Migration costs and rewarding schemes in spatial public goods games

Rashid Ibrahimli,[1,2] Marco Tomassini,[3] and Alberto Antonioni[1]
[1]*Grupo Interdisciplinar de Sistemas Complejos (GISC), Department of Mathematics, Carlos III University of Madrid, 28911 Leganés, Spain*
[2]*Department of Data Science, Caja Blanca Datos SL, 28013 Madrid, Spain*
[3]*Department of Information Systems, University of Lausanne, CH-1015 Lausanne, Switzerland*

(Received 17 January 2024; accepted 3 April 2024; published 3 May 2024)

This study explores the influence of migration costs and rewarding schemes on cooperation through the implementation of computational behavioral models in spatial public goods games. The former involves a cost for agents to migrate to a neighboring group, while the latter rewards them for remaining in the same group for multiple rounds. By analyzing these mechanisms separately and in combination, we unveil their effects on cooperative behavior. The grid-based game dynamics begins with equal size groups, and agents can adjust their contributions each round, with the option to migrate if unsatisfied. Our findings reveal that when considered separately, the rewarding scheme is not as effective in achieving full cooperation as the migration cost scheme. Combining migration costs and rewards instead yields high cooperation levels with low public goods game enhancement factors and migration probability. Our results offer valuable insights for contexts where promoting cooperative behavior is crucial, such as community engagement development and public policies.

DOI: 10.1103/PhysRevE.109.054303

I. INTRODUCTION

Public goods games (PGGs) illustrate social situations in which individuals are called upon to make contributions, incurring personal costs, ultimately benefiting the community. The central issue in PGGs is the emergence of free-riding as a dominant strategy, presenting a classic social dilemma [1,2]. Generally, the rational strategy for an individual in a public goods game (PGG) would be to refrain from contributing and instead benefit from the contributions of others. However, when a substantial number of agents adopt this self-interested approach, it results in the failure to generate any public good. The game dynamic is significantly influenced by the number of iterations or repetitions [3,4]. Additionally, group size and information also play pivotal roles in shaping this dynamic [5]. As the game progresses, agents tend to reduce their contributions, causing the overall average contribution to converge towards zero. This phenomenon aligns with the concept of the *last-round effect*, representing the Nash equilibrium in a one-shot PGG where contributing nothing is the optimal strategy. Despite this theoretical perspective, in real-life scenarios, we often observe a high level of cooperation within PGG frameworks [6].

Empirical studies have consistently demonstrated that individuals learn from their mistakes as the game unfolds [7]. Over successive iterations, agents acquire the skill to maximize their payoffs, leading to behavioral adjustments. An analysis of linear PGG conducted over multiple rounds reveals a noteworthy trend: the mean contribution in the first round is approximately half of the endowment [8]. However, as the number of rounds increases, agents tend to adopt more strategic, payoff-maximizing approaches with reduced contributions. The main purpose of behavioral experiments carried out with human subjects is to design and to test which game setup can achieve the higher contribution level. One of the most prevalent strategies involves the implementation of institutional or peer-to-peer incentives, as well as sanctions based on agent contributions [3,4,9]. Institutional mechanisms involve the establishment of formal rules and authorities to administer punishment and rewards. On the other hand, peer-to-peer schemes rely on interactions and agreements among individuals within the group to administer punishment and rewards. These approaches effectively drive increased contributions by reshaping the strategies of the participants [3,10–12]. In some cases, both reward and punishment are implemented in combination [13]. The main idea of these incentives is to impact the self-interest of individuals, which is, in fact, to pressure the agent to contribute [10]. In order to add a more realistic dimension, the exploration of PGGs in different networked frameworks has been a significant research focus, including spatial structures [14–17]. In spatially structured games, agents are positioned in designated nodes of lattices, interacting exclusively with neighboring nodes [18,19]. Studies incorporating diverse spatial PGG topologies have demonstrated the effectiveness of implementing reward and punishment mechanisms in enhancing cooperation [20,21].

