Letter

Phase chimera states on nonlocal hyperrings

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Chimera states are dynamical states where regions of synchronous trajectories coexist with incoherent ones. A significant amount of research has been devoted to studying chimera states in systems of identical oscillators, nonlocally coupled through pairwise interactions. Nevertheless, there is increasing evidence, also supported by available data, that complex systems are composed of multiple units experiencing many-body interactions that can be modeled by using higher-order structures beyond the paradigm of classic pairwise networks. In this work we investigate whether phase chimera states appear in this framework, by focusing on a topology solely involving many-body, nonlocal, and nonregular interactions, hereby named *nonlocal d-hyperring*, $(d+1)$ being the order of the interactions. We present the theory by using the paradigmatic Stuart-Landau oscillators as node dynamics, and we show that phase chimera states emerge in a variety of structures and with different coupling functions. For comparison, we show that, when higher-order interactions are "flattened" to pairwise ones, the chimera behavior is weaker and more elusive.

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Introduction. Chimera states are an intriguing dynamical phenomenon occurring in systems of coupled oscillators where some units oscillate synchronously, forming coherent domains that exist alongside other domains characterized by incoherent oscillations. Since their first numerical observation by Kaneko [1], the emergence of these patterns has raised the interest of many scholars in nonlinear science. In later studies, these patterns have been identified in experimental setups involving Josephson junctions [2], laser systems [3], mechanical systems [4], electronic circuits [5,6], nanoelectromechanical oscillators [7], neuroscience [8], unihemispheric sleep in birds, mammals, and reptiles [9], as well as various forms of pathological brain states [10]. They have been also found in numerical experiments involving systems of nonlocally coupled oscillators, such as Ginzburg-Landau systems, Rössler oscillators, logistic maps [11–15], and identical phase oscillators [16], prior to being named "Chimera states" in Ref. [17].

Chimera states have been extensively investigated over the past years, leading to exploration along various intriguing avenues [18,19]. From a theoretical point of view, there has been considerable effort to characterize the different types of chimera states that may appear: from phase chimeras [16] to amplitude chimeras [20], amplitude mediated chimeras [21], chimera death states [22], traveling chimeras [23], and globally clustered chimeras [24,25]. Let us observe that the recently defined phase chimera [26] differ from the ones presented in Ref. [16], despite bearing the same name. Indeed, by writing the dynamics of the jth oscillator as $a_j(t)\exp[i(2\pi\Omega_j t+\theta_j)]$, Kuramoto and Battogtokh [16] defined a phase chimera as a state where $a_j(t)$ is constant for all j while the angles $\phi_j=2\pi\Omega_j t+\theta_j$ can be divided into two groups, one for which Ω_j is constant and a second one for which Ω_j depends on the node index. Both behaviors can be appreciated after a sufficiently large t. Differently, Zajdela and Abrams [26] defined a phase chimera as a state for which both $a_j(t)$ and Ω_j are constant, i.e., do not depend on the node index, and only the phases θ_j exhibit coherent and incoherent behaviors. The latter is the framework we will hereby consider.

Another avenue of research on chimera states has focused on understanding the network topologies and coupling mechanisms that can facilitate their emergence. Although chimera states have been proved to arise in 1D rings where every node is connected to its two neighbors (a condition named *local coupling* [27,28]) as well in the "opposite" case of a complete network where each node is connected to all the other ones (*global coupling* [29,30]), the *nonlocal coupling* configuration proved to be particularly important for the onset of chimera states. Indeed, a large amount of literature has demonstrated that chimera states are commonly observed within such setting [16,19]. The nonlocal coupling corresponds to a 1D ring where, beside first order neighbors, each node is also connected in a regular way to $2(k-1)\geqslant 2$ other ones, i.e., each node is connected to k nearby nodes counted in anti-clockwise manner and k in the other direction. The network is regular by

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construction, all the nodes having the same degree $2k$, and it has a complete group of automorphisms, i.e., it is invariant for any possible translation. Beside those regular cases, chimera states have been also found on networks with diverse nonregular topologies [31–36]. Nonetheless, the identification of chimera states has been proven to be challenging, given their transient nature [37] and strong dependence on the initial conditions [38]: hence, researchers have also endeavored to propose mechanisms that can lead to the emergence of robust and persistent chimeras [39,40].

