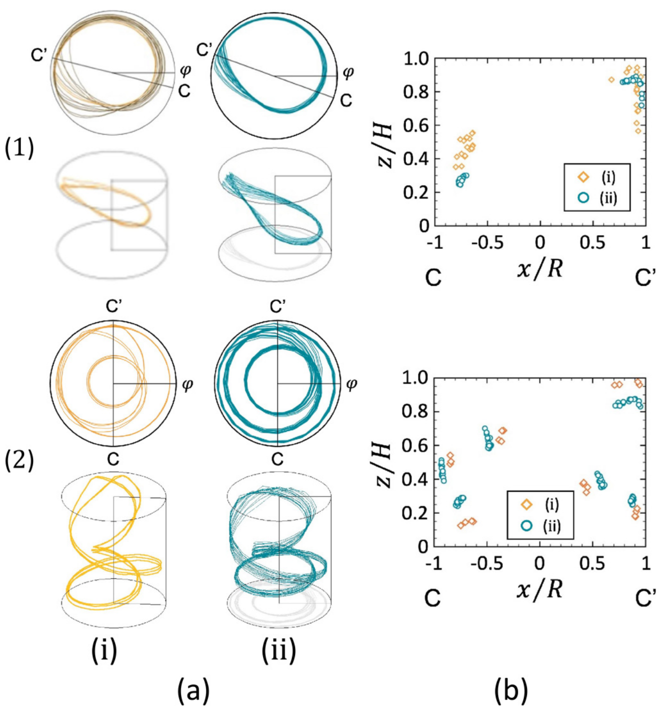

FIG. 4. (a) Particle trajectories (i) observed and reconstructed by experiments and (ii) generated by the present model as depicted in Figs. 3(iii)(C) ($\omega_\mathrm{p}/\widehat{\Omega} \approx 1$) for Row (1) and in Figs. 3(iii)(A) ($\omega_\mathrm{p}/\widehat{\Omega} \approx 1/3$) for Row (2). (b) Spatial distributions of coherent structure within specific cross-sectional plane C-C$'$.

coherent structures previously observed in experiments [9–13] and numerical analyses [18,30,32–36]. Further, these results are also consistent with the predictions by the PL model proposed for the cases of $m = 2$ or 3 [14]. It is emphasized that, in the case of (B) where $\omega_\mathrm{p}/\widehat{\Omega} \approx 1/2$, previous studies have identified the presence of this type of coherent structure known as "PAS with a spiral" in HTW of $m_\mathrm{HTW} = 1$ [15,16] as introduced. Through this evaluation, we can define the structure that completes two revolutions in the azimuthal direction ($p = 2$) and then closes (or, a period of two) in a single period of turnover ($q = 1$) as a coherent structure with a fractional wave number $m_\mathrm{PAS} = 1/2$. The model further predicts the existence of a coherent structure that completes three revolutions in the azimuthal direction and then closes (or, a period of three) as shown in (A) $\omega_\mathrm{p}/\widehat{\Omega} \approx 1/3$, which has not been identified in previous research. Additionally, local maxima for "resonant mode" [14] are realized in the distribution of K when $\omega_\mathrm{p}/\widehat{\Omega}$ becomes a rational number q/p ($q \neq 1$) [denoted as (a) $\sim$ (f) in Fig. 3(iii)]. Structures such as (b) and (c) exhibit three-dimensionally closed particle trajectories, which correspond to those predicted numerically by Barmak *et al.* [18]. The structures (c) were also identified by on-ground experiment [37]. Based on these findings, it is unveiled that the addition of an azimuthal component to the toroidal vortex in a steady flow leads to the twisting of particle trajectories, resulting in the formation of coherent structures of rational azimuthal wave numbers in the liquid bridges. A qualitative comparison is presented in Fig. 4 between experimental data [columns (a)–(i)] and the coherent structures predicted by the present model [(a)–(ii)]. Row (1) depicts the case of $\omega_\mathrm{p}/\widehat{\Omega} \approx 1$, where the occurrence of the coherent structure was confirmed experimentally [37]. Row (2) indicates the case of $\omega_\mathrm{p}/\widehat{\Omega} \approx 1/3$, which is newly predicted and experimentally validated in this paper. Note that this new type of PAS is also demonstrated to satisfy rigidily the correlation $m_\mathrm{PAS} = m_\mathrm{HTW}/W$. In each frame, both a top view (above) and a bird's-eye view (bottom) in the rotating frame of reference are provided. The shape of the predicted coherent structures [subcolumn (ii)] resulting from the turnover motion of a single particle subjected to an azimuthal flow with angular velocity Ω closely matches the structures illustrated in the rotating oscillatory flow field obtained by the experiments [subcolumn (i)]. The differences in the existing region of the coherent structures in the z axis arise from the disparities in the particle motion during particle turnover between the cases of (i) the real coherent structures in the time-dependent flow under higher Re_γ as observed in the experimental results, and (ii) the predicted ones by the particles in the steady toroidal flow under lower Re_γ as adopted in the model. Paying attention to the spatial distribution of coherent structures within a specific cross-sectional plane (Poincaré section) [Fig. 4(b)], a grouping of Poincaré points corresponding to the pairs of revolution number p are observed in each region of $x/R \geqslant 0$ and $x/R \leqslant 0$. Three-dimensional distribution of coherent structures formed by small particles exhibits a strong correlation with the Kolmogorov-Arnold-Moser (KAM) tori in the convective field of the enclosed space [18,20]. This suggests that the model can provide predictions regarding the distribution of KAM tori, which act as attractors, in the formation of coherent structures. The above discussion examines the correlation between the trajectory of a single particle and the coherent structure. To form the coherent structures by the multiple particles [11–13,15,16], the synchronous turnover motion of the particles at different azimuthal positions in the liquid bridge is indispensable [37]. When considering the particle trajectories shown in Figs. 3(i)(a), instead of viewing them as the motion of individual particles, a perspective of multiple particles within a steady flow is approved. Specifically, the ratio $\omega_\mathrm{p}/\widehat{\Omega}$, originally considered in the context of the motion of a single particle, is adapted to multiple particles by establishing a rational correlation between the phase shift in turnover motions Δt_θ and the azimuthal position difference $\Delta\theta_\mathrm{p}$, which leads the correlation between the motion of multiple particles and the formation of coherent structures.

IV. CONCLUDING REMARKS

Low-Stokes-number particles aligning into one-dimensional coherent structures, known as particle accumulation structures (PASs), have been observed within traveling-wave-type convection. We experimentally verify the 'phase locking model' [14], which describes the formation of coherent structures in thermocapillary liquid bridges: Synchronous coupling of the toroidal vortex and the azimuthally traveling wave is achieved for a range of azimuthal wave numbers when the particles form coherent structures in time-periodic flows. The frequency ratio considering the Doppler shift, W, converges to unity in the case of integer m_{PAS}, whereas it becomes $1/m_{\mathrm{PAS}}$ otherwise. We predict coherent structures with rational wave numbers in the azimuthal direction by adding azimuthal motion to a particle with two-dimensional turnover motion, and we unveil these experimentally. We propose a correlation between the azimuthal wave numbers of PAS and HTW, which is rigorously demonstrated for the coherent structures of fractional wave numbers predicted in the present study as well as those investigated experimentally and numerically.

ACKNOWLEDGMENTS

This work was partially supported from the Japan Society for the Promotion of Science (JSPS) by Grant-in-Aid for Challenging Research (Exploratory) (Grant No. 20K20977).

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