In this study, we explore through numerical simulations the migration mechanism and rewarding incentives in PGGs played on a lattice structure. Unlike previous research [18,22–25], in our approach, the nodes of the lattice represent distinct groups rather than individual agents. Other studies exploring evolutionary games and mobility within a two-dimensional space focus on pairwise interactions between agents at specific distances, rather than grouping them into nodes of a network structure [26–29]. Migration, in our context, refers to agents moving to neighboring groups in the case of dissatisfaction with their payoff. While some studies [30,31] have explored and underscored the importance of heterogeneity in mimicking real-life scenarios and enhancing cooperation, the prevalence of homogeneous games persists. This preference is based on previous findings that indicate no significant qualitative difference in average contributions in repeated PGGs [32]. Our main objective is to examine the impact of the migration scheme within the spatial group structure, referred to as the baseline model, and to assess how punishment and reward schemes influence this baseline model. In this context, punishment is defined as a migration cost, requiring agents to incur a cost when they migrate to neighboring groups. Conversely, the rewarding scheme can be described as a loyalty reward, wherein payoffs increase until reaching the maximum amount when agents remain within their group. To align the simulation with experimentally observed scenarios, fractional contributions were employed, as has been utilized in another previous study [33].

The structure of the paper continues with an explanation of the methodology, followed by a presentation of the results, and will conclude with a discussion of these.

II. METHODS

A. Spatial group structure and game interactions

In contrast to our previous study [33], which focused on a group structure with no spatial constraints, the current study introduces a bidimensional grid structure with periodic boundary conditions (i.e., toruslike topology). Here, each node of the grid represents a group of agents engaging in a public goods game (PGG) and it is connected to its four spatial neighbors, determined by the Manhattan distance. Connecting edges correspond to the potential migration options available to the agents dissatisfied with their current group. The dimensions of the implemented grids are $m \times m$, where the product $G = m^2$ represents the total number of groups. On the other hand, the total number of agents, denoted as N, is chosen to be divisible by G to ensure a uniform initial distribution of agents among the groups. At the beginning of the simulations, each group includes $k = \frac{N}{G}$ agents.

Regarding the game interactions, the total contributions made by the agents in each separate group are summed and multiplied by an enhancement factor $r \geqslant 1$; the resulting amount is then divided equally among all participants. The payoff π_i of agent i after each round is determined by the following equation:

$$\pi_i = (E - c_i) + \frac{r}{n_i} \sum_{j=1}^{n_i} c_{g_{i,j}},$$

where E represents the agent round endowment, which is set to $E = 1$; c_i is the contribution of agent i in each round, with $0 \leqslant c_i \leqslant 1$; n_i is the number of agents in the group g_i of agent i; and $c_{g_{i,j}}$ is the contribution of all agents in the group g_i. The first term on the right-hand side, $d_i = E - c_i$, denotes the amount remaining from the endowment E after agent i plays its contribution. When there is only one agent in a group ($n_i = 1, c_{g_{i,j}} = c_i$), no dilemma arises, with $r \geqslant 1$. Instead, the dilemma intensifies as the number of agents in the group increases.

Initially, as in [33], agent contributions are randomly selected from a predefined list of values, namely, $\{0, 0.25, 0.5, 0.75, 1\}$. The maximum allowable contribution corresponds to the total round endowment received by the agent. This set of contribution options mirrors, in a simplified way, standard laboratory experiments where participants, typically endowed with around 10 tokens, can choose integer values for their contributions.

B. Game dynamics and setups

In our settings, introduced in a previous work as the blind model [33], agents lack any knowledge about the contributions and payoffs of their neighbors. After each round, when agents make their own contributions and receive payoffs, they must decide on their subsequent action based on certain satisfaction criteria dictated by the rules of the game setup. Previously [33], we included more information in agents' satisfaction rules implementing the rational model heuristics, although the current study is limited to the blind model scenario. Here, if an agent's benefit is equal to or greater than their contribution, they are considered satisfied. In such cases, there is a 0.5 probability that the agent will increase their contribution by 0.25 in the next round, or, with the same probability, they may choose to keep their contribution unchanged. The agent's benefit, denoted as b_i, is calculated as the individual's gain from the game without considering the initial endowment E received by the agent at the start of each round. It is determined by the formula $b_i = \frac{r}{n_i} \sum_{j=1}^{n_i} c_{g_{i,j}}$. Conversely, if an agent's benefit is less than their contribution, they are deemed unsatisfied. In this case, they opt to randomly migrate to another group among those directly connected to the agent's group, with a probability of p, maintaining their contribution amount and having the same endowment as other agents for the following round. Instead, with a probability $1 - p$, they decrease their contribution by 0.25 for the subsequent round without migrating.

