Letter

Arrhenius law for interacting diffusive systems

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Finding the mean time it takes for a particle to escape from a metastable state due to thermal fluctuations is a fundamental problem in physics, chemistry, and biology. Here, we consider the escape rate of interacting diffusive particles, from a deep potential trap within the framework of the macroscopic fluctuation theory—a nonequilibrium hydrodynamic theory. For systems without excluded volume, our investigation reveals adherence to the well-established Arrhenius law. However, in the presence of excluded volume, a universality class emerges, fundamentally altering the escape rate. Remarkably, the modified escape rate within this universality class is independent of the interactions at play. The universality class, demonstrating the importance of excluded volume effects, may bring insights to the interpretation of escape processes in the realm of chemical physics.

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Introduction. The celebrated Arrhenius law (AL) is a cornerstone in physics, chemistry, and biology, capturing the activation time of a system from a metastable state. Thermally induced activation processes are ubiquitous in nature, e.g., in chemical reactions, protein folding, gene expressions to name but a few. Although each variant has its unique and intriguing features, the universality of AL with regard to the activation barrier and the surrounding temperature nonetheless is remarkable.

The AL is manifested within the Kramer's reaction rate theory [1–3]. Usually there, one is interested in the time taken by an overdamped particle to escape from a trapping potential $U(x)$ while coupled to a thermal bath at temperature T [4]. For the particle to escape the trap, it needs a fluctuation to grant it an excess energy ΔU—the energy difference between the bottom of the potential to the escape point at the top. The AL states that for weak thermal fluctuations $D_0 = k_B T \ll \Delta U$, the inverse of the mean escape time—the escape rate—is

$$\Phi = \tau_0^{-1} e^{-\Delta U/D_0}. \tag{1}$$

τ_0 provides a microscopic timescale, which may be assessed due to arguments by Eyring [5,6]. Importantly, τ_0 is nonuniversal as it depends on the shape of $U(x)$. The wide applicability of the AL can be attributed to the universal exponential decay $e^{-\Delta U/D_0}$. It suggests that by varying the temperature, the experimentally accessible mean escape time allows to infer the activation energy ΔU, independent of the nonuniversal prefactor τ_0. To understand this better, imagine a diffusive process on a multidimensional, rugged energy landscape that can imitate chemical reactions in a network [7–9] or protein conformational dynamics from unfolded to natively folded state via misfolded states [10]. Presence of a variety of local minima surrounded by energy barriers $\Delta U \gg k_B T$ renders a natural separation of timescales in these systems—fast fluctuations in the well followed by slow and rare fluctuations between the wells. In other words, the enzyme fluctuates many times within a well (typical trajectories) before leaving it (rare trajectories). This is a key assumption behind Eq. (1) in generic activation processes [3,7,8,11].

Despite many years of study, discoveries are still being made around the AL and exciting applications continue to be found such as activation in the presence of viscoelastic medium [12,13] or escape dynamics of active particles [14,15], temperature-dependent activation energies [16,17], multiple metastable states [14], experiments with colloids [18,19], semiclassical transition rate theory [20], and inference methods from barrier crossings [9,21]. Yet, one frontier that remains surprisingly less explored is the *validity of AL in many-body systems*. Indeed, even for two particles with short-range interactions, finding the activation time seems to be a formidable challenge. See Ref. [22], which sketches out a formulation for linearly interacting systems. In addition to the existing slow and fast timescales, the "nature" of interaction also sets another timescale in the problem. Consequently, the universal exponential decay in AL may no longer hold true due to the complex interplay of different agents and their interactions [23–26]. This is illustrated in the breakdown of the AL for active particles [27], in the case of infinite-range interactions [28,29] as well as in low temperature glassy dynamics [17].