While there have been numerous studies focusing on chimera states within network structures, these states have received significantly less attention in systems where the units interact not only in pairs, but also in groups of three or more units. Studying chimera states in the presence of these types of interactions, known as higher-order or many-body [41,42], is particularly important. In fact, on the one hand, there is an increasing evidence that many systems, e.g., in neuroscience [43–46], naturally exhibit higher-order interactions. On the other hand, several studies on dynamics emerging in hypergraphs and simplicial complexes have already shown a significant impact of higher-order interactions on the collective behavior of the system; the latter include, e.g., studies on synchronization and pattern formation [47–54], random walks [55,56], and contagion processes [57–61]. Previous works on chimera states in higher-order structures have already lead to several interesting results [7,62–65]. Chimera states have been found experimentally by considering triadic interactions [7] and, theoretically, while developing a general theory for the study of synchronization patterns in higher-order systems [62,63]; despite dealing with small hypergraphs and specific settings, these works have pointed out that chimera states may also appear in the presence of higher-order interactions. A more systematic study of the impact of higher-order interactions has been done in Refs. [64,65], where phase oscillators coupled through simplicial complexes are studied; both works show that higher-order interactions, when added to paiwise ones, enhance the likelihood of observing chimera states in the system.

In this work, we further corroborate the claim that chimera states are boosted by the presence of many-body interactions by focusing on a higher-order structure that we call *nonlocal hyperring* and consists exclusively of higher-order interactions. This is an important difference with respect to the setup studied, for instance, in Ref. [65], where simplicial complexes are considered, or in Ref. [66], where a generalization of the nonlocal coupling is extended to higher-order interactions through a continuous limit [67]: in both settings, the effects of pairwise and three-body interactions cannot be fully disentangled. The topology that we consider is nonlocal and nonregular, i.e., nodes have different hyper degree. We consider a d-uniform hypergraph, where all the hyperedges have the same size, $d+1$, and are connected two-by-two, in a periodic structure, by junctions nodes shared by every two consecutive hyperedges. In this way, we have a nonregular extension of the notion of nonlocal rings to higher-order structures. Since nonlocal hyperrings only include higher-order interactions, the effect of the latter on the emergence of chimera states can be better identified. Let us observe that, despite the lack of regular topology, the studied systems always admit a global synchronous solution, that thus "competes" with the possible chimera state. For the sake of definiteveness, we numerically investigate the behavior of nonlocal hyperrings when the nodal dynamics is given by the Stuart-Landau oscillator, which has been widely used as a paradigmatic model of oscillatory dynamics to study chimera states [18], given that this system is the normal form of the Hopf-Andronov bifurcation and thus it presents the general features of any limit-cycle oscillator close to such bifurcation [68]. We then compare the results with the ones observed on a pairwise network, obtained by projecting the higher-order structure on a pairwise network: namely, we consider that every node has a nonweighted pairwise connection with all the nodes part of the same hyperedge, i.e., they form a clique. The higher-order setting exhibits a considerable enhancement of chimera patterns, meaning that they are present for a wider range of coupling strengths and they have a longer life span. Finally, we show that not only phase chimera states, but also other interesting dynamical patterns, such as amplitude chimeras [69], can be observed in the novel setting.

Stuart-Landau oscillators on nonlocal hyperrings. We consider a system made of n interacting Stuart-Landau units, a paradigmatic model of oscillatory dynamics. In the absence of any interaction, each unit j ($j=1,\dots,n$) of the system is described by the following equations:ital

$$\begin{aligned}\dot{x}_j &= \lambda x_j - \omega y_j - \left(x_j^2+y_j^2\right)x_j = f(x_j,y_j),\\ \dot{y}_j &= \omega x_j + \lambda y_j - \left(x_j^2+y_j^2\right)y_j = g(x_j,y_j),\end{aligned} \tag{1}$$

where λ is a bifurcation parameter controlling the onset of a limit cycle of amplitude $\sqrt{\lambda}$, for $\lambda>0$, and ω is the natural frequency of the oscillator. Let us observe that we assume those parameters to be the same for all nodes to ensure the existence of a global synchronous solution.