Theoretical and experimental research on PGG suggests that the introduction of punishment and rewarding rules in the game has a positive effect on curbing free-riding [34–36]. Some researchers have gone as far as proposing that punishment and rewards are crucial mechanisms for achieving sustainable cooperation [10]. In the literature, two types of incentive structures have been explored: peer and institutional. In the peer incentive structure, agents may enforce rewards or punishments on other agents within the group. The second structured mechanism involves institutions, where game rules impose incentives based on predefined conditions. In this work, we focus on simulating the latter incentive system through a rewarding scheme, which incorporates both punishment and reward components. Here, each incoming agent receives an individual discount rate for their total earnings, starting at $\delta_i = 0.6$. This rate increases by 0.1 per round until $\delta_i = 1$. If an agent migrates to a new group, their discount rate resets to the initial value of 0.6. Unlike the baseline model, the introduction of a discount factor introduces a dilemma even with a single player in a group, particularly for low values of r. The payoff equation in this scenario becomes $\pi_i = (E - c_i) + r\delta_i c_i$.

Previous studies have indicated that institutional punishment has a more significant impact on achieving higher contribution levels compared to rewarding strategies [16]. In contrast to the typical method of implementing punishment, we introduce a different approach where punishment

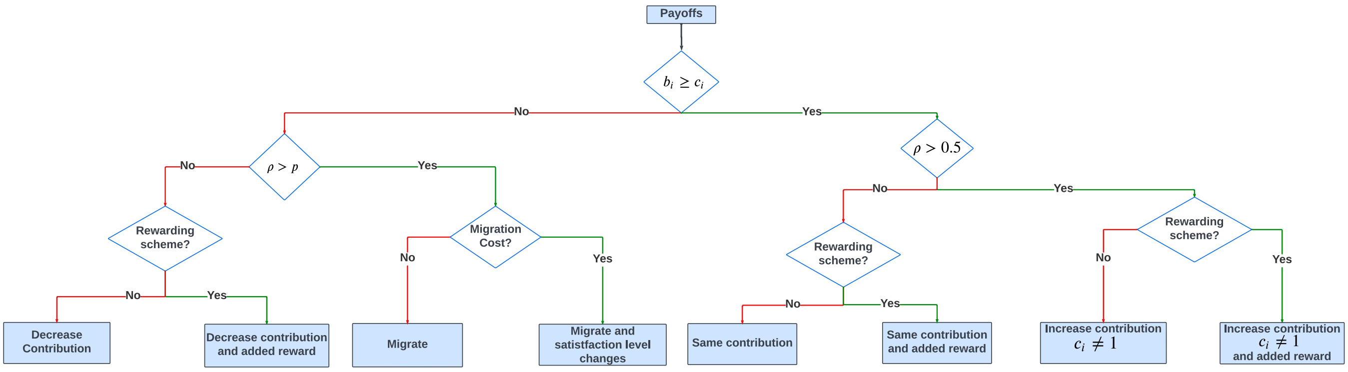

FIG. 1. Treelike representation of simulated PGG models, incorporating migration cost and a rewarding scheme. In this illustration, b_i represents the benefit of agent i, c_i is the contribution of agent i, r denotes a random number generated between 0 and 1, and p signifies the migration probability of agents.

is applied through migration. Some studies allowed agents to freely switch groups without consequences if they were dissatisfied with their payoff at the end of each round [32]. Although integrating migration within groups can facilitate increased levels of cooperation, incorporating migration costs into the equation has the potential to further enhance this cooperation and is more realistic. As previously mentioned, an agent is considered satisfied if its benefit from the round is equal to or exceeds its contribution. However, when an agent is unsatisfied and willing to migrate to a neighboring group, the condition can be adjusted to $c_i - c_m \leqslant b_i$, where c_m is the migration cost. In this context, the migration cost is chosen to be $c_m = 0.2$, which is a reasonable value.

To summarize, we investigate four game setups:

(1) Baseline: Agents play the PGG with the option to migrate to a randomly chosen adjacent group when unsatisfied.

(2) Rewarding scheme: In addition to the baseline setup, agents are rewarded for staying in the same group for multiple rounds.

(3) Migration cost: In addition to the baseline setup, migrating to a neighboring group incurs a cost of $c_m = 0.2$.

(4) Combined: Both the rewarding scheme and migration cost are present in this setup. Figure 1 shows a complete overview of different model setups, presented in a structured tree format.