In this letter, our aim is to delve deeper into the AL for interacting systems. However, before doing that, it is insightful to derive the AL for M noninteracting particles. Notably, the AL can be computed from the large time survival probability which can be derived by mapping the problem to an

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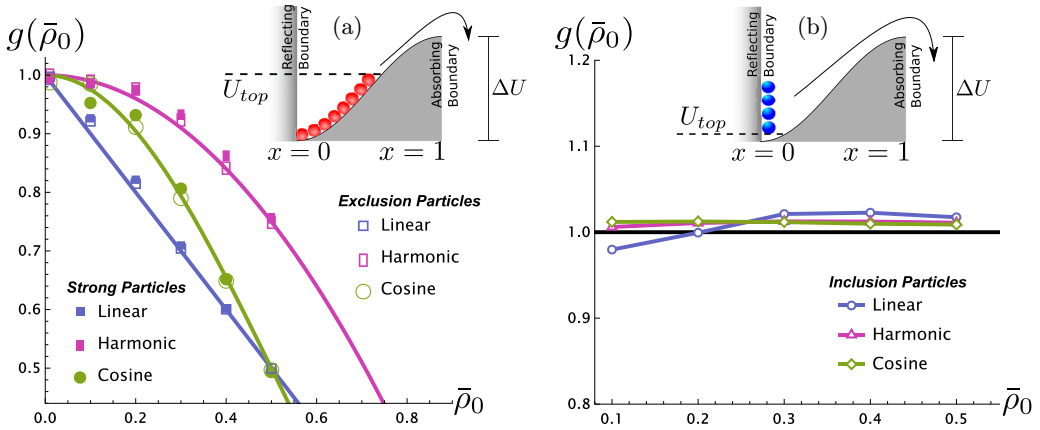

FIG. 1. The generalized AL for interacting diffusive systems. The minimal energy configuration for the SEP, strong particles model, and SIP is sketched in panels (a) and (b) correspondingly. For the excluded volume models, the particles pack tightly, leading to $U_{\rm top} = U(\overline{\rho}_0)$. For the SIP, all the particle can accumulate on the potential minimum at $x = 0$, leading to $U_{\rm top} = 0$. The g function is tested numerically, as a function of $\overline{\rho}_0$, with three potentials $U(x)$: linear $\Delta U x$, harmonic $\Delta U x^2$ and cosine $\frac{1}{2}\Delta U(1-\cos(\pi x))$. We confirm numerically, for the SEP and the strong particles model $g = 1 - U(\overline{\rho}_0)/\Delta U$ (left panel) and for the SIP $g = 1$ (right panel), as predicted by Eq. (2). Both predictions are well fitted up to the expected numerical errors. We refer to Ref. [32] for more details.

effective Schrödinger equation with absorbing boundaries–see the textbooks [4,30] for this standard procedure. Technically, the survival probability $S(t) \asymp \exp[-t\Phi]$ [31], where the escape rate $\Phi = M\Lambda$ and Λ is the ground state energy of the Fokker-Planck Hamiltonian $\hat{H}_{\rm FP} = -D_0\partial_{xx} + \frac{D_0}{4}(\partial_x U/D_0)^2 - \frac{1}{2}\partial_{xx}U$ [4,32,33] (also see [34,35]). Here we stress that the escape rate is the inverse of the mean activation of a single particle [36]. Evidently, the AL universality still prevails for noninteracting particles. What happens to this universality when we incorporate many-body interactions? Here, we try to address this important question.

Consider an extended 1D system of interacting diffusing particles in a monotonically increasing potential $U(x)$ with initial mean density $\overline{\rho}_0 = M/L$, where M is the number of particles and L is the domain size. We show in this letter that the mean time it takes for a particle to escape from the potential is given by the following generalized AL:

$$\Phi \asymp e^{-\Delta U g(\overline{\rho}_0)/D_0}, \tag{2}$$

where g depends both on the density $\overline{\rho}_0$ and on the interparticle interactions. Once again, we stress that Φ is the particle escape rate, its inverse indicates the mean escape time of a single particle. By minimizing the particle configuration energy, one can show $g = 1 - U_{\rm top}/\Delta U$ where $U_{\rm top}$ is the highest energy a particle can attain in the minimum energy configuration. The exponential form of the AL is still preserved. However, the salient point is the emergence of two universality classes. Whenever $\Delta U/D_0$ is large, and no excluded volume interactions take place, the minimum energy configuration compresses all the particles at the origin, keeping the AL universality intact with $g = 1$. However, excluded volume effects change the minimum energy configuration and thus g strongly depends on the particle density. This implies, on the one hand, the breakdown of the AL universality as g depends on the shape of the potential. On the other hand, notice that g is independent of the dynamics at play. That is, we have identified a *new universality class*—the excluded volume universality.

To illustrate the generalized AL, consider the paradigm of the simple exclusion process (SEP) where the interparticle interaction is hard-core exclusion [37]. Minimizing the particle configuration energy implies that they are tightly packed around the potential minimum, resulting in $g < 1$ [see Fig. 1(a), details later]. Therefore, the excluded volume imposes a shorter mean escape time. Eq. (2) is the central result of this letter and we will demonstrate it for a class of diffusive interacting systems, within the framework of the macroscopic fluctuation theory (MFT) [38].

A hydrodynamic theory for diffusive systems. Over the past two decades, the MFT has been instrumental to understand nonequilibrium fluctuations in diffusive systems at the hydrodynamic level [39–54]. Quite recently, the MFT was also used to capture the survival probability of interacting diffusive particles from a domain [55]. Here, we extend this formalism in the presence of a potential which naturally allows us to compute the survival probability and study the generalized AL.

To set the stage, let us consider a 1D system of size L. The system is occupied with interacting diffusive particles that satisfy the continuity equation $\partial_s\rho = -\partial_x j$ with the density and current density $\rho(x,s)$ and $j(x,s)$, respectively. Here, $x \in [0,1]$ and $s \in [0,t]$ are diffusively rescaled [47]. The fundamental formula of the MFT asserts that the path probability is

$$\begin{aligned} \text{Prob}[\{\rho, j\}] &\asymp \exp\left[-\frac{L}{4D_0}\int_0^1 dx \int_0^t ds\, \frac{(j - J(\rho))^2}{\chi(\rho)}\right] \\ J(\rho) &= -D(\rho)\partial_x\rho - \chi(\rho)\partial_x\tilde{U}, \end{aligned} \tag{3}$$

where we have defined $\tilde{U} = U/D_0$ as the rescaled potential. $D(\rho)$ and $\chi(\rho)$ are the density-dependent diffusivity and mobility which encapsulate the diffusive dynamics. In other words, all the interparticle interactions of the dynamics are captured in D, χ. In Eq. (3), the continuity equation is implicitly assumed.

Moreover, we assume that the particles are constrained between one reflecting and one absorbing wall at $x = 0$ and $x = 1$, respectively. Thus, $S(t)$, the survival probability of the particles to stay inside the region up to time t can be written as a conditional sum over all the paths Prob$[\{\rho, j\}]$ that satisfy the following boundary conditions

$$J(\rho)|_{x=0} = \rho|_{x=1} = 0, \tag{4}$$

and the mass conservation in the system $L\int dx\,\rho(x,t) = M$.

Within the MFT, one expects $S(t) \asymp e^{-t\,\Phi}$, where computation of Φ reduces to a minimization problem of finding an optimal fluctuation $\{\rho, j\}$ that satisfy the above mentioned constraints. Note that the optimal fluctuation governs Φ due to the large L saddle dominated probability in Eq. (3). However, the minimization problem still remains hard. To address that, we use the additivity principle, that was introduced in [56] and proved as a useful tool in evaluating large deviations [38] (also see Refs. [41,42,57–59]). The additivity principle posits that the optimal fluctuation density is time-independent, reducing the complexity of finding the optimal fluctuations.

For a 1D system, the additivity principle assumption $\rho(x,t) = \rho(x)$ implies $j(x,t) = \text{const}$ due to the continuity equation. However, const $= 0$ since the current vanishes at the boundaries. Finding Φ then reduces to the following minimization problem:

$$\Phi = \frac{D_0 L}{4}\min_{\rho(x)}\int dx\,\mathcal{L}(\rho, \partial_x\rho),$$
$$\mathcal{L} = \frac{J(\rho)^2}{\chi} - 4\Lambda(\rho - \overline{\rho}_0), \tag{5}$$

where $\rho(x)$ is subjected to Eq. (4) and Λ is a Lagrange multiplier ensuring mass conservation $\overline{\rho}_0 = \int dx\,\rho(x)$.

The minimization problem in Eq. (5) then boils down to solving an Euler-Lagrange (EL) equation with the Lagrangian $\mathcal{L}$. Using the transformation $F(\rho) = \int_0^\rho dz \frac{D(z)}{2\sqrt{\chi(z)}}$ (see Refs. [55,60]), the resulting EL reads

$$-\partial_{xx}F + \frac{1}{8}(\partial_x\tilde{U})^2\chi' - \frac{1}{2}\sqrt{\chi}\partial_{xx}\tilde{U} = \Lambda\frac{\sqrt{\chi}}{D}, \tag{6}$$

where $\chi = \chi(\rho[F])$ and $\chi' = \delta\chi/\delta F$. The resulting boundary conditions inherited from Eq. (4) are $J_F|_{x=0} = \rho[F]|_{x=1} = 0$ with a rescaled current $J_F = \partial_x F + \frac{1}{2}\chi^{1/2}\partial_x\tilde{U}$. Thus, the survival probability can be estimated via Eq. (5) as

$$\Phi = D_0 L\int dx\,J_F^2, \tag{7}$$

where F is the solution of Eq. (6). Before discussing the interactions, it is instructive to re-derive the results for noninteracting particles. Indeed, here $D = 1$, $\chi = \rho$ so that $\rho = F^2$ and Eq. (6) recovers $\hat{H}_{\text{FP}}F = \Lambda F$, which is the eigen-value problem for the noninteracting system. Skipping details, one can show that $\Phi = L\overline{\rho}_0\Lambda$ [32] which implies that Λ is the ground state energy of $\hat{H}_{\text{FP}}$ as found earlier.

For short range interactions, and in the limit of a dilute system, i.e., $\overline{\rho}_0 \to 0$, suggests interactions become negligible. Indeed in this limit, $F \to 0$. Thus, Eq. (6) can be linearized in F, leading to the noninteracting case. Thus for dilute systems, the AL is recovered. In what follows, to justify the generalized AL in Eq. (2), we turn to study several interacting systems.

Excluded volume universality. The SEP is a lattice gas model where each lattice site x has an occupancy $n_x = \{0, 1\}$. In the SEP, a particle is allowed to hop to a nearest neighbor with a fixed rate, provided the target site is empty. Despite its simplicity, the SEP has been widely studied in the context of nonequilibrium statistical mechanics and has been used as a paradigmatic model for understanding the behavior of interacting particle systems in many physical and biological systems, e.g., traffic flow, tracer dynamics, protein synthesis, and gene regulation [37,61]. The SEP typically demonstrates genuine nonequilibrium behavior, e.g., a nonproduct steady state measure and long range nonequilibrium correlations [38,47]. Furthermore, the SEP is susceptible to exact solutions; microscopically via the Bethe ansatz and macroscopically through the MFT [41,42,44,46,62].

For the SEP, $D = 1$, $\chi = \rho(1-\rho)$ [47]. The transformation to F leads to $\rho = \sin^2 F$. We restrict F to the range $[0, \pi/2]$ to ensure a positive $\chi^{1/2} = \frac{1}{2}\sin 2F$. While χ, χ' in Eq. (6) becomes explicit, a solution for arbitrary potential is challenging. Fortunately, an analytical solution can be obtained for the linear potential $\tilde{U}(x) = \Delta\tilde{U}x$. In that case, Eq. (6) is simply an autonomous equation. With the transformation $y = \cos 2F$, and by assuming that the density as well as y are monotonous functions, Eq. (6) can be reduced to a first-order differential equation [32],

$$\frac{dy}{dx} = \frac{1}{2}\Delta\tilde{U}\sqrt{1-y^2}\sqrt{1-y^2+8\lambda(y-y_0)},$$
$$y_0 = y|_{x=0}, \quad y_1 = y|_{x=1} = 1, \tag{8}$$

where $\lambda = \Lambda/\Delta\tilde{U}^2$. Note that $-1 \leqslant y_0 \leqslant 1$. Here $y_0 \to 1(-1)$ implies that the density at the reflecting boundary is approaching zero (unity).

Equations (7) and (8) constitute an implicit solution of the survival probability, denoted by Φ_{SEP}, for the SEP. However, it is worth stressing that one need not explicitly solve y (and thus F) to obtain Φ_{SEP}. Skipping details from [32], we obtain

$$\frac{\Phi_{\text{SEP}}}{D_0 L} = \frac{C_0}{2}(y_0 - 1 + C_0 - C_2 + 4\lambda(C_1 - y_0 C_0)),$$
$$\text{where} \quad C_k = \int_{y_0}^{1} dy \frac{y^k}{\sqrt{1-y^2}\sqrt{1-y^2+8\lambda(y-y_0)}}, \tag{9}$$

for $k = 0, 1, 2$. Fortunately, the C_k integrals are analytical and involve elliptic functions. Furthermore, Φ_{SEP} is given in terms of (λ, y_0) which are implicit functions of $\Delta\tilde{U}$ and $\overline{\rho}_0$.

To make the relations explicit, we notice that direct integration of (8) leads to $C_0 = \Delta\tilde{U}/2$. Also, recalling that $\overline{\rho}_0 = \int dx \sin^2 F$ implies that $2\overline{\rho}_0 = 1 - C_1/C_0$. The regime of large $\Delta\tilde{U}$ is supported by $0 < 1 + y_0 \ll \lambda \ll 1$. In these limits and to leading order $\lambda = 2e^{-\Delta\tilde{U}(1-\overline{\rho}_0)}$ and $(1 + y_0) = e^{-\Delta\tilde{U}\overline{\rho}_0}$. Finally, we have

$$\Phi_{\text{SEP}} = D_0 L A e^{-\Delta\tilde{U}(1-\overline{\rho}_0)}, \tag{10}$$

where A is a polynomial function of $\overline{\rho}_0$ and $\Delta\tilde{U}$ [32].

At this point, we connect the above with our announced result (2). Equation (10) implies $g = 1 - \overline{\rho}_0$ for the linear potential. In the exclusion process, the system's energy is minimized if the particles are ordered as close to the potential minimum as possible. Consequently, the minimum energy

occurs when the system is fully occupied between $x = 0$ and $x = \overline{\rho}_0$, and therefore $U_{\rm top} = U(x = \overline{\rho}_0)$ [see Fig. 1(a)].

Although solving Eq. (6) is hard for generic monotonous potentials, the same physical arguments can be given to infer the generalized AL. Indeed, the minimum energy attained by tight packing the exclusion particles in monotonous potentials, lead to $U_{\rm top} = U(x = \overline{\rho}_0)$. To demonstrate this behavior, we employ a standard shooting method algorithm to solve the eigen-problem (6) for various potentials (see Ref. [32] for more details). The numerical results shown in Fig. 1 provide an excellent agreement with Eq. (2) thus validating the generalized AL for arbitrary monotonous potentials with excluded volume effects. In the absence of excluded volume effect, Eq. (2) implies $g = 1$. This is indeed the case for the reflected Brownian motion, where particles cannot overtake one another, following from Jepsen line arguments [63–65]. The reflected Brownian motion is precisely the limit of the SEP at vanishing $\overline{\rho}_0$, in agreement with $g = 1$.

To further cement our claim that within the excluded volume universality, g is independent of the dynamics for processes, we study the strong particles model [32,58,66,67]. The strong particles model is a lattice gas model, where k particles, filling k consecutive lattice sites, can hope one step to an empty neighbor with rate r_k. Choosing $r_k = 1/k^2$ to simplify the F transformation, we find $\chi = \rho, D = 1/(1-\rho)$. Indeed, Fig. 1 confirms that g is identical for the SEP and the strong particles model, revealing they both belong to the excluded volume universality.

Arrhenius universality. We now turn our attention to the symmetric inclusion process (SIP) where the interaction among the particles is attractive and particles do not mutually exclude. Naively, one might guess that attractive interactions decrease the escape rate and therefore ultimately result in $g > 1$. In contrast, we demonstrate here that $g = 1$ as expected from Eq. (2). Namely, the SIP, lacking excluded volume effects belongs to the Arrhenius universality class and satisfies Eq. (1).

The jump rate of a SIP particle from site x with occupancy n_x to a neighboring site y with occupancy n_y is defined as $n_x(1 + n_y)$. Thus, multiple particles can occupy a given site and the dynamics makes particle clustering favorable [68,69,70].

For the SIP, one finds $D = 1, \chi = \rho(1 + \rho)$. The F-transformation implies $\rho = \sinh^2 F$. Notice that unlike the SEP, F is unbounded for the SIP. Similarly to the SEP analysis, one can explicitly write the Euler-Lagrange equation (6), but an explicit solution is again challenging for an arbitrary potential. However, some analytical progress can be made for the linear potential. Skipping details, we find [32]

$$\Phi_{\rm SIP} = D_0 L A e^{-\Delta\tilde{U}}, \tag{11}$$

where A once again is a polynomial function of $\overline{\rho}_0$, $\Delta\tilde{U}$ [32].

Equation (11) remarkably coincides with the single particle AL noting that there is no slowdown in the escape. This indeed stems from the underlying physics of the SIP, where the attractive interactions imply that the minimum energy configuration is attained when all the particles bunch at the potential minimum. Comparing $\Phi_{\rm SIP}$ with Eq. (2) results in $g = 1$. In fact, this holds for any monotonous potential. To demonstrate this fact, we employ our shooting algorithm to solve Eq. (6) for different potentials. The numerical results are illustrated in Fig. 1 validating the AL for SIP regardless of the particle density $\overline{\rho}_0$. We should stress that the simplicity of Eq. (11) can be deceptive since there is no *simple* or *trivial* way to derive it.

Finally, we would like to argue that the escape time does not depend on the nature of the interactions. To demonstrate that, we consider another canonical interacting process: The Zero-Range process—a class of lattice models, where particle jump rates depend solely on the on-site occupancy. ZRPs can accommodate both attractive and repulsive interactions, but with no excluded volume [71]. Macroscopically, this corresponds to $D = \partial_\rho \chi$ where χ increases with the density [32,56,72]. Assuming χ to be polynomial in ρ, the ZRP indeed conforms to Eq. (2) with $g = 1$. In Ref. [32], this prediction is verified numerically for a set of ZRP models.

Discussion. Understanding escape times in a thermal activation process while accounting for many-body interactions is at the heart of this letter. To this end, we have studied the survival probability of interacting particles with diffusive dynamics in a potential trap. In the limit of weak thermal fluctuations $\Delta U \gg D_0$, we show, using the seminal macroscopic fluctuation theory, that the Arrhenius law can be generalized [Eq. (2)] with $\Delta U g$ an effective activation energy. Importantly, we reveal that the escape problem in lattice gas models branches into two universality classes. We demonstrate our result in five distinct paradigmatic models. Particle models with excluded volume exhibit a decrease in g leading to a facilitated escape from the potential. Importantly g is independent of the interaction. However, whenever excluded volume effects are not present, we reach quite surprisingly and nontrivially to an effective single particle problem. Thus, the canonical AL applies for processes without excluded volume effects. It should be stressed that albeit its technical simplicity and intuitively appealing physical interpretations, any attempts to generalize the insight gained from single particle AL to many particles with arbitrary interactions remains exorbitantly challenging. Furthermore, the generalized AL holds in store quite a few surprises.

Naively, one expects that the generalized AL would be more restrictive from an experimental stand point. However, Eq. (2) suggests the opposite with two immediate interesting directions within the excluded volume universality class. Consider a narrow tube serving as a 1D transport channel for colloids, where both the trap and the interactions can be manipulated externally. Here, the new universality class suggests that such changes would not affect escape rates from the potential trap. However, changes in the colloids density, or varying the shape of the potential (modulated by, e.g., optical tweezers), while keeping ΔU fixed, will result in different escape rates. Therefore, measurement of escape rates allows to infer the particle density in the trap as we change the potential, or vice versa. See Refs. [73–76] and especially Ref. [18] for similar inference problems.

Beyond these applications, this work opens new avenues of research. First, it will be important to extend Eq. (2) to include arbitrary potential landscape, possibly also to higher dimensions [77,78].

A yet unexplored path is to consider the AL for multi-species particles, where D, χ become matrices [79]. It will be worth investigating whether such inter-species interactions can speed up the escape rate of a complex activation process.

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