Here, as in Refs. [47,70], we study nonlinear many-body coupling functions that cannot be decomposed into a combination of weighted pairwise interactions [71]. To model the higher-order interactions we use hyperedges, whose structure can be encoded by using adjacency tensors, that are a generalization of the adjacency matrix for networks [41]. We adopt the convention that a hyperedge involving $(d+1)$ nodes [and, thus, encoding a $(d+1)$-body interaction] is called a d-hyperedge. Such notation is more common in the literature dealing with simplicial complexes [72], while, in the context of hypergraphs, often a d hyperedge encodes a d-body interaction [41]. For example, $A^{(3)}=\{A^{(3)}_{i,j,k,l}\}$ is the third-order adjacency tensor, encoding the four-body interactions, with $A^{(3)}_{i,j,k,l}=1$ if units i, j, k, l have a group interaction (namely, nodes i, j, k, l are part of the same three hyperedge), and 0 otherwise. Using these tensors, the generalized d degree (or hyperdegree), $k_j^{(d)}$, representing the number of d hyperedges of which node j is part, can be computed as

$$k_j^{(d)} = \frac{1}{d!}\sum_{j_1,\dots,j_d=1}^{N} A^{(d)}_{jj_1\dots j_d}. \tag{2}$$

Finally, we assume that the coupling terms act on both the dynamical variables describing the Stuart-Landau oscillator (1) and involve only the first component of the state vector of the oscillator, as in Ref. [6]; let us observe that this working assumption is not restrictive and other couplings can also be considered. Taking into account all these considerations, the model of Stuart-Landau equations coupled via a generic $(d+1)$-body interactions reads

$$\dot{x}_j = f(x_j, y_j) + \epsilon \sum_{j_1,\dots,j_d=1}^{n} A^{(d)}_{j,j_1,\dots,j_d}\big(h^{(d)}(x_{j_1},\dots,x_{j_d}) - h^{(d)}(x_j,\dots,x_j)\big),$$
$$\dot{y}_j = g(x_j, y_j) + \epsilon \sum_{j_1,\dots,j_d=1}^{n} A^{(d)}_{j,j_1,\dots,j_d}\big(h^{(d)}(x_{j_1},\dots,x_{j_d}) - h^{(d)}(x_j,\dots,x_j)\big), \quad (3)$$

where $\epsilon > 0$ is the coupling strength and we assumed that the coupling is diffusivelike [47]. The model encompasses only multibody interactions (specifically, $(d+1)$-body interactions encoded in the dth order adjacency tensor $A^{(d)}$), as we aim to analyze a pure many-body framework.

Let us observe that higher-order interactions are often considered together with pairwise interactions [64,65]; however, in this setting it is not clear how to distinguish the different types of contributions. For this reason, we decided to consider hypergraphs instead of simplicial complexes, as the latter structures contain interactions of any order, from the highest to the lowest (i.e., order one, which is the pairwise case). Moreover, to the best of our knowledge, structures representing the higher-order counterpart of nonlocal rings have not yet been considered in previous works on chimera states. Our goal is to fill this gap by considering a ring encompassing m hyperedges of order d. These hyperedges are placed on a circle and labeled in counterclockwise order as $1, 2, \dots, m$. In the simplest case, the m hyperedges share with each other only a single node. Furthermore, each shared node may belong at most to two different hyperedges, meaning that the generic hyperedge j has two shared nodes, one with the hyperedge $(j-1)$ and one with the hyperedge $(j+1)$. The remaining $d-2$ nodes are only part of the hyperedge j. Let us observe that more general structures could be handled as well.

In the following, we will focus on the case of four-body interactions, but similar results have been obtained for three-body, as well as for five-body and six-body interactions (see Supplemental Material (SM) [73]). In our model, the coupling functions have been chosen to be cubic and diffusivelike, as previously done in Ref. [47], namely

$$h^{(3)}(x_{j_1}, x_{j_2}, x_{j_3}) = x_{j_1}x_{j_2}x_{j_3}. \quad (4)$$

In the SM, we illustrate other choices, i.e., diffusivelike functions that are not cubic. In this specific case, an example of nonlocal ring with $m = 5$ hyperedges of size 4 is shown in Fig. 1(a). The structure is nonregular, as some nodes are part of two hyperedges, i.e., they have hyperdegree 2 [see Eq. (2)], while others interact only within a single hyperedge, and thus exhibit hyperdegree 1. In general, for a nonlocal ring with d-body interactions and n nodes, the nodes that are shared by two hyperedges, i.e., nodes $\{1, 1+(d-1), \dots, n-d-(d-3), n-(d-2)\}$ have generalized dth degree [74], equal to 2, while all the other nodes have generalized dth degree equal to 1. In the following, we will be interested in comparing

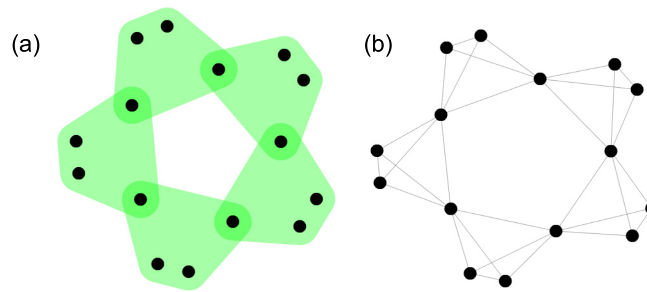

FIG. 1. Nonlocal hyperring vs clique-projected network. (a) A nonlocal three hyperring with $m = 5$ hyperedges and 15 nodes. (b) Corresponding clique-projected network obtained by transforming each hyperedge into a clique.

the dynamical behavior of SL oscillators coupled via higher-order structures with the one resulting from a pairwise one. To have a reliable comparison, we decide to work with the network obtained by projecting each hyperedge in a clique, as shown in Fig. 1(b), to obtain the so-called clique-projected network (cpn). Namely, each couple of nodes in the hyperedge is connected with an unweighted pairwise link. In this way, we end up with $d+1$ cliques connected through junction nodes, that are part of two consecutive cliques, exactly as the hyperedges are disposed in the hyperring. Let us point out that this is not the only possible projection onto a network, as one could also consider projection based on one-cell complexes, which are topological objects obtained by gluing together cells in different ways. In this sense, they can be viewed as generalizations of simplicial complexes [72] that have recently found applications in several field, from machine learning [75] to synchronization dynamics [76]. In the SM we have analysed a different projection, specifically, mapping any d hyperedge as a regular polygon having $(d+1)$ vertices. When the topology is pairwise, the coupling functions take only one variable, we thus define $h^{\text{cpn}}(x_j) = h^{(3)}(x_{j_1}, x_{j_2}, x_{j_3})|_{x_{j_1}=x_{j_2}=x_{j_3}=x_j}$. In our example of four-body interactions, the coupling functions (4) can be replaced on the cpn by

$$h^{\text{cpn}}(x_\ell) - h^{\text{cpn}}(x_j) = x_\ell^3 - x_j^3.$$

To make the difference between the pairwise and higher-order coupling more visible, in the SM, we explicitly write the equations for a node shared between two hyperedges (junction node) and a node belonging to only one hyperedge (nonjunction node).

Higher-order versus pairwise interactions. We now proceed to the analysis of chimera states observed in the nonlocal hyperring, while also comparing them to the pairwise case. In all the simulations, we have considered hyperrings and networks of $n = 204$ nodes, hence $m = 68$ hyperedges of size 4 (i.e., 3-hyperedges), and we have set initial conditions to $(x_i = +1, y_i = -1)$ for nodes indexes $j \leqslant n/2$ and $(x_j = -1, y_j = +1)$ for $j > n/2$, as in Ref. [18].

In Fig. 2, we compare the dynamics emerging by using higher-order and pairwise interactions. In Figs. 2(a) and 2(b), we show the behavior of the system of SL oscillators coupled with a nonlocal three hyperring, while Figs. 2(c) and 2(d) depict the same system of oscillators, but now coupled via the clique-projected network (pairwise interactions). In Fig. 2(a),