III. RESULTS

In this section, we present the results obtained from the numerical simulations. Beginning with the baseline model, as illustrated in Fig. 2, the graph depicts the temporal progression of the agents' mean contribution over 800 consecutive rounds. This setup consists of 144 groups, each comprising 10 agents at the beginning of the simulation and a migration probability set at $p = 0.5$. Notably, when the condition $r > 2$ is satisfied, full contribution is achieved in very few rounds. Similarly, for the specific case of $r = 2$, a high level of cooperation is attained after 100 rounds. However, it is crucial to note that values below this threshold result in inevitable free-riding, as all agents become unsatisfied. The threshold value $r = 2$, together with migration probability $p = 0.5$, tested for the baseline model, can be used as a reference point for determining when a setup can be considered more cooperative for lower enhancement factors or other values of the migration probability.

Figure 3 illustrates the average contribution of the agent population after 800 rounds, considering a migration probability ranging from 0 to 1 and an enhancement factor spanning from 1 to 3. The grid size was 12×12, having 144 groups with 10 agents in each one at the beginning. The reported values were obtained after 800 rounds and averaged over 10 Monte Carlo simulations. Furthermore, we report in Fig. 4 the normalized average number of migrations for the same game setups. For a proper comparison, the normalization is calculated taking the number of migrations performed during the entire simulation and divided by the maximum number of migrations achieved in the four setups. We also tested

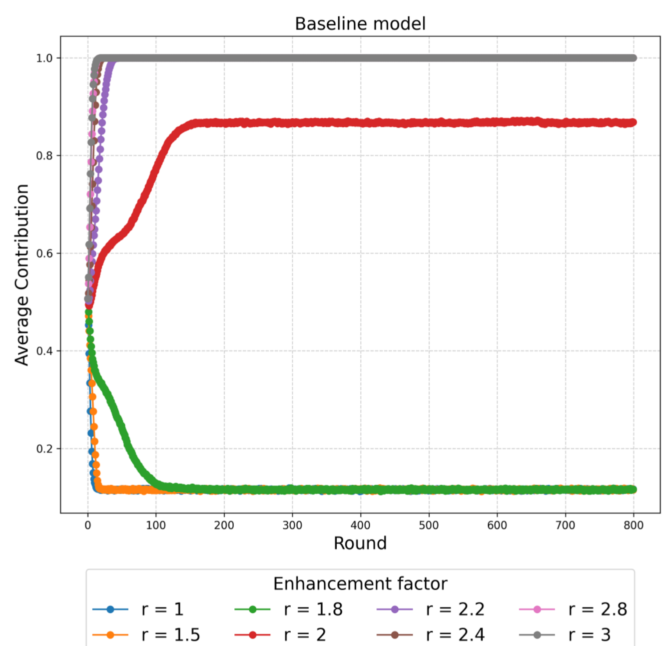

FIG. 2. Average contribution as a function of the round number in the baseline model for the enhancement factors provided in the inset. The simulation includes 144 groups (12×12 grid), with each group initially comprising 10 agents and a migration probability of $p = 0.5$. The model is simulated over 800 rounds and averaged over 10 simulations.

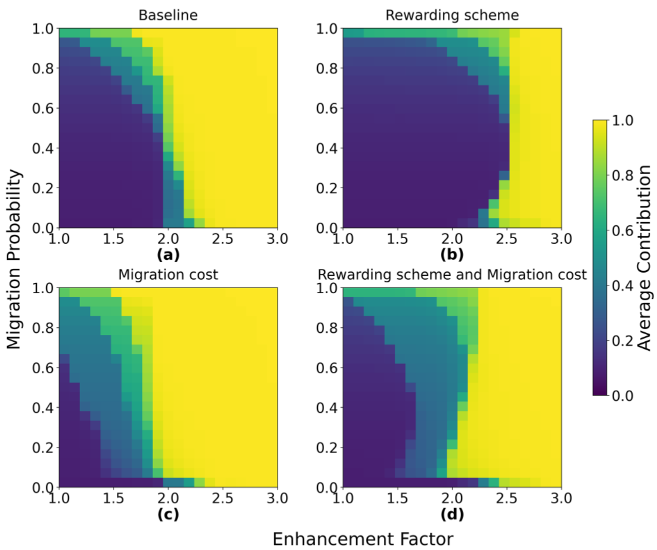

FIG. 3. Final average contribution for the four game setups, PGG enhancement factors, and migration probabilities. The simulations were generated with 144 groups (12×12 grid), each with 10 agents initially and 800 rounds. The results were averaged over 10 Monte Carlo simulations.

the four scenarios with the initial group sizes reduced to 5. These results are not included as the only significant difference occurred in the migration cost scenario, where, due to the diminished strength of the dilemma with respect to the migration cost, full cooperation was achieved for any nonzero value of the migration probability.

