


Rough neutron fields and nuclear reactor noise

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Nuclear reactor cores achieve sustained fission chain reactions through the so-called “critical state”—a subtle equilibrium between their material properties and their geometries. Observed at macroscopic scales during operations, the resulting stationary neutron field is tainted by a noise term that hinders various fluctuations occurring at smaller scales. These fluctuations are either of a stochastic nature (whenever the core is operated at low power) or related to various perturbations and vibrations within the core, even operated in its power regime. For reasons that are only partially understood using linear noise theory, incidental events have been reported, characterized by an increase of the power noise. Such events of power noise growth, sometimes up to seemingly unbounded levels, have already led in the past to voluntary scrambling of reactors. In this paper, we will use a statistical field theory of critical processes to model the effects of neutron power noise. We will show that the evolution of the neutron field in a reactor is intimately connected to the dynamic of surface growths given by the Kardar-Parisi-Zhang equation. Recent numerical results emerging from renormalization-group approaches will be used to calculate a threshold in the amplitude of the reactor noise above which the core enters a new criticality state, and to estimate the critical exponents characterizing this phase transition to rough neutron fields. The theoretical model of nonlinear noise built in this paper from *ab initio* statistical mechanics principles will be correlated and compared to data of misunderstood reactor noise levels and reactor instabilities and will be shown to provide both qualitative and quantitative insights into this long-standing issue of reactor physics.

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I. INTRODUCTION

A. Phenomenology of nuclear instabilities

More than 430 nuclear reactors are currently in operation worldwide, the vast majority of them being thermal water-moderated reactors: neutrons are slowed down through successive scattering by protons and induce fissions on heavy nuclei once thermalized. Among these reactors, the two main technologies are pressurized water reactors (PWRs) and boiling water reactors (BWRs). While both technologies benefit from a long and extensive experience feedback—dating back to the 1970s—neutron flux fluctuations measured by in-core and ex-core instrumentation have attracted much attention for two opposite reasons. On the one hand, this so-called “neutron noise” is a powerful diagnostic tool to characterize and eventually locate reactor core perturbations [1]. On the other hand, large noise baselines in PWR reactors and instability events with sometimes sudden increases in noise in BWRs have themselves been a recurrent cause for concern. Originally termed reactor instabilities, the phenomenology of these largely unexplained sudden neutron noise patterns soon caught the attention of reactor physicists (see, for instance, the OECD international benchmark on the Swedish BWR Ringhals 1 core [2]). Since then, several such events have occurred in BWRs, noticeably in Sweden and Mexico [3]. The attention devoted to neutron noise has recently been renewed, taking the form of international collaborations [4], as recent measurements in various PWR reactors have identified drifts and contained increases of their neutron noise during operations. Furthermore, many of these reactors exhibit a high baseline of noise during normal operation, such as in

pre-Konvoi and Konvoi PWRs in Germany [5], Switzerland [6], or Spain [7,8], and even more recently in the first operating EPRs [9]. Although normal noise levels depend on reactors (they are close to 0.2% [10] for Westinghouse PWRs, 2% for Konvoi, and 5% for pre-Konvoi [11]), both a progressive increase exceeding 10% [12] and sudden unstable events developing on a ~ 1 min timescale have been recorded, particularly on BWRs [3,13]. In these last cases, fluctuations would swiftly exceed these values, leading to immediate reactor scrambling [14]. Depending on local regulations and industrial procedures, some reactors now have a reduced nominal power to avoid the development of such instabilities, with a threshold usually set between 8% and 10% [11]. On top of that, many reactor automatic protection systems are based on the calculation of the time derivative of the core power to eventually prevent unwanted power excursion. This strategy can be affected by high neutron noise, thus translating a commercial matter into a nuclear safety concern.

B. Modeling of stochastic noise

The current understanding and modeling of neutron noise is based on neutron transport theory, which aims at studying the behavior of neutron gas evolving in multiplicative media. Neutron transport theory is key to reactor physics, as it constitutes the foundation upon which simulation codes are developed and used to support all industrial activities revolving around reactors, such as radiation shielding and criticality safety studies, fuel cycle management, and reactor operations, just to name a few. It supposes that the average behavior of the dilute neutron gas is faithfully described by a linear

Boltzmann equation, clearing the path to the development and use of various approximation schemes. The most widespread of these schemes is the so-called diffusion theory: neutrons are considered at thermal equilibrium so that the spatiotemporal characteristics of their population—and the associated core power distribution—are properly reproduced by a simple heat equation [15]. With proper adjustments of the coefficients of diffusion equations, the numerical schemes are so accurate that they are implemented in the automatic protection systems of many commercial reactors for core operations and scramming procedures. However, power fluctuations can affect the overall statistics of neutrons within the reactor. At start-up, for example, the so-called zero-power state of the core can be subjected to stochastic fluctuations, arising from the random path of neutron flights and from the stochastic nature of the fission process, producing variable numbers of neutrons each time heavy nuclei are fissioned. The fission process in fact equips the random walk with a branching structure [16–18]. Building on a number of theoretical studies [19–21], recent experimental results [22] have provided a concrete example of such low-power phenomena developing in the core: the emergence of spatiotemporal correlations, dubbed “neutron clustering,” offers a concrete example of an *in situ* mechanism ultimately leading to deviations from the expected neutron distribution.

C. Modeling of power noise: The linear noise hypothesis

At high reactor power, the need to cool the core imposes high coolant flow rates, leading to various vibrations of its components such as the internals, the fuel rods, or the assemblies. Because the water serves as a neutron moderator, these vibrations therefore translate in a random noise on local reactivities (i.e., related to the net production of neutrons) because induced fissions are extremely sensitive to the incoming neutron energies. The observable consequence is that the neutron flux, measured by in-core or ex-core detectors, also develops fluctuations around its mean value. As previously pointed out, the neutron noise can be used as a diagnostic tools to characterize and eventually locate reactor core perturbations [1], but it is also perceived as a safety concern. Indeed, on the one hand, reactor automatic protection systems calculate in real time a derivative of the flux to appreciate its variations. This enables detection, for instance, of accidental power transients: blinding such a protection system may jeopardize the safety strategy. On the other hand, more worryingly, high reactor noise levels and instabilities sometimes appear: power fluctuations and oscillations progressively or abruptly might increase to require, ultimately, reactor scramming procedures [14]. Such events have attracted a great deal of attention in the past 50 years to the present day [4] and have triggered theoretical [23] and numerical [24] investigations. Known to be associated with fluctuations in coolant temperature and vibrations of reactor components, these events and the parameters that influence them are still only partially understood [5].

Linear noise theory is the main theory upon which this understanding of neutron noise is based. Its derivation relies on fundamental principles of neutron transport and assumes a linearity in the summation of neutron noise sources. Under this assumption, both a theoretical description of its properties

and numerical schemes can be devised, either using deterministic (i.e., mesh based) codes, or using Monte Carlo transport codes. Many such codes have been built over the years, such as the CORE-SIM simulation tool [25], just to name the most recent one. In the context of a simplified two-group scheme, this tool is a meme to estimate eigenvalues, eigenvectors, and first-order neutron noise capable of robust predictions (see [26,27] for instance). More or less recent works using complex numbers to describe the neutron noise sources have also progressively been conceived and improved, also leading to accurate first-order noise numerical schemes, such as the exact weight cancellation [24]. However, since the 1960s, it is known that self-induced (i.e., nonlinear) power oscillations can lead to reactor instabilities, particularly in BWRs [28]. In an attempt to describe such events, a few numerical schemes were elaborated, the most recent one being based on a ladder expansion technique implemented over a Galerkin finite-element method [29]. The rarity of such numerical codes, which rely on heavy approximations and assumptions, can partly be explained by the difficulties to build any theoretical models of nonlinear noise that could help in validating them. Also, because the theoretical models themselves contain different degrees of phenomenological observations, no analytical predictions have helped as of yet to confirm the validity of the numerical tools. One of these theoretical models of nonlinear noise recently published [30] hinges on a common experimental observation made during various noise incidents/accidents mentioning abrupt changes in the noise following a smooth variation of the reactor control parameter (such as, for instance, the boron carbide control rods), and it indeed suggests there may be a relationship between nonlinear neutron noise and a Hopf bifurcation consecutive to a phase transition. Such a formulation describing neutron noise in terms of phase transition was recently devised [31] thanks to recent developments in the statistical field theory of reaction-diffusion processes [32]. While limited to stochastic neutron noise, the fluctuations of the neutron population arising from their branching random walk were formally related to critical phenomena, even in the presence of stabilizing feedback (such as the Doppler effect [33]). These fluctuations were proved to be related to a time-directed percolation process and to shift the effective random walk of neutrons from a diffusive to a superdiffusive behavior [31]. Because such counterreactions do not occur at the “zero power” stage of the reactor, this model was limited to describing numerical artifacts occurring during the simulation of the neutron transport in multiplicative media (such codes implement a feedback through the so-called population control mechanism [21,34] that stabilizes the neutron statistics [35]).

This model, while limited to describing stochastic fluctuations, nevertheless paved the way to a proper modeling of real neutron noise occurring during the power regime of reactors, using a similar statistical field theory of critical processes. In Sec. II we will describe the statistics of neutrons in fissile media using a stochastic heat equation with multiplicative noise. Relating this equation to the celebrated Kardar-Parisi-Zhang (KPZ) equation, we will show that the reactor might be subjected to a roughening phase transition, where the neutron field becomes rough and is described by the exponents of the KPZ universality class, even when reactor feedbacks are

considered. Using results from both perturbative and non-perturbative numerical approaches to estimate both the phase transition threshold and the fixed point value, Sec. III will first briefly present the phenomenology of reactor instabilities and then select two reactor instability events so as to correlate the roughening transition model with reactor experimental data.

II. NUCLEAR REACTOR MODEL AND THE KPZ EQUATION

At start-up, the power in nuclear reactors can be extremely small—around 1 kW—and the associated neutron statistics is therefore close to 10^6 cm^{-3} , lending itself to stochastic fluctuations. The power regime is instead associated with high neutron densities: indeed, when the reactor reaches its nominal power, the neutron densities are 10^{13} cm^{-3} , and stochastic fluctuations have been totally washed out. At this point, fission reactions release gigawatts of thermal power: such a heat source must be transferred from the primary to the secondary loop through increased flow rates of the coolant primary pumps. The subsequent temperature fluctuations and induced vibrations are therefore intrinsically associated with high neutron densities and high fuel temperatures, and feedback effects might need to be taken into account for a proper description of the neutron gas. In this section, we will first build a simple reactor model in the intermediate temperature regime—where thermal feedback mechanisms can safely be neglected—and then in the high-temperature regime, adding a quadratic field to model the Doppler temperature effect.

A. Nuclear reactor model

While research and development reactor physics codes rely on solving the transport equation on fine energy meshes (such as 281 energy groups for the EDF COCAGNE code [36]), industrial numerical schemes dedicated to *in situ* operation of light-water reactors in real time most often rely on the two-group diffusion approximation [37]: thermal neutrons induce fissions and outgoing neutrons belong to a fast energy group prior to their slowing down to the thermal group. Both groups are simple diffusion equations with proper experimentally adjusted coefficients. Even a one-group diffusion equation allows us to grasp the main spatial features of the neutron field. The temporal behavior is also adequately described by a double timescale: most neutrons produced during fissions are called prompt neutrons and are emitted on a short (10^{-14} s) timescale, while a small fraction of these neutrons, which are called delayed neutrons, are produced following electroweak interactions within fission fragments on rather long timescales (from 1 s to 1 min). However, the study of stationary spatial distribution can safely be based on only one effective timescale, denoted Λ . Finally, as is usually done, the resulting one-group diffusion equation is supplemented by a stochastic term that models the random temporal and spatial fluctuations within the core. The resulting equation can be written as

$$\frac{\partial}{\partial t} n(\vec{x}, t) = D \Delta n(\vec{x}, t) + \frac{\rho}{\Lambda} n(\vec{x}, t) + \sigma \xi(\vec{x}, t) n(\vec{x}, t), \quad (1)$$

where $n(\vec{x}, t)$ denotes the neutron density field, D is the diffusion coefficient, Δ is the three-dimensional Laplace

operator, ρ is the dimensionless reactivity (a net balance between neutron productions by fissions and neutron disappearance by captures), and $\sigma \xi(\vec{x}, t)$ is an uncorrelated white noise of strength σ^2 given by

$$\langle \xi(\vec{x}, t) \xi(\vec{x}', t') \rangle = \delta^3(\vec{x} - \vec{x}') \delta(t - t') \quad (2)$$

[δ being the standard Dirac delta function and $\xi(\vec{x}, t)$ the normalized component of the uncorrelated white noise]. The multiplicative white noise term reflects the various periodic fluctuations in the power regime (see, for instance, Ref. [38] among many others) that occur on different spatial scales: for instance, in pressurized water reactors (PWRs) the smallest perturbation size a is such that $a \sim 1 \text{ cm}$, and it is associated with temperature fluctuations in the coolant flow [8], while mechanical vibrations of the core components range from intermediate scales up to the whole reactor typical size L —between two and three orders of magnitude above. In fact, both scales introduce natural regularization of the white noise: a in the UV sector and L in the IR sector. Noticeably, stochastic fluctuations occurring on microscopic scales cannot be adequately grasped by such a multiplicative noise model but can instead be described by additive noise [31]. The potential contribution of such a multiplicative stochastic noise to significant nonlinear effects in reactor physics, potentially leading to instabilities, has been recently acknowledged [30] through the use of catastrophe theory as a possible phenomenological explanation. In the next subsection, a comprehensive approach to solving the power noise equation is proposed based on a close connection between reactor power noise and the dynamic of surface growths.

B. The roughening transition model

1. Reactor power noise and dynamics of surface growth

Known in statistical physics literature as the stochastic heat equation (SHE) with multiplicative noise, Eq. (1) can be reformulated using the following change of variables:

$$v = D, \quad \eta = \frac{2v}{\lambda} \sigma, \quad c = 2D \frac{\rho/\Lambda}{\lambda}, \quad (3)$$

where a free positive parameter λ was introduced. Then, associating the neutron density field $n(\vec{x}, t)$ to a height field $h(\vec{x}, t)$ through a Cole-Hopf transformation $n(\vec{x}, t) = \exp[\frac{\lambda}{2v} h(\vec{x}, t)]$ allows us to map our stochastic heat equation to a λ -parametrized family of Kardar-Parisi-Zhang (KPZ) equations supplemented by an additive term c ,

$$\frac{\partial}{\partial t} h(\vec{x}, t) = v \Delta h + \frac{\lambda}{2} (\vec{\nabla} h)^2 + \eta \xi(\vec{x}, t) + c, \quad (4)$$

where v , η , and c will be referred to, respectively, as the diffusion coefficient, the strength of the noise, and the speed of the surface growth in the KPZ equation. Whenever the reactor is stationary, ρ is close to 0 and the constant term c also becomes small. Conversely, the KPZ equation is mapped onto the SHE through

$$\rho/\Lambda = \frac{\lambda c}{2v}, \quad D = v, \quad \sigma = \frac{\lambda}{2v} \eta. \quad (5)$$

This connection between the power noise equation of reactor physics and the KPZ equation, which relates the power fluctuations of a reactor to the dynamics of surface growth, went

largely unnoticed in the reactor physics community. To better understand it, and since the choice of a particular Cole-Hopf transformation fixes the relative amount of stochasticity and of nonlinearity in the KPZ equation, it is possible to choose the natural gauge $\lambda/\nu = 2$, which leads to the equation

$$\frac{\partial}{\partial t} h(\vec{x}, t) = D\Delta h + D(\vec{\nabla}h)^2 + \sigma \xi(\vec{x}, t) + \rho/\Lambda. \quad (6)$$

This particular form underlines a simple alphabet connecting the neutron density field to surface growth. In fact, the speed of surface growth is given by the reactivity state of the reactor: when the reactor is supercritical with constant positive reactivity ρ , the surface grows at a speed ρ/Λ . Also, while the neutron diffusion coefficient D drives both the smoothing by evaporation and the lateral growth of the surface in the same proportions (in this particular gauge), the multiplicative noise of the reactor in the power regime becomes an additive noise in the KPZ equation. In a spirit of generality, in the following we will not fix any particular gauge, and we will use Eqs. (3) and (5) to connect the KPZ equation and the SHE equation. We will, however, set ρ to 0 when numerical applications will be tackled as only reactors close to criticality will be considered.

2. KPZ renormalization in dimension $d = 3$

Surprisingly, when looking at characterizing the KPZ universality class, it is customary to use the inverse Cole-Hopf transformation so as to get rid of λ nonlinearity in the KPZ equation. In this context, the multiplicative noise equations describing reactor fluctuations are used, in a mirror approach, as a means to solve surface growth equations, and no gauge related to the Cole-Hopf transform needs to be fixed. In an effort to briefly summarize the large literature on the subject, the study of fluctuations in statistical field theory can be reduced to three steps: (i) the white noise is first regularized introducing a momentum scale κ , (ii) a field theory in the manner of Janssen–De Dominicis–Martin-Siggia-Rose (JDMSR) [39–42] is employed to relate the calculation of observables to an effective κ -regularized action S_κ , and finally (iii) a renormalization-group (RG) analysis is performed to define hypothetical fixed points in the RG flow procedure and to calculate the critical exponents of these fixed points. In our case, the white noise regularization can be done following Ref. [43] by taking the UV limit $\kappa^{-1} \rightarrow 0$ of a spatially colored noise,

$$\langle \xi(\vec{x}, t) \xi(\vec{x}', t') \rangle_\kappa = \delta(t - t') V_\kappa(\vec{x} - \vec{x}'), \quad (7)$$

with $V_\kappa(\vec{r}) = \langle \xi(\vec{x}, t) \xi(\vec{x} + \vec{r}, t) \rangle_\kappa = (\kappa/\sqrt{\pi})^3 e^{-(\kappa\vec{r})^2}$ being the momentum-dependent spatial correlation of the noise model, such that $V_\kappa(\vec{r}) \xrightarrow{\kappa \rightarrow \infty} \delta(\vec{r})$. Interestingly enough, this noise regularization is extremely close to physical noise models built to reproduce flow temperature fluctuations in reactor physics [8]. Observables $\langle O[n] \rangle$ can then be expressed as a functional integral over noise configurations through

$$\langle O(n) \rangle_\kappa = \int \mathcal{D}\xi \mathcal{O}[n] P[\xi], \quad (8)$$

where the normalization constant has been intentionally omitted and where $P[\cdot]$ is the probability of a given noise

configuration ξ , assumed to be Gaussian in the following. Still following the JDMSR approach, this functional integral is shown to be equal to

$$\langle O[n] \rangle_\kappa = \int \mathcal{D}n \mathcal{D}\bar{n} O[n] \exp^{-S_\kappa[n, \bar{n}]}. \quad (9)$$

In this expression, the auxiliary field $\bar{n}(\vec{x}, t)$ has been introduced, and the effective action S_κ for a critical medium is given by

$$\begin{aligned} S_\kappa[n, \bar{n}] = & \int d\tau d\vec{x} \bar{n}(\vec{x}, \tau) \left[\partial_\tau - \frac{\Delta}{2} \right] n(\vec{x}, \tau) \\ & - \frac{g}{4} \int d\tau d\vec{x} d\vec{x}' \bar{n}(\vec{x}, \tau) n(\vec{x}, \tau) V_\kappa(\vec{x} - \vec{x}') \bar{n}(\vec{x}', \tau) \\ & \times n(\vec{x}', \tau) \end{aligned} \quad (10)$$

with a time rescaling $\tau = 2Dt$, and using the notation $g = \frac{\sigma^2}{D}$ for the bare coupling constant. Taking the UV limit and neglecting the power of the fields higher than quadratic (considered to be irrelevant under the renormalization group [43]) finally leads to

$$S_\kappa[n, \bar{n}] \xrightarrow{\kappa \rightarrow \infty} \int d\tau d\vec{x} \left[\bar{n}(\vec{x}, \tau) \left[\partial_\tau - \frac{\Delta}{2} \right] n(\vec{x}, \tau) - \frac{g}{4} (n\bar{n})^2 \right]. \quad (11)$$

Restraining the action to $(n\bar{n})^2$ powers of the neutron field suggests a mapping between the intuitive picture of two-boson attractive interactions in particle physics [43], and the contact interaction of traveling noise sources (seen as perturbation of the neutron field) in reactor physics. Past a certain threshold in the amplitude of the vibrations/fluctuations encoded in the coupling constant g , the noise perturbations traveling in the reactor become “sticky” and start to add up and to roughen the neutron field. Unlike in linear noise theory, this nonlinear interaction opens the door to potential beyond-mean-field effects that can adequately be analyzed using renormalization-group techniques, as in the following.

3. Roughening phase transition

Introducing the dimensionless running coupling

$$S_\kappa[n, \bar{n}] \xrightarrow{\kappa \rightarrow \infty} \int d\tau d\vec{x} \left[\bar{n}(\vec{x}, \tau) \left[\partial_\tau - \frac{\Delta}{2} \right] n(\vec{x}, \tau) - \frac{g}{4} (n\bar{n})^2 \right], \quad (12)$$

$\hat{g} = \frac{\sigma^2}{D} \kappa$, the renormalization of \hat{g} is achieved by requiring the scattering amplitude (between two noise perturbations) to be κ (i.e., cutoff) -independent, which takes the form of fixed points in the dimension $d = 3$ Callan-Symanzik equation

$$\frac{\partial \hat{g}}{\partial \kappa} = \hat{g} - \frac{\hat{g}^2}{(4\pi)^{3/2} \Gamma(3/2)}. \quad (13)$$

(Γ is the standard Gamma function.) This equation possesses two fixed points. The fixed IR point $\hat{g} = 0$ corresponds to $\kappa \rightarrow 0$, set in practice by the IR cutoff $\kappa^{-1} = L$. The fixed UV point $\hat{g}_c = (4\pi)^{3/2} \Gamma(3/2)$ corresponds to $\kappa \rightarrow \infty$, obtained in practice for $\kappa^{-1} = a$. In agreement with Refs. [44–46], when considering numerical applications, we will set the minimum spatial scale a close to unity [46] since $a = \zeta/\pi$, where ζ is the crossover length [46] below which no fluctuations are included [47]. As discussed in Sec. II A, in the power regime

the minimal spatial scale of fluctuations is considered to correspond to temperature perturbations with $\zeta \simeq 1$ cm [8]. Using these notation, the bare noise coupling σ_{rt} corresponding to the roughening transition threshold can be expressed in terms of the bare diffusion coefficient of Eq. (1) following

$$\sigma_{\text{rt}}^2 = 4\sqrt{\pi}D\zeta. \quad (14)$$

4. Renormalized parameters and critical exponents

Below this neutron noise threshold, the behavior of the reactor should be given trustably by the mean field. Above this threshold, the RG flow pushes the coupling \hat{g} and the coefficients of Eq. (1) toward renormalized values associated with the so-called strong noise regime. Therefore, the critical state within the reactor shifts to the KPZ universality class. The renormalized value \hat{g}^* of the UV fixed point in dimension $d = 3$ is shown to be close to $\hat{g}^* \simeq 5/2 \hat{g}_c$ [47], which leads to a (spatial) scale-dependent expression of the renormalized parameters σ_κ and D_κ given by

$$\sigma_\kappa^2 = 10\pi^{3/2} \frac{D_\kappa}{\kappa}. \quad (15)$$

When κ^{-1} increases from the microscopic scale a to the macroscopic scale L , the parameters acquire effective meaning: σ_κ and D_κ are, respectively, the observed/effective noise and the diffusion coefficient at the spatial scale κ^{-1} . σ_κ and D_κ can be expressed in the form of power laws characterized by the universal dynamical exponent z and the universal roughness exponent χ ,

$$D_\kappa \sim \kappa^{-(2-z)}, \quad (16)$$

$$\sigma_\kappa^2 \sim \kappa^{-(3\chi+1)}, \quad (17)$$

with $z + \chi = 2$ (which is enforced by the Galilean invariance). Numerically, different estimates of critical exponents are found in the literature, such as $\chi = 0.33$ [48], $\chi = 0.3$ [49], or $\chi = 0.17$ [47], and in the following we will retain $\chi \simeq 0.3$, thus fixing $z \simeq 1.7$. From a reactor physics point of view, a scaling following $(\Delta x)^z \propto \Delta t$ with z sensitively less than 2 tends to shift the random walk of neutrons from a diffusive behavior to a semiballistic one and might drastically change the spatial distribution of neutrons within the core, thus unambiguously signing its strong noise criticality regime.

5. Rough neutron fields

The fact that nonlinear addition of noise sources tends to roughen the neutron field through the persistence of such sticky noise perturbations is nicely described using the exponent χ , defined as the roughness W_h of the interface in the KPZ equation, which scales as $W_h \sim L^\chi$ for sufficiently large L and for large times. Although no equivalent exponent emerges from the study of the SHE equation with multiplicative noise, it is possible to estimate the “roughness” of the neutron field under mild assumptions. Indeed, most nuclear reactors are considered decoupled in the sense that their typical size L (a few meters) is very large compared to the neutron’s typical traveled path length before capture, which is a few centimeters [50], ensuring the validity of the L^χ scaling of the roughness of the KPZ interface. In the parametric sector where $\lambda \ll \nu/h$ (which can be set by tuning λ since the SHE is connected

to a λ -parametrized family of KPZ equations, and assuming that h is bounded), the relation between the neutron field $n(\vec{x}, t)$ and $h(\vec{x}, t)$ becomes linear, thus ensuring a linearity (at first order) between the roughness of the neutron field W_n and the roughness of the KPZ interface through $W_n = (\frac{\lambda}{2\nu})^2 W_h = (\frac{\lambda}{2\nu})^2 L^\chi$. The noise term η is, however, enhanced by this reparametrization (we have chosen a small nonlinearity/high noise gauge), which acts as a prefactor but leaves the scaling law valid: at first order, the roughness of the neutron field should also scale as L^χ . In the general case, independently of any particular approximation, and without calculating a precise scaling law, the neutron density field is by definition an exponential function of a potentially rough height field. This indicates that decoupled nuclear reactors develop in the power regime the equivalent of neutron clustering [22] in the stochastic regime, and that the neutron field of the SHE also presents a roughness if the roughening transition of the KPZ equation is attained.

C. Stochastic heat equation with feedbacks

1. Multiplicative noise with a wall

When the reactor power is increased, stabilizing feedbacks kick into action. The most prominent one is the Doppler broadening of the cross-sections: in a thermal reactor, where neutrons are slowed down by elastic collisions in the moderator (water plus additional nuclei such as boron), the broadening of these cross-sections under a temperature increase enlarges the heavy nuclei resonances and constitutes a trap for neutrons that are captured by the nuclei. Unable to participate in the chain reaction, the reactivity ρ is sensitively diminished. Most models capture this effect using a term proportional to $-n^2$ [51–53]. In [31], a stochastic noise model was built taking feedbacks into account. Paradoxically, this zero-power noise model of nuclear reactors adequately describes the maximal power attained by Monte Carlo simulations, which, as of today, can simulate—in terms of neutrons—only a few Watts on modern parallel computing architectures [22]. This model was therefore convenient for understanding simulation artifacts. Now, the same approach can be applied to the power noise model of a real reactor, given by Eq. (1) and supplemented by the $-\alpha n^2$ term. This model—of a stochastic heat equation with multiplicative noise plus powers of the field—has already been extensively studied in the literature, and, in the following, the main results demonstrated in Refs. [48,54] are recalled. Whenever it is supplemented with a feedback term of strength characterized by the coupling constant α , the stochastic equation with multiplicative noise Eq. (1) becomes

$$\begin{aligned} \frac{\partial}{\partial t} n(\vec{x}, t) = & D\Delta n(\vec{x}, t) + \sigma \xi(\vec{x}, t)n(\vec{x}, t) + \frac{\rho}{\Lambda} n(\vec{x}, t) \\ & - \alpha n(\vec{x}, t)^2. \end{aligned} \quad (18)$$

Using as before the Cole-Hopf transform, and using the natural gauge $\frac{\lambda}{2\nu} = 1$, the resulting equation is identical to Eq. (6) minus a supplementary term,

$$\frac{\partial}{\partial t} h(\vec{x}, t) = D\Delta h + D(\vec{\nabla} h)^2 + \sigma \xi(\vec{x}, t) + \frac{\rho}{\Lambda} - \alpha e^{2h}, \quad (19)$$

which is a KPZ equation with a wall. This wall acts as a pinning structure constraining the evolution of the neutron density field, and the criticality of the reactor corresponds to a pinning/depinning phase transition.

2. Common features with the KPZ equation

A renormalization-group analysis of the multiplicative noise equation (with a wall) and the simple KPZ equation (i.e., without a wall), however, indicates that they share several common points [54]: both have two fixed points separating a low noise Gaussian phase and a strong noise phase. Also, the transition point to the strong noise regime is still given by the same value of the dimensionless running coupling constant \hat{g}_c calculated in Eq. (14), and, in the case of the strong noise phase, the critical exponent z that governs the spatiotemporal scaling is also unchanged with respect to the KPZ equation, indicating an effective superdiffusive behavior of the neutron random walk. Consequently, χ should also be unaffected, as well as the κ -scaling of the effective parameters σ_κ and D_κ . But supplementary critical exponents are associated with the pinning/depinning phase transition (via the presence of the wall), and the reader is referred to [48,54] for more information.

III. REACTOR NOISE AND INSTABILITIES

We have seen that decoupled reactors might reach a new criticality regime dictated by the universality classes of the KPZ equation and the multiplicative noise SHE equation for high noises given by Eq. (14). In this section, we will provide more quantitative insight on the previous findings, and we will correlate them with two particular events that exhibited strong noise patterns and power instabilities.

A. Phenomenology

The state of the art in the understanding of neutron noise relies on the analysis of the characteristic frequencies at which fluctuations occur. The typical frequency spectrum is different on BWRs and PWRs: while peaked at frequencies below 1 Hz (often close to 1/2 Hz), the spectrum is rather flat for BWRs, which develop stronger and more frequent noise events, and the white noise hypothesis is hence more realistic. In contrast, the spectrum of PWRs is closer to a decaying exponential (still in the frequency domain) [55]. For both reactors, fluctuations can be either out-of-phase (or regional) whenever they have opposite signs in different regions of the core (e.g., upper/lower part of the core) or in-phase (or global) when they are synchronized throughout the core. This leads to C- or S- shaped vibration of the assemblies [56], connecting the neutron noise to mechanical vibrations of the core components between 1 and 50 Hz (of fuel rods and assemblies and thermal shield below 25 Hz, and of the reactor vessel and internals below 50 Hz). Below 1 Hz, they are believed to be connected to temperature fluctuations in the primary loop. The two mechanisms have different control pathways on the neutron population: in the case of coolant temperature fluctuations, both the density and the Doppler effect modify the probability to slow down neutrons and hence their capacity to reproduce through fissions, while in the case of mechanical

vibrations, it is the interassembly coolant thickness that is modified.

In the next two subsections, a neutron noise model relying on the strong noise hypothesis will be built, and correlated on two sets of experimental data—one corresponding to a PWR and the other to a BWR—so as to grasp at a minimal cost the wide variety of neutron noise events.

B. Almaraz-Trillo PWR noise data

Because of their commercial nature, data released from power plants in general and concerning neutron noise in particular are relatively scarce. Among the published data available, experimental measurements associated with increased noise levels from the 1000 MW Almaraz Trillo PWR reported in 2011 [12,57] present an interesting scheme, as can be observed in Fig. 1, giving the time evolution of the root mean square of the neutron noise measured by ex-core detectors (left plot), as proposed by these references, and a projection of the noise amplitudes in a histogram aiming at presenting both the average noise and its typical deviations (right plot). In fact, throughout the cycle, this noise is progressively amplified by a factor of 2 between cycles 8 and 25, with intracycle variations ranging from 4% to 14% at their extremum. In this section, this data sample will be used for comparison with the roughening phase transition model derived above, looking first at the overall amplitude of the noise and then at the variations of the noise. At cycle 20, no more general trend seems to affect cycle-to-cycle fluctuations, and the noise response of the reactor seems stationary. A histogram that reports the noise amplitude from that cycle is shown in Fig. 1. The average noise value extracted from this graph is estimated to be close to 10%. The large periodic oscillations in the noise itself can be attributed to intracycle effects. Indeed, to compensate for the reactivity loss due to fuel burnup effects, boron is used at the beginning of the cycle in an amount adjusted in such a way that the reactivity through the cycle stays constant. The effect of this intracycle variation of the boron concentration will be discussed in the last paragraph of this subsection.

1. Noise amplitude from the roughening transition

Making the assumption that reactor instabilities are related to the roughening transition, the noise threshold at which the transition occurs can also be numerically evaluated, in a context where spatial patterns of noise, while presenting involved structures, as analyzed, for example, in the linear noise theory [56], are neglected. The noise given by Eq. (14) is associated with Eq. (1), which, once solved, gives access to the neutron density field $n(\vec{x}, t)$. Closely following the approach detailed in [23], and since we are only interested in temporal fluctuation of a spatially averaged noise, Eq. (1) can be rewritten factorizing the purely time-dependent amplitude $P(t)$ out of a remaining spatial part of the neutron density that is considered constant, $n(x, t) \approx n_0$. Equation (1) hence takes the simplified form

$$\frac{\partial}{\partial t} P(t) = \frac{\rho}{\Lambda} P(t) + \sigma \xi(\vec{x}, t) P(t). \quad (20)$$

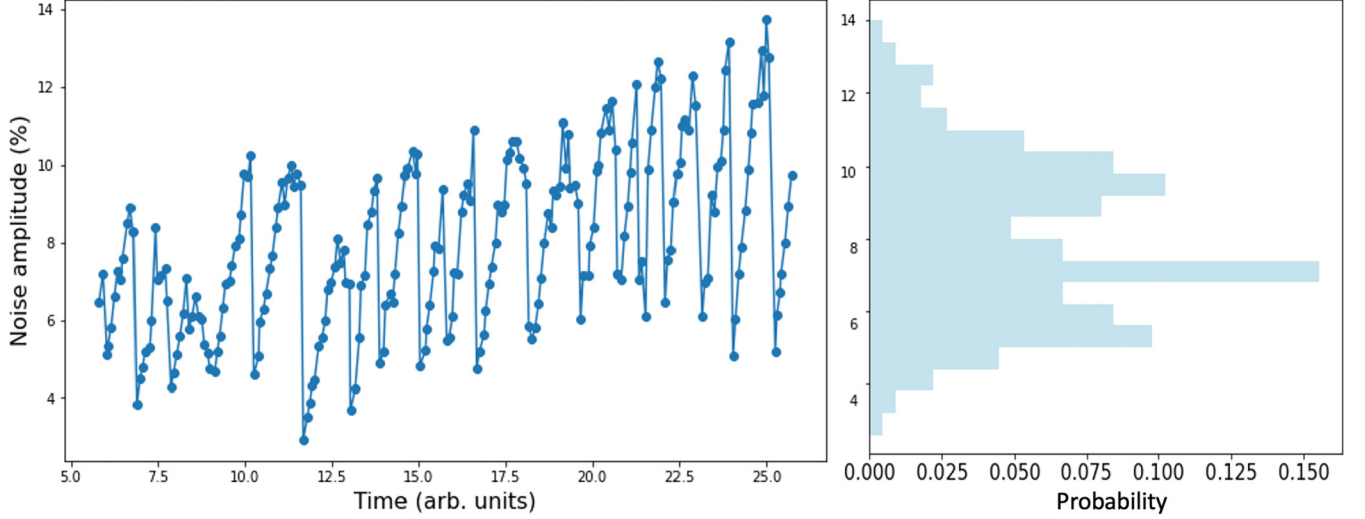


FIG. 1. Almaraz-Trillo PWR data from [12,57]. Left plot: RMS of the neutron noise amplitude (in %) as a function of time (in arbitrary units). Local minima correspond to BOC measurements and local maxima to EOC measurements. Right plot: noise amplitude histogram of all measurements. The estimated average noise is 8%.

In addition, the spatiotemporal noise term $\eta(\vec{x}, t) = \sigma \xi(\vec{x}, t)$ and its intensity σ were implicitly related through the definition

$$\langle \eta(\vec{x}, t) \eta(\vec{x}', t') \rangle = \sigma^2 \delta^3(\vec{x} - \vec{x}') \delta(t - t'), \quad (21)$$

the spatial dependence of which can be readily integrated over the whole reactor of volume $V = \int dx^3$ still assuming that we are only interested in the temporal component of the noise. Therefore, the resulting temporal noise takes the form

$$\langle \eta(t) \eta(t') \rangle \int dx^3 = \sigma^2 \delta(t - t') \int dx^3 \delta^3(\vec{x}), \quad (22)$$

$$\langle \eta(t) \eta(t') \rangle = \frac{\sigma^2}{V} \delta(t - t'), \quad (23)$$

hence recasting Eq. (20) in a classical white noise equation

$$\frac{\partial}{\partial t} P(t) = \frac{\rho}{\Lambda} P(t) + \frac{\delta \rho}{\Lambda} P(t) \quad (24)$$

[where $\delta \rho$ is the first-order perturbation of the reactor reactivity, and where we have used the notation $\delta \rho / \Lambda = \frac{\sigma}{\sqrt{V}} \xi(t)$ for the volume-rescaled noise coupling]. To derive an equation for the noise from this equation as suggested by Ref. [23], $P(t)$ is also split in a (time) fluctuating component $\delta P(t)$ plus a constant term P_0 . Second-order terms are neglected, and the system is considered to be close to criticality so that one obtains the equation for the fluctuations,

$$\frac{\partial}{\partial t} \delta P(t) = \frac{\delta \rho}{\Lambda} P_0 = \frac{\sigma}{\sqrt{V}} \xi(t) P_0, \quad (25)$$

or equivalently in the frequency domain w , after a Fourier transform

$$\left| \frac{\delta P(w)}{P_0} \right| = \frac{\sigma^2}{V w}. \quad (26)$$

This equation connects the fluctuations in the amplitude factor of the neutron density, i.e., the neutron noise $\delta P/P_0$, to the reactivity noise $\delta \rho$, which was formulated as a function of the noise coupling term σ . Following Eq. (14), the neutron noise

at which the roughening transition occurs, noted $|\delta P/P_0|_{\text{rt}}$, is given by

$$\left| \frac{\delta P}{P_0} \right|_{\text{rt}} = \frac{\sigma_{\text{rt}}^2}{V w} = 4\sqrt{\pi} \frac{D \zeta}{V w}. \quad (27)$$

A numerical estimation of this term can be worked out by setting the observation frequency w to 1 Hz—close to its maximum—and using a cylindrical reactor volume of typical radius $r = 150$ cm and $H = 400$ cm to calculate V . Since the criticality state of the reactor is controlled by thermal neutrons, the bare diffusion coefficient D is set to $10^5 \text{ cm}^2 \text{ s}^{-1}$ as suggested by the thermal energy PWR data of [8]. Also in accordance with this reference, and as stated in the previous section, $\zeta \simeq 1$ cm is the minimal spatial scale below which no power fluctuations occur, and it can be associated with a temperature-induced change of the fuel absorption cross-section. For these parameters, the RMS of the noise amplitude is $\sqrt{|\delta P/P_0|_{\text{rt}}} \simeq 15\%$. As can be observed in the right plot of Fig. 1, the experimental data present a bimodal distribution, the noise amplitude progressively drifting within the cycle from a local minimum value to a local maximum value. The average value (over all available cycles) is, as discussed in the previous paragraph, close to 10%. Given the level of detail of the roughening transition model, its value can be considered to be very close to these experimental data. Indeed, any increase/decrease in the minimal correlation length ζ linearly affects the noise threshold estimation. But this length differs sensitively from a few mm to a few cm depending on scarce experimental measurements and widely varying theoretical modeling [8,23]. Beyond this agreement, it should be noted that Eq. (27) sets the dependence of the noise at which the transition occurs on other physical parameters: the bigger the reactor volume V —or the smaller the diffusion coefficient D —and the smaller the noise (i.e., the vibration level) at which the reactor meets the rough regime. This prediction could also lend itself to experimental confirmation.

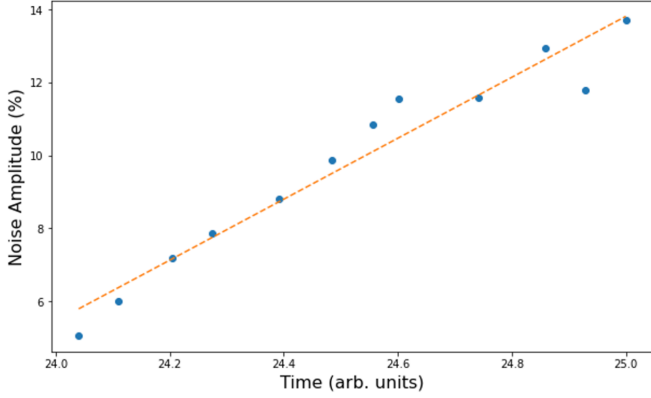


FIG. 2. Zoom on cycle 24 of the Almaraz-Trillo PWR data. The noise amplitude (in %) is represented as a function of time (in a.u.). The data (blue dots) are compared to linear behavior predicted by the roughening phase transition of the KPZ equation (orange dashed line).

2. Cycle effect and boron dilution

In addition to the progressive cycle-to-cycle drift in the reactor noise amplitude, Fig. 1 also exhibits a strong intracycle variation of this amplitude, leading to the bimodal distribution observed: points in the histogram characterized by small noise values are measurements at the beginning of the cycle (BOC), while points characterized by large values correspond to end of cycle (EOC) measurements. A zoom on cycle 24 given in Fig. 2 shows the intracycle behavior with a linear increase in the noise, ranging from less than 6% at BOC to 14% at EOC. While more contained in the previous cycles, this linear increase is reproduced in each cycle. Once again, it is possible to resort to the roughening transition model to understand and quantify these variations. Indeed, at BOC boron is diluted into the moderator to anticipate reactivity effects during cycle: the large capture cross-section σ_a^B of this isotope in the thermal range ensures that excess reactivity of the fuel is compensated by the proper amount of BOC boron dilution. During the cycle, boron is consumed by the neutron flux ϕ following a Bateman equation for the boron atom density $N_B(t)$,

$$\frac{dN_B}{dt} = -\sigma_a^B N_B \phi, \quad (28)$$

which can be readily integrated to find the boron concentration time dependence $N_B(t) = N_B(0)e^{-\sigma_a^B \phi t}$ [where the neutron flux ϕ is assumed to be constant, and where $N_B(0)$ is the boron concentration at BOC]. Insofar as the boron absorption macroscopic cross-section Σ_a^B is related to the boron absorption microscopic cross-section through $\Sigma_a^B(t) = N_B(t)\sigma_a^B$ (N quantities stand for atom densities in cm^{-3} and σ_i for microscopic cross-sections in cm^2), it is therefore possible to estimate the time-dependent total boron absorption cross-section through

$$\Sigma_a^B(t) = \Sigma_a^B(0)e^{-\sigma_a^B \phi t}. \quad (29)$$

Now, Eq. (26) indicates that the neutron noise $|\frac{\delta P}{P_0}|$ is proportional to the diffusion coefficient D . Since neutron transport theory tells us $D = \frac{1}{3\Sigma_t}$ for isotropic media, with $\Sigma_t = \Sigma_s + \Sigma_f + \Sigma_a^B(t)$ (Σ_t being the total macroscopic cross-section,

Σ_s is the total scattering cross-section, and Σ_f is the total fission cross-section of the medium), it follows that injecting Eq. (29) into the neutron noise expression and performing a double first-order development at small times t of the resulting expression allows us to obtain the time variation of the neutron noise within a cycle via

$$\left| \frac{\delta P}{P_0} \right| = c_1 + c_2 \sigma_a^B \phi t + O(t^2). \quad (30)$$

This equation, where constants c_1 and c_2 have been introduced, was hence derived supposing that the boron macroscopic absorption cross-section dominates other absorption cross-sections while being small compared to the macroscopic scattering cross section. This is supported by PWR exploitation data since $\Sigma_a^B \approx 0.1 \text{ cm}^{-1}$ and Σ_s is roughly two orders of magnitude above. As a consequence, the intracycle noise amplitude variation predicted by our model should at first order follow a linear increase in time. Results of this model are superimposed on the data of cycle 24 (see Fig. 2), showing a very good agreement.

In this subsection, we considered a—close to—stationary reactor with a long-term high noise level, and we showed that its noise amplitude was compatible with the noise associated with the roughening transition. To better assess a potential switch from a Gaussian behavior to a rough behavior in a nuclear reactor, the ideal situation would be to consider a hypothetical event in which the reactor switches, on small timescales, from standard operating conditions (assumed to belong to the Gaussian fixed point) to a degraded state of the reactor with a high neutron noise level (that could be correlated to the strong noise regime) and to try to associate the shift in the critical parameters to the universal exponents of the phase transition. Such situations of abrupt neutron noise increase are more frequently met with BWRs, and therefore, in the next subsection, the Laguna Verde BWR incident is discussed.

C. Laguna Verde emergency incident

1. BWRs

Noisy baselines and instability events occur more frequently in BWRs than in PWRs, and they also seem to develop on smaller timescales or equivalently with heavier tails in the frequency spectra. By comparison to PWRs, BWRs have a slightly harder neutron spectrum: a proper modeling of their noise would require the so-called two-group approximation, where two coupled noisy diffusion equations should be used for each energy group. But their particularity is believed to be associated with the two-phase coolant flow: liquid water is injected at the bottom of the reactor and subject to a liquid-vapor phase transition during its travel through the core. The resulting scale-free structure of bubble sizes during the boiling crisis has recently been experimentally put into evidence and shown to belong to the percolation universality class [58]. The minimal size of the bubbles is considered to be $\approx 0.01 \text{ cm}$ [59,60] and defines the minimal correlation length ζ , which, recalling Eq. (27), lowers the threshold in the noise amplitude associated with the roughening transition from an order of magnitude compared to PWR, thus eventually occurring at $\sqrt{|\delta P/P_0|_{\text{rt}}} \simeq 1.5\%$. This is relatively consistent with empir-

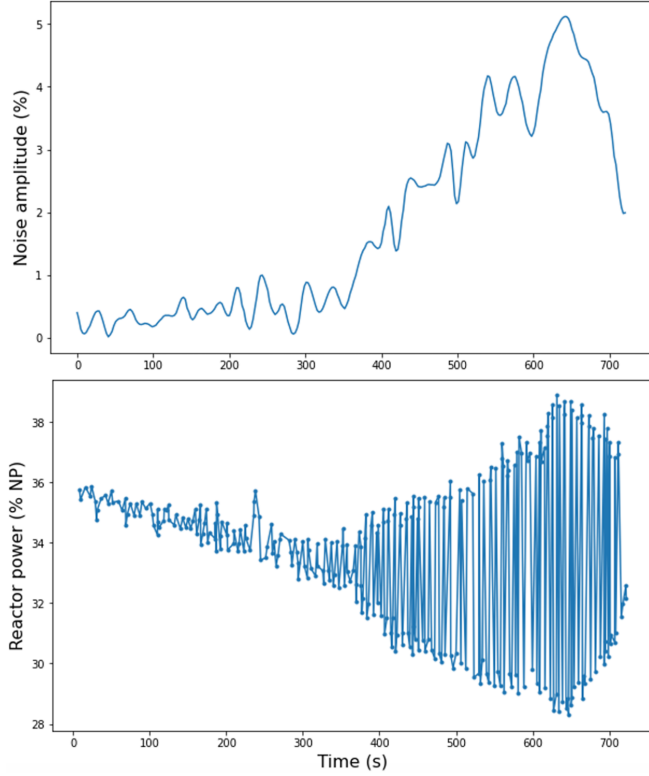


FIG. 3. Bottom plot: experimental data of the Laguna Verde BWR 1995 instability event extracted from Refs. [3,59,61], presenting the average power level vs time, as recorded by APRM detectors. Top plot: linear interpolation of the neutron noise amplitude during the event.

ical observations that concern more frequent and abrupt noisy events in BWRs than in PWRs.

2. The event

To provide more quantitative insight into such phenomena and to illustrate such an event occurring on short timescales, the 1995 Laguna Verde emergency incident is now considered [3,59,61,62]. It happened in January of 1995, during the startup of the reactor, and it lasted 723 s. During that time, power oscillations grew apparently unbounded, even though the power was slightly decreased in an attempt to control their amplitude (see the bottom plot of Fig. 3, where an extraction of visible data points from the previous references has been performed). As the maximum peak-to-peak value of the oscillations reached 10%, a scrambling procedure was triggered. The top plot of Fig. 3 presents the growth of the noise during the event, using an interpolation of the raw data after subtracting the average decrease trend in power. As can be observed, the initial noise amplitude was below 1%, indicating stable behavior of the reactor, but an apparent exponential growth was initiated with a period close to 100 s.

3. Shift in power spectra due to roughening transition

The roughening transition to a strong noise regime could once again provide information on this particular phenomenology. Indeed, the scale-free behavior of the boiling

[58] discussed above mirrors a scale-free correlation length κ^{-1} and could enforce the scale invariance hypothesized by the renormalization procedure. As discussed before, the roughening transition threshold is lower due to the minimal size of the bubbles, explaining that BWRs might be more prone to enter a rough noise regime. In the case of Laguna Verde, the tripping of the reactor occurred during a change in equilibrium where the power was rather constant on average (it was initially lowered during the first stage of the incident), but where the noise amplitude was promptly increasing. Unlike the PWR case in the previous subsection, where a strong noise equilibrium was hypothesized, the emergency procedure most probably prevented such a regime (if any) in Laguna Verde. Since noise amplitudes characteristic of this regime cannot consequently be estimated, interpreting this change in the behavior of the reactor requires us to use other tools, such as a frequency spectrum analysis. Equation (26) can be rewritten as

$$\left| \frac{\delta P(w)}{P_0} \right| = \delta \rho(w) G_0(w), \quad (31)$$

where the reactivity perturbation $\delta \rho$ is defined as before, and where $G_0(w) = \frac{1}{w\Lambda}$ is the so-called zero-power reactor transfer function [63]. For more realistic transfer functions that take into account complex space-dependent thermal-hydraulic and neutron phenomena, the differential Eq. (24) is sensitively modified, and the effective transfer function $G(w)$ could present substantial deviations from G_0 . In any case, auto-power spectrum density (APSD) signals are often characterized by power laws for the w spectrum, which allow one to understand rapid kinetic changes in the reactor. However, trying to appreciate the hypothetical change in APSD power laws associated with the roughening transition can be based on the fact that the change from normal behavior to a strong noise regime is only dictated by the shift in the tails of $\delta \rho(w)$. For a white noise signal, $\delta \rho$ is constant. This corresponds to the temporal white noise $\eta(\vec{x}, t)$ associated with the bare noise coupling σ . In the strong noise regime, σ_κ acquires an effective meaning and depends on the observation scale κ^{-1} : the strength of the effective noise coupling is crucially dependent on this scale. The scale-dependent change in the effective noise amplitude is given as a power law associated with a critical exponent, and, using Eq. (17), it takes the form

$$\sigma^2(\kappa) \propto \kappa^{-\eta_D^*}, \quad (32)$$

with $\eta_D^* = 3\chi + 1$ and $\chi \simeq 0.3$ (see Sec. II B 4). Now, since perturbations in the case of BWR are believed to be mainly correlated with void effects arising from the two-phase flow, a simplified model can be postulated that inversely relates the frequency scale w at which the reactor is observed to the spatial scale κ^{-1} at which it is observed. In fact, the coolant transports perturbations at a constant velocity $v \approx 100 \text{ cm s}^{-1}$. At a given position, the duration associated with a perturbation is hence given by $w^{-1} = 2\pi\kappa^{-1}/v$, from which the proportionality between w and κ is hypothesized. In this model, a small perturbation of $\kappa^{-1} \simeq 1 \text{ cm}$ corresponds to 10 Hz, whereas a quarter-core perturbation of $\kappa^{-1} \simeq 100 \text{ cm}$ corresponds to 1 Hz: these perturbations will define the dynamic frequency range for the observations. Finally, it is possible to appreciate the change in the spectrum of the reac-

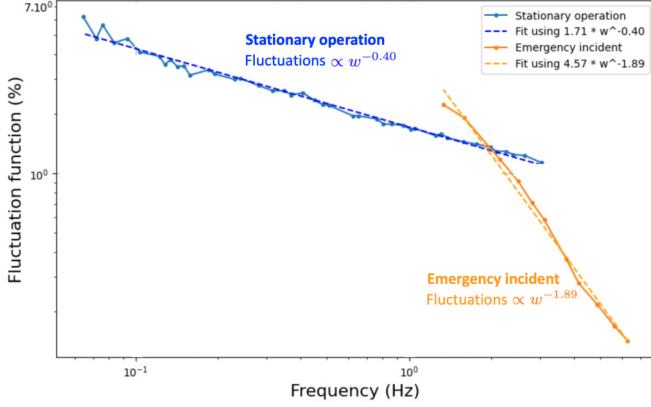


FIG. 4. Experimental data of the Laguna Verde BWR 1995 instability event [3,59,61]. Fluctuation function as proposed by Ref. [59] presented vs frequency w , during a stationary operation of the Laguna Verde reactor (data: blue solid line, fit: blue dashed line) and during the emergency incident (data: orange solid line, fit: orange dashed line).

tor transfer function $G(w)$ when the core shifts from normal operating conditions to a strong noise regime, using

$$\left| \frac{\delta P(w)}{P_0} \right| \propto \frac{G(w)}{w^{3\chi+1}}. \quad (33)$$

Therefore, the RMS of the noise amplitude should approximately scale as $w^{-1/2}$ under normal conditions [with $G(w) = G_0(w) \propto 1/w$], while the scaling associated with a strong noise regime should be given by $w^{-[1/2+(3\chi+1)/2]} \approx w^{-3/2}$. To appreciate whether such a change had occurred during the Laguna Verde incident, data were reanalyzed according to the methodology of Ref. [59]. This paper compares a standard sequence of recorded data of Laguna Verde at nominal power (NP) to the data recorded during the incident. Since the reactor was operated at 30% NP at the outbreak of the incident and, furthermore, during a power decrease, Alvarez-Ramirez *et al.* proposed a detrended noise analysis of the sequence. The results of the analysis, consistent with Ref. [59], appear in Fig. 4, where the fluctuation values of the experimental data are extracted from Ref. [59] but presented as a function of the frequency w and adjusted only by focusing on large values w (over a 700 s signal). The blue points represent the fluctuation values associated with the Laguna Verde reactor operating under normal conditions [the $G(w)$ function], while the orange data points are experimental data recorded during the incident. The noise amplitude estimated by the detrended fluctuation function clearly follows a different w scaling. The results of the adjustment using power laws of both signals are indicated in the plot. The normal sequence, labeled “stationary operating conditions,” indeed exhibits a frequency power law of 0.4, close to the $1/\sqrt{w}$ associated with a white noise signal. This anticorrelation most probably indicates the presence of feedbacks that keep the signal stationary [59]. Concerning the incidental signal, labeled “emergency incident,” the fit of the slope on a log-log scale is also coherent with [59], and equal to 1.89 with a 2% confidence interval, when observed in the dynamic range of 1–10 Hz. This value is sensitively higher than the value calculated for the strong noise regime

but still coherent given the level of simplifications of the model. It shall be noted that, while various perturbative and nonperturbative estimates agree on χ values close to 0.3 or above [43,48,49,64], Ref. [47] finds a value of 0.17—hence sensitively lower—resulting in a scaling $w^{-1.25}$, differing even more from the Laguna Verde data. This estimation of χ uses the BMW numerical scheme [65], which proved to be efficient for $O(N)$ models. As a matter of fact, if the real value of χ in dimension 3 was below 0.2, not only would the w scaling get closer to normal functioning of the reactor, but also the laws of neutron transport would get closer to the standard diffusive behavior (as z would be above 1.8). As a final remark, $1/w^2$ noise spectra were also characterized in this range for the APSD of high-noise events observed in PWRs [5], which could indicate that the insights provided by the strong noise-induced shift in the frequency power law calculated for BWRs could also contribute to understanding open issues of PWR noise measurements.

IV. CONCLUSION

Small and limited power fluctuations are key to the safety of nuclear reactors. On the one hand, automatic protection systems rely on them to assess the reactor power time derivative and take appropriate actions. On the other hand, neutron noise measurements are sometimes used to assess the condition of operating reactor cores. However, for reasons that are only partially understood using linear noise theory (on which both theoretical and numerical modeling of neutron power noise are based), the fluctuations themselves can sometimes increase to unusual levels, as in some PWRs, or grow unboundedly, as has been observed in some BWRs, and hence pose direct safety concerns.

Leveraging on the intimate connection between reactor fluctuations models and the celebrated KPZ equation, a field-theoretic approach to the neutron noise phenomenon has been proposed in this paper. Using results from renormalization approaches, this nonlinear noise model could be solved and predicted that, above a given threshold in the amplitudes of the vibrations or perturbations affecting the core, the reactor could switch to a new criticality state, profoundly different from standard criticality. The neutron fields characterized by this phase transition are indeed rough, in direct analogy to the dynamics of surface growth, even when feedback mechanisms are taken into account.

Two specific events were then considered to benchmark this neutron noise model. First, the noise amplitude measured during a noise increase of the Almaraz-Trillo PWR was compared to the noise threshold of the roughening transition, showing agreement within a few percent. Second, the small minimal correlation length related to the two-phase flow of BWR was shown to lower this threshold in the noise amplitude even further down to values currently met during operations, hence eventually explaining why these reactors are more prone to develop high noise events. Data from the Laguna Verde BWR accident in 1995 were used in particular to show that the spectral frequency shift separating standard and incidental operation of this

reactor could also be related to the roughening transition model.

From the perspective of reactor physics, this theory, if further supported by data, could help to characterize and understand instabilities, especially those occurring in BWRs, and hopefully could be used to design diagnostic tools to prevent their appearance. For example, the use of largest Lyapunov exponents has been proposed [13] in the context of real-time analysis of BWR plant data, which typically implies fractional exponents similar to those derived using the roughening transition model. This seems to be all the more feasible as the Lyapunov exponents of the SHE with multiplicative noise have recently been derived, albeit in lower dimensions [66]. For PWRs, long-term trends (over many cycles) in noise amplitude have been measured in various reactors. Such phenomena may also benefit from interpretation in light of the roughening phase transition. Indeed, if, as has been calculated, temperature fluctuations and core component vibrations put the reactor close to the phase transition limit, any small-scale persistent perturbation would explain such long-lasting trends.

To be quantitatively conclusive, different improvements of the current work will be required. In particular, the numerical values of the coupling constant and of universal critical exponents used in this work were extracted from the BMW renormalization-group numerical scheme [65], based on various assumptions. A scheme dedicated to reactor physics would require us to remove some of these assumptions and in particular to take into account finite-size media with absorbing boundaries, and to solve the two-energy group coupled diffusion equation.

Finally, on a different note, from the perspective of statistical physics, BWR noise events could represent a first measurement of critical exponents of the KPZ universality class in dimension $d = 3$, thus confirming the adage that every cloud has a silver lining.

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- [1] C. Demazière and I. Pázsit, Development of a method for measuring the moderator temperature coefficient by noise analysis and its experimental verification in ringhals-2, *Nucl. Sci. Eng.* **148**, 1 (2004).
 - [2] T. Lefvert, Ringhals 1 stability benchmark, Final Report OECD/NEA, 1996, <https://www.oecd-nea.org/upload/docs/application/pdf/2022-07/nsc-doc96-22.pdf>.
 - [3] O. A. Olvera-Guerrero, A. Prieto-Guerrero, and G. Espinosa-Paredes, Non-linear stability analysis of real signals from nuclear power plants (boiling water reactors) based on noise assisted empirical mode decomposition variants and the shannon entropy, *Entropy* **19**, 359 (2017).
 - [4] V. Verma, C. Demazière, P. Vinai, G. Ricciardi, R. Jacqmin, Assessment of the neutron noise induced by stationary fuel assembly vibrations in a light water reactor, *International Conference on Mathematics and Computational Methods applied to Nuclear Science and Engineering, Portland, United States* (2019).
 - [5] M. Seidl, K. Kosowski, U. Schüler, and L. Belblidia, Review of the historic neutron noise behavior in German KWU built PWRs, *Prog. Nucl. Energy* **85**, 668 (2015).
 - [6] G. Girardin, R. Meier, L. Meyer, A. Aalander, and F. Jatuff, Detailed measurements and analyses of the neutron flux oscillation phenomenology at Kernkraftwerk Goesgen, *Atw. Int. Z. Kernenergie* **63**, 452 (2018).
 - [7] *Almaraz Trillo Report, 2012, Neutron Noise Status in Trillo NPP, CO-12/043*, Technical Report Almaraz, 2012.
 - [8] F. Zylbersztejn, H. N. Tran, I. Pázsit, C. Demazière, and H. Nylén, On the dependence of the noise amplitude on the correlation length of inlet temperature fluctuations in PWRs, *Ann. Nucl. Energy* **57**, 134 (2013).
 - [9] N. Jiang, G. Min, Z. Fang, and H. Qi, Investigation on vibration response characteristics and influencing factors of the fuel rods of EPR, *Front. Energy Res.* **10**, 990525 (2022).
 - [10] In terms of standard deviation of the core power.
 - [11] J. A. Bermejo, C. Montalvo, and A. Ortego, On the possible effects contributing to neutron noise variations in KWU-PWR reactor: Modelling with S3K, *Prog. Nucl. Energy* **95**, 1 (2017).
 - [12] M. Viebach, Rapport stage, Ph.D. thesis, Technische Universität Dresden, 2021.
 - [13] C. J. Gavilán-Moreno and G. Espinosa-Paredes, Using largest lyapunov exponent to confirm the intrinsic stability of boiling water reactors, *Nucl. Eng. Technol.* **48**, 434 (2016).
 - [14] L. A. Torres, D. Chionis, C. Montalvo, A. Dokhane, and A. García-Berrocal, Neutron noise analysis of simulated mechanical and thermal-hydraulic perturbations in a PWR core, *Ann. Nucl. Energy* **126**, 242 (2019).
 - [15] Sometimes the two-groups diffusion theory is also used, which couples two diffusion equations in two energy domains so as to grasp spectral phenomena.
 - [16] T. E. Harris, *The Theory of Branching Processes* (Springer, Berlin, 1963).
 - [17] A. Zoia, E. Dumonteil, and A. Mazzolo, Residence time and collision statistics for exponential flights: The rod problem revisited, *Phys. Rev. E* **84**, 021139 (2011).
 - [18] A. Zoia, E. Dumonteil, and A. Mazzolo, Collision densities and mean residence times for d-dimensional exponential flights, *Phys. Rev. E* **83**, 041137 (2011).
 - [19] E. Dumonteil, F. Malvagi, A. Zoia, A. Mazzolo, D. Artusio, C. Dieudonné, and C. De Mulatier, Particle clustering in Monte Carlo criticality simulations, *Ann. Nucl. Energy* **63**, 612 (2014).
 - [20] A. Zoia, E. Dumonteil, A. Mazzolo, C. de Mulatier, and A. Rosso, Clustering of branching Brownian motions in confined geometries, *Phys. Rev. E* **90**, 042118 (2014).
 - [21] T. M. Sutton and A. Mittal, Neutron clustering in Monte Carlo iterated-source calculations, *Nucl. Eng. Technol.* **49**, 1211 (2017).
 - [22] E. Dumonteil, R. Bahran, T. Cutler, B. Dechenaux, T. Grove, J. Hutchinson, G. McKenzie, A. McSpaden, W. Monange, M. Nelson, N. Thompson, and A. Zoia, Patchy nuclear chain reactions, *Commun. Phys.* **4**, 151 (2021).

- [23] *Handbook of Nuclear Engineering*, edited by D. G. Cacuci (Springer, New York, 2010).
- [24] H. Belanger, D. Mancusi, and A. Zoia, Exact weight cancellation in Monte Carlo eigenvalue transport problems, *Phys. Rev. E* **104**, 015306 (2021).
- [25] C. Demazire, CORE SIM: A multi-purpose neutronic tool for research and education, *Ann. Nucl. Energy* **38**, 2698 (2011).
- [26] S. Yum, M. Hursin, A. Vasiliev, P. Vinai, A. G. Mylonakis, C. Demazière, and R. Macián-Juan, Uncertainty analyses of neutron noise simulations in a Zero-Power reactor, *Ann. Nucl. Energy* **174**, 109157 (2022).
- [27] A. Vidal-Ferrándiz, D. Ginestar, A. Carreño, G. Verdú, A. Dokhane, V. Verma, Y. Perin, J. Herb, A. Mylonakis, C. Demazière, and P. Vinai, Modelling and simulations of reactor neutron noise induced by mechanical vibrations, *Ann. Nucl. Energy* **177**, 109300 (2022).
- [28] J. Thie, Boiling-water-reactor instability, *Nucleonics* **16**, 102 (1958).
- [29] S. M. Ayyoubzadeh, S. A. Hosseini, and N. Vosoughi, Higher order power reactor noise analysis: The multigroup diffusion model, *Ann. Nucl. Energy* **111**, 354 (2018).
- [30] I. Pázsit, V. Dykin, H. Konno, and T. Kozłowski, A possible application of catastrophe theory to boiling water reactor instability, *Prog. Nucl. Energy* **118**, 103054 (2020).
- [31] B. Dechenaux, T. Delcambre, and E. Dumonteil, Percolation properties of the neutron population in nuclear reactors, *Phys. Rev. E* **106**, 064126 (2022).
- [32] R. Garcia-Millan, J. Pausch, B. Walter, and G. Pruessner, Field-theoretic approach to the universality of branching processes, *Phys. Rev. E* **98**, 062107 (2018).
- [33] When the temperature in the fuel increases, the Doppler broadening of neutron cross-sections tends to increase capture resonances and hence constitutes one of the main feedback mechanisms.
- [34] C. De Mulatier, E. Dumonteil, A. Rosso, and A. Zoia, The critical catastrophe revisited, *J. Stat. Mech.: Theor. Expt.* (2015) P08021.
- [35] Monte Carlo simulations of reactors, even when deployed on peta-scale architectures, can only simulate neutron statistics met during the first stages of real reactor startups, when the power is low.
- [36] A. Calloo, D. Couyras, F. Févotte, M. Guillo, C. Brosselard, B. Bouriquet, A. Dubois, E. Girardi, F. Hoareau, M. Fliscounakis, H. Leroyer, E. Noblat, Y. Pora, L. Plagne, A. Ponçot, and N. Schwartz, COCAGNE: EDF new neutronic core code for ANDROMÈDE calculation chain, in *Proceedings of M&C 2017 - International Conference on Mathematics & Computational Methods Applied to Nuclear Science & Engineering* (Korean Nuclear Society, Jeju, Republic of Korea, 2017).
- [37] S. Marguet, *The Physics of Nuclear Reactors*, Springer Link (Springer International, Cham, Switzerland, 2017).
- [38] M. M. R. Williams, *Random Processes in Nuclear Reactors* (Pergamon Press, New York, 1974).
- [39] H. Janssen, On a lagrangian for classical field dynamics and renormalization group calculations of dynamical critical properties, *Z. Phys. B* **23**, 377 (1976).
- [40] C. De Dominicis, Techniques of field renormalization and dynamics of critical phenomena, *J. Phys. Colloq.* **37**, C1-247 (1976).
- [41] P. C. Martin, E. D. Siggia, and H. A. Rose, Statistical dynamics of classical systems, *Phys. Rev. A* **8**, 423 (1973).
- [42] J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena* (Oxford University Press, Oxford, 2021).
- [43] Y. Nakayama and Y. Nishida, Efimov effect at the Kardar-Parisi-Zhang roughening transition, *Phys. Rev. E* **103**, 012117 (2021).
- [44] M. Kardar, G. Parisi, and Y. C. Zhang, Dynamic scaling of growing interfaces, *Phys. Rev. Lett.* **56**, 889 (1986).
- [45] E. Medina, T. Hwa, M. Kardar, and Y. C. Zhang, Burgers equation with correlated noise: Renormalization-group analysis and applications to directed polymers and interface growth, *Phys. Rev. A* **39**, 3053 (1989).
- [46] T. Nattermann and L.-H. Tang, Kinetic surface roughening. I. The Kardar-Parisi-Zhang equation in the weak-coupling regime, *Phys. Rev. A* **45**, 7156 (1992).
- [47] L. Canet, H. Chaté, B. Delamotte, and N. Wschebor, Non-perturbative renormalisation group for the Kardar-Parisi-Zhang equation: general framework and first applications, *Phys. Rev. E* **84**, 061128 (2011).
- [48] W. Genovese and M. A. Muñoz, Recent results on multiplicative noise, *Phys. Rev. E* **60**, 69 (1999).
- [49] E. Marinari, A. Pagnani, and G. Parisi, Critical exponents of the KPZ equation via multi-surface coding numerical simulations, *J. Phys. A* **33**, 8181 (2000).
- [50] The neutron's path length in a diffusive medium with diffusion with a (macroscopic) absorption cross section Σ_a is given by $6 \frac{D}{\Sigma_a}$.
- [51] G. Bell and S. Glasstone, *Nuclear Reactor Theory* (U.S. AEC, Washington DC, 1970).
- [52] Z. A. Akcasu, G. S. Lellouche, and L. M. Shotkin, *Mathematical Methods in Nuclear Reactor Dynamics* (Academic, New York, 1971), p. 460.
- [53] J. J. Duderstadt and W. R. Martin, *Transport Theory* (Wiley, New York, 1979), p. 613.
- [54] M. A. Muñoz, Multiplicative noise in non-equilibrium phase transitions: a tutorial, in *Advances in Condensed Matter and Statistical Physics* (Nova Science, New York, 2003).
- [55] A. Rouchon, Analyse et développement d'outils numériques déterministes et stochastiques résolvant les équations du bruit neutronique et applications aux réacteurs thermiques et rapides, Ph.D. dissertation, Université Paris-Saclay, 2016.
- [56] C. Demazière, A. Rouchon, and A. Zoia, Understanding the neutron noise induced by fuel assembly vibrations in linear theory, *Ann. Nucl. Energy* **175**, 109169 (2022).
- [57] W. Kraus and K.-M. Haendel, Consequences and experiences of the transition from the core design of the first core to an advanced fuel management in Pwrs, in *Proceedings of the International Meeting on Fuel Design and Licensing of Mixed Cores for Water Cooled Reactors* (IAEA, Vienna, 2011), pp. 12–14.
- [58] L. Zhang, J. H. Seong, and M. Bucci, Percolative scale-free behavior in the boiling crisis, *Phys. Rev. Lett.* **122**, 134501 (2019).
- [59] J. Alvarez-Ramirez, G. Espinosa-Paredes, and A. Vazquez, Detrended fluctuation analysis of the neutronic power from a nuclear reactor, *Physica A* **351**, 227 (2005).
- [60] T. Uga, Determination of bubble-size distribution in a BWR, *Nucl. Eng. Des.* **22**, 252 (1972).

- [61] J. Blazquez and J. Ruiz, The laguna verde bwr instability event. some lessons learnt, [Prog. Nucl. Energy](#) **43**, 195 (2003).
- [62] J. Blázquez, C. Montalvo, A. García-Berrocal, and M. Balbás, Searching the beginning of BWR power instability events with the Hilbert Huang transform, [Ann. Nucl. Energy](#) **54**, 281 (2013).
- [63] Which is modified as $G_0(w) = \frac{1}{w*(1+\frac{\beta/\Lambda}{iw+\lambda})}$ when delayed neutrons are taken into account, with β the delayed neutron fraction and λ the precursor decay rate.
- [64] G. Ódor, B. Liedke, and K.-H. Heinig, Directed d-mer diffusion describing the Kardar-Parisi-Zhang-type surface growth, [Phys. Rev. E](#) **81**, 031112 (2010).
- [65] J. P. Blaizot, R. Méndez-Galain, and N. Wschebor, A new method to solve the non-perturbative renormalization group equations, [Phys. Lett. B](#) **632**, 571 (2006).
- [66] P. Ghosal and Y. Lin, Lyapunov exponents of the SHE under general initial data, [Ann. Inst. H. Poincaré Probab. Statist.](#) **59**, 476 (2023).