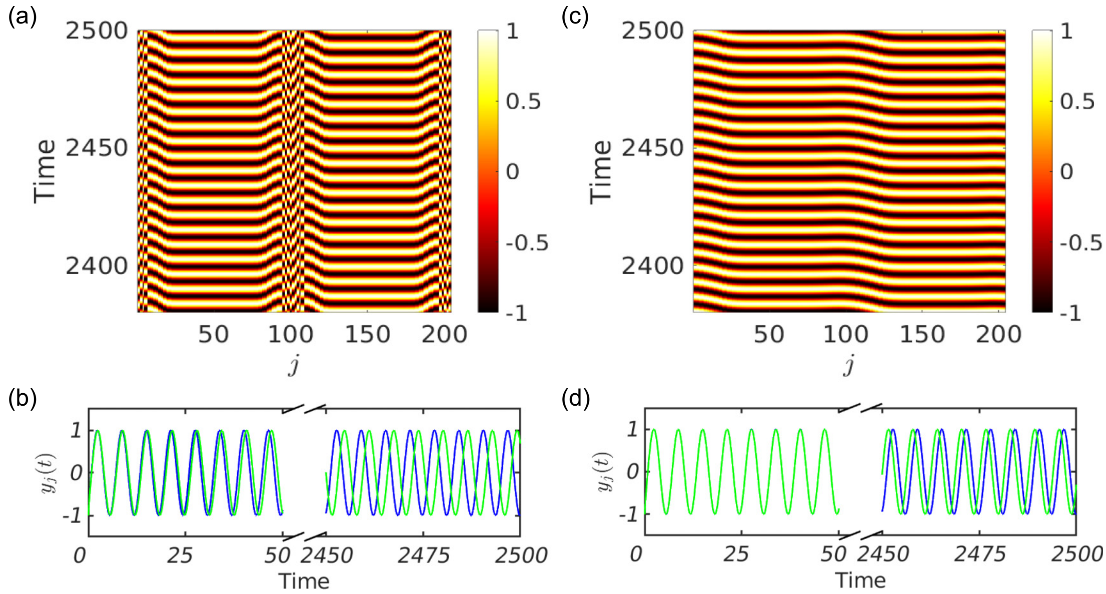

FIG. 2. Nonlocal hyperring vs clique-projected network. Left panels show spatiotemporal patterns (a) and time series (b) for variables $y_j(t)$ of the SL oscillators coupled with a nonlocal three hyperrings with $n = 204$ nodes and $m = 68$ hyperedges; the emerging dynamical behavior is a phase chimera state with two heads, i.e., there are regions of regular behavior, separated by two regions of decoherence. Right panels (c) and (d) show the analogous quantities on the clique-projected network. Panels (b) and (d) show the time series for nodes 50 (blue) and 101 (green). For panels (a) and (c), on the horizontal axis we set the node index j while in the vertical one, the time. The coupling strength is fixed at $\epsilon = 0.01$ and model parameters are $\lambda = 1$ and $\omega = 1$.

we can appreciate a phase chimera behavior, whereas in Fig. 2(c) there are no domains with incoherent behaviors. Figures 2(b) and 2(d) show the time series for short and long times for nodes $j = 50$ (blue) and $j = 101$ (green). First of all, we observe that in both cases the orbits have the same amplitude and the same period, hence same frequency. However, in the case of the higher-order coupling, nodes initially synchronized are eventually found in phase opposition [see blue and green curves in Fig. 2(b)]; On the other hand, in the clique-projected network, nodes develop a small phase lag [see blue and green curves in Fig. 2(d)]. We are thus facing phase chimeras [26] in the hyperring, but not on the clique-projected network. To better analyze the obtained pattern, we cast the SL variables into a single complex number, one for each node, $z_j(t) := x_j(t) + iy_j(t)$, and we then rewrite the latter as $z_j(t) = a_j(t)\exp[i(2\pi\Omega_j t + \theta_j)]$, where a_j is the signal amplitude, Ω_j the frequency and θ_j the constant phase.

By using the Fast Fourier Transform and a sliding window setting, we numerically compute these quantities for all nodes after a sufficiently long transient interval. The results reported in Fig. 3 support the claim that we are dealing with phase chimeras according to Ref. [26]: indeed, the amplitudes, a_j, and the frequencies, Ω_j, are almost constant, i.e., their values do not depend on the node indexes, while the constant phases, θ_j, are node-dependent. In the case of higher-order coupling [Fig. 3(c)], the phases can be divided into two regions: a first one, where the phase value does not depend on the node index, and a second one, where the opposite holds true. Moreover, in this second region the phases are scattered in $[-\pi, \pi)$. The clique-projected network has a similar behavior: amplitudes and frequencies have constant values, while the phases exhibit smooth transitions between two "flat" regions, where there is no node dependence of the phase θ_j. The latter difference, which can be quantified through the *normalized total phase variation* (see SM, which includes also Refs. [78–80]), is indeed the key: while for the system with higher-order coupling we can see a clear chimera behavior, the aforementioned smooth variation cannot be considered a chimera state, as there is no coexistence between coherent and incoherent states. In the SM, we show that such difference between a higher-order topology and a pairwise one persists also when considering different pairwise projections, such as the aforementioned boundary of the cell complex. Although a full correspondence between the two cases cannot be established, as we are dealing with interactions that are intrinsically distinct, the difference is striking and it is consistently observed when the coupling strength is varied. Moreover, it also persists for different hyperrings, such as the two, four, and five hyperrings, as shown in the SM numerically and by mean of the normalized total phase variation.

Conclusions. In this work, we have studied the effect of pure many-body interactions on chimera states, showing that the presence of a higher-order topology enhances the emergence of phase chimeras. We have introduced a nonlocal nonregular (pure) higher-order topology, which we called nonlocal hyperring, and observed that the comparison between the dynamics on the latter and on the clique-projected network shows a remarkable difference in the behaviors and allows to draw a conclusion that is consistent with the existing literature, namely that higher-order interactions facilitate the emergence of chimera states in systems of coupled oscillators. Moreover, nonlocal hyperrings allow for the emergence of other phenomena, as we show in the SM, such as a hybrid state of amplitude chimeras and chimera death, and pure chimera death [22]. We believe that the emergence of such type of chimera states, widely studied for pairwise interactions, but

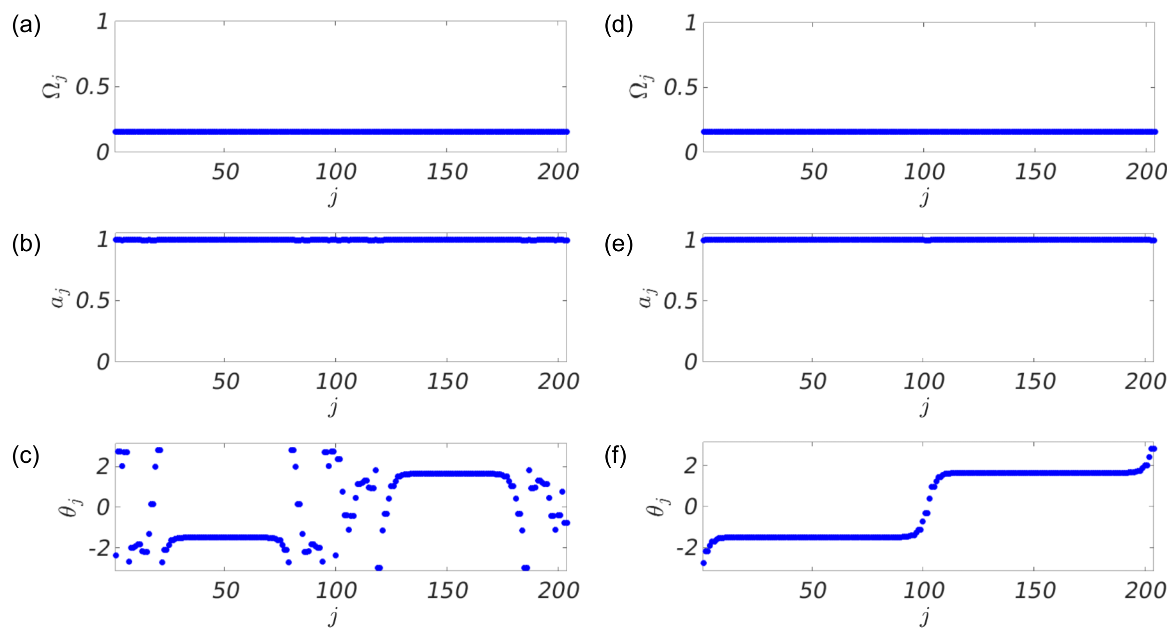

FIG. 3. Dynamical quantities associated to nonlocal hyperring and clique-projected network. Left panels show frequency (a), amplitude (b), and phase (c) of the Stuart-Landau oscillators computed by using the fast Fourier transform (FFT) on the complex variables $z_j(t) = a_j(t)\exp[i(2\pi\Omega_j t + \theta_j)]$, as function of the node index, j, at 2500 time units. The left panels refer to quantities whose evolution is determined by a nonlocal three hyperring with $n = 204$ nodes and $m = 68$ hyperedges. The right panels show the analogous quantities for Stuart-Landau oscillators coupled via the clique-projected network. The coupling strength is $\epsilon = 0.01$ and the model parameters are $\lambda = 1$ and $\omega = 1$.

until now never observed with higher-order ones, makes our framework promising and will inspire future studies in this direction.

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