The results of the baseline model in Fig. 3(a) reveal that the migration rule positively influences achieving full contributions for any given enhancement factor. In scenarios where there is no migration, indicated by a migration probability $p = 0$, the average contribution tends to remain at or below 0.2 until the enhancement factor r reaches approximately 2. Full contribution becomes possible only after the enhancement factor surpasses approximately 2.4. As the migration probability increases, the required enhancement factor threshold for achieving full contribution decreases. For example, when the entire population has a migration probability of 0.5, full contribution becomes attainable with an enhancement factor of around 2.1 (see, also, Fig. 2). The underlying concept is that higher migration probabilities among unsatisfied agents lead to increased overall contributions. Even for low enhancement factor values ($r < 2$), where all unsatisfied agents migrate, the game dynamics shows higher average contributions. As the enhancement factor rises, full contribution levels are quickly achieved. Unsatisfied agents, when migrating to new groups, consistently make higher contributions, leading to increased average contributions in these groups. Conversely, satisfied agents with low contributions, i.e., defectors, stay in their original groups. However, following the agent updating rules, satisfied agents maintain or increase their contributions in subsequent rounds, resulting in the convergence towards higher contributions.

In the rewarding scheme setup [see Fig. 3(b)], an enhancement factor of 2.3 or lower leads to decreasing average contributions below 0.8 migration probabilities. Surprisingly, higher (above 0.6) and lower (below 0.2) migration probabilities correlate with increased average contributions, showcasing the migration's role in fostering cooperation. In particular, in the simulations depicted in Fig. 5, where games are simulated with an enhancement factor of 2.5 and varying migration probabilities over 2000 rounds, contributions gradually converge to the full amount for low migration probabilities. On the other hand, higher migration stabilizes average contributions around the full amount. In summary, while the rewarding system boosts contributions in specific scenarios, especially at low and high migration probabilities, it is less effective than the baseline setup.

In the migration cost setup shown in Fig. 3(c), it is noteworthy that even with a minimal enhancement factor, introducing migration costs can significantly boost cooperation. For low enhancement factors ($1.5 \leqslant r < 2$) and higher migration probabilities, almost full cooperation is achieved. This phenomenon can be explained by the agents' tendency to migrate to neighboring groups until they reach satisfaction, as the migration cost gradually becomes tolerable enough for them to stay satisfied within the group. Above $r = 2$, full cooperation is consistently attained, with the exception being cases of zero migration probability.

In the combined setup depicted in Fig. 3(d), the resulting contribution level appears to be a combination of the results from the previous two setups. Consequently, there is an improvement in the average contribution within the enhancement factor interval where cooperation was initially low due to the rewarding scheme. This improvement can be attributed to the influence of migration cost on the rewarding scheme. However, it is important to note that overall, achieving a high level of cooperation may still be challenging due to the negative impact of the rewarding scheme. Noting the increased number of migrations in this final setup [see Fig. 4(d)], it is evident that migration proves beneficial once again in attaining higher levels of cooperation in regions where the rewarding scheme alone fails to yield any contributions.

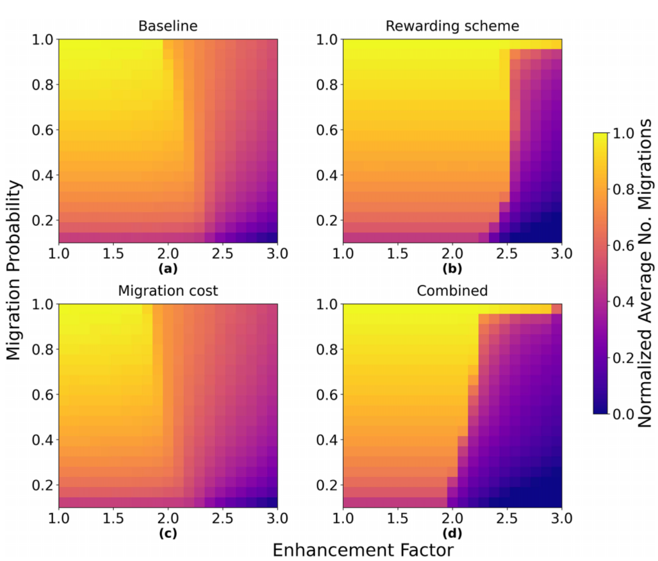

FIG. 4. Normalized average migrations for the four game setups, PGG enhancement factors, and migration probabilities. Results were obtained using the same setups of Fig. 3.

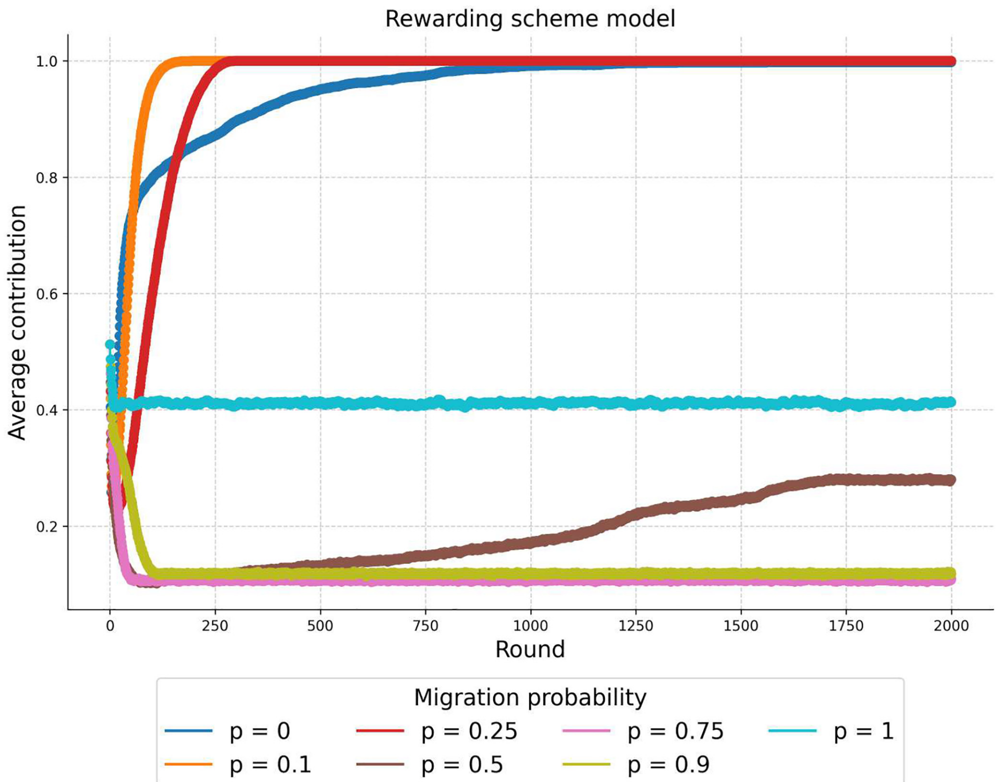

FIG. 5. Evolution of the average contribution for the rewarding scheme setup. Results were obtained using the same setups of Fig. 3, except for the number of runs which is 2000, with curves corresponding to different migration probabilities and enhancement factor of $r = 2.5$.

Finally, Fig. 6 visually illustrates agent mobility and final configurations for the four setups. Significant agent dynamics in neighboring group movement were notable, particularly in the initial 100 rounds. Subsequently, the migration rate considerably slowed. Additionally, distinct population imbalances within adjacent groups emerged, with one group having more fully contributing (and satisfied) agents than its neighboring groups. Comparing the effects of a rewarding scheme to a migration cost scheme, it becomes clear that migration costs can promote higher cooperation levels, even in scenarios with low enhancement factors and migration probabilities. This observation supports the notion that punishment mechanisms may be more effective than reward mechanisms in fostering higher contribution levels.

The initial conditions determining the stability of a group (i.e., being fully composed of satisfied agents) can be defined by examining the initial state. A group is initially formed by N agents, each contributing, on average, 0.5, following a uniform distribution of initial contributions. Consequently, the benefit equals $\delta r/2$. If this benefit exceeds the agent's contribution c_i, the agent is satisfied and is more likely to increase their contribution, increasing group stability and contribution level. This condition is easily met when $r > 2$ and $\delta = 1$.

To characterize the observed absorbing states, we analyze the different final configurations obtained in Fig. 6. The first scenario occurs when the enhancement factor r is too low relative to the game setup, particularly evident in the rewarding scenario (third row panels). Here, agents moving to a new group receive a payoff discounted by $\delta \leqslant 1$, leading to dissatisfaction (i.e., contributing less or moving again to another group). This heuristic further decreases the average contribution level of the group, rendering most agents unsatisfied and uncooperative. Another absorbing state arises when migration costs are incorporated into the previous scenario, resulting in a reduced migration rate (fourth row panels). Migration costs make a few groups composed of satisfied agents, enhancing their cooperation rate for a medium-high enhancement factor. Once these groups stabilize, new incoming unsatisfied agents from other groups can join and become satisfied, further reinforcing the group's cooperation level. The same absorbing state is observed in the scenario with migration costs only (second row panels), where forming cooperative groups becomes easier due to the higher satisfaction threshold in the absence of the rewarding scheme. Finally, the baseline scenario presents an intermediate state between the migration cost and combined scenarios, characterized by greater agent mobility (first row panels). To summarize, the observed absorbing states can be divided into two main outcomes: mostly unsatisfied and not fully cooperative agents, or fully satisfied and cooperative agents forming static groups. In the second outcome, the size of the static groups that are formed can vary. This variation is influenced by the specific setup of the game and the initial conditions of group formation, as described above.

IV. CONCLUSIONS

In our study, we incorporated migration features into spatially structured dynamic public goods games, along with punishment and rewarding mechanisms to explore their

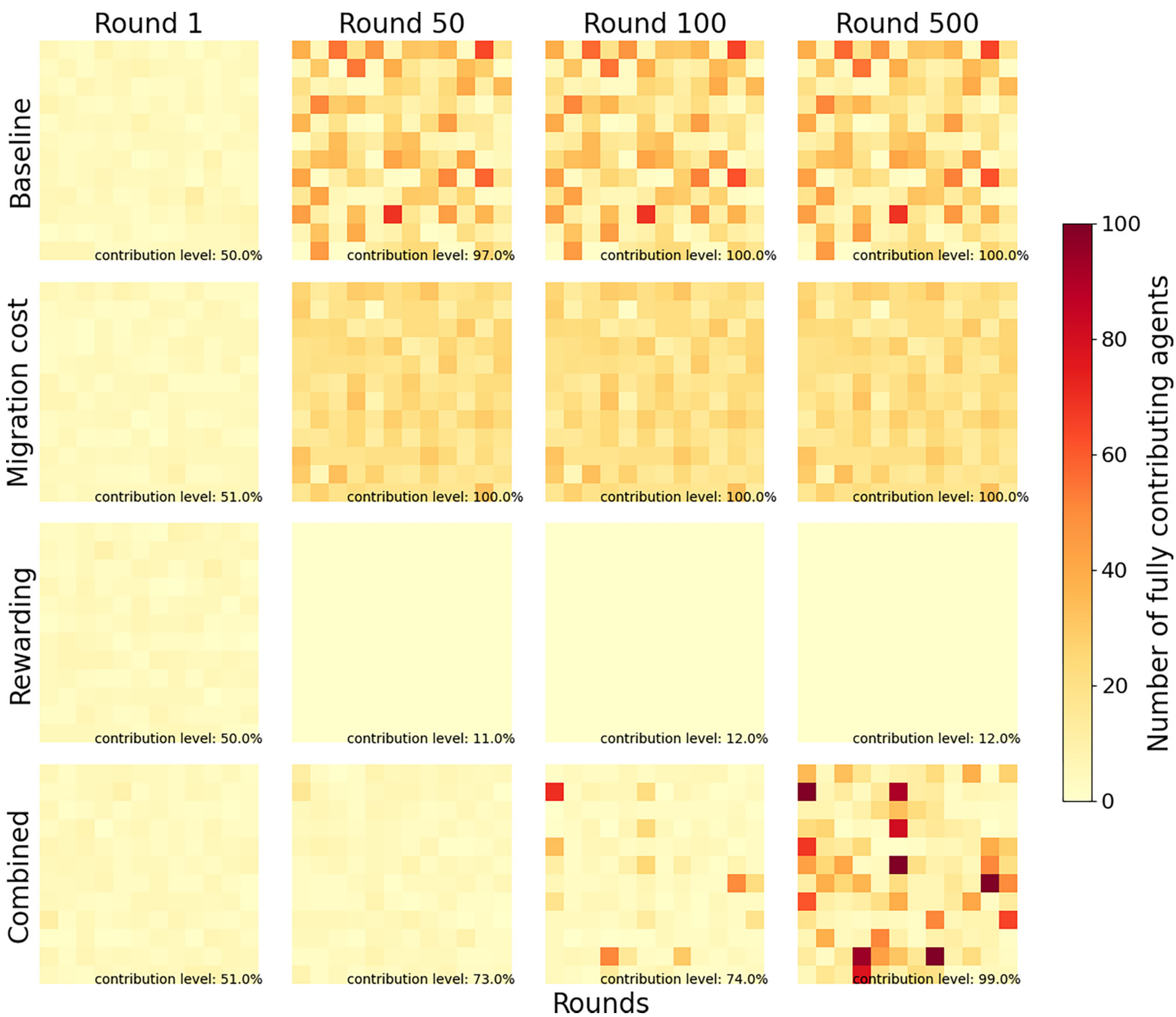

FIG. 6. The spatial evolution for the four game setups showing the number of fully contributed agents in each group, i.e., a cell of the grid, with snapshots at rounds 0, 50, 100, and 500. Results were obtained using the same setups of Fig. 3. Simulation parameters were set to $r = 2.2$, $p = 0.5$, and $c_m = 0.2$.

impact on cooperation. These mechanisms were institutional procedures, simulating that an overseeing authority administered all rewards and penalties. Punishment occurred when an agent moved to a neighboring group, while in the rewarding scheme, each agent's contribution was incrementally boosted each round if they stayed within their group.

We investigated four game setups: the baseline without additional mechanisms, the inclusion of migration costs, the implementation of a rewarding scheme, and a combined mechanism. Interestingly, even without any mechanisms, the migration feature alone led to a high level of cooperation and effectively curbed free-riding, even at lower enhancement factors. The introduction of migration costs further amplified cooperation, especially at lower enhancement factors. In this scenario, while migration costs may lower the frequency of migration, it simultaneously enabled agents to lower their satisfaction threshold, thereby enhancing their willingness to contribute. Conversely, the rewarding scheme had a contrasting effect in this spatially structured public goods game, resulting in an increased number of free-riders and a decrease in cooperation. In summary, we introduced migration costs and a rewarding scheme to incentivize agents to remain with their groups and enhance cooperation. Our simulations indicate that with migration costs, agents tend to move to other groups and eventually become satisfied with their contributions, even when enhancement factors are low.

As a next step in our research, we plan to conduct experiments in a laboratory and online setting with human participants simulating a framework that is similar to our simulations. This will further validate and expand upon our findings, bridging the gap between theoretical simulations and real-world applications. It will also provide valuable insights into how these mechanisms interact with human decision-making.

ACKNOWLEDGMENTS

This research has been supported by Ministerio de Ciencia e Innovación (Spain/PEICTI) through grants PID2022-141802NB-I00 (BASIC) and IJC2019-040967-I.

[1] E. Ostrom, *Governing the Commons: The Evolution of Institutions for Collective Action* (Cambridge University Press, Cambridge, 1990).
[2] K. G. Binmore, *Game Theory and the Social Contract*, Vol. 1 Playing Fair (MIT Press, Massachusetts, 1994).
[3] E. Fehr and S. Gächter, Am. Econ. Rev. **90**, 980 (2000).
[4] M. Kosfeld, A. Okada, and A. Riedl, Am. Econ. Rev. **99**, 1335 (2009).
[5] M. Pereda, I. Tamarit, A. Antonioni, J. A. Cuesta, P. Hernández, and A. Sánchez, Sci. Rep. **9**, 15023 (2019).
[6] D. G. Rand and M. A. Nowak, Trends Cognit. Sci. **17**, 413 (2013).
[7] M. N. Burton-Chellew, H. H. Nax, and S. A. West, Proc. R. Soc. B. **282**, 20142678 (2015).
[8] J. Zelmer, Exptl. Econ. **6**, 299 (2003).
[9] A. Alventosa, A. Antonioni, and P. Hernández, J. Econ. Behav. Organiz. **185**, 513 (2021).
[10] Y. Dong, B. Zhang, and Y. Tao, Sci. Rep. **6**, 28809 (2016).
[11] O. Gurerk, B. Rockenbach, and I. Wolff, SSRN Electron. J. **5**, 1 (2010).
[12] H.-C. Peng, Ann. Public Cooperat. Econ. **93**, 533 (2021).
[13] J.-J. Wu, C. Li, B.-Y. Zhang, R. Cressman, and Y. Tao, Sci. Rep. **4**, 6421 (2014).
[14] I. Okada, H. Yamamoto, E. Akiyama, and F. Toriumi, Sci. Rep. **11**, 7642 (2021).
[15] Q. Su, L. Wang, and H. E. Stanley, New J. Phys. **20**, 103030 (2018).
[16] X. Wang, R. Ding, J. Zhao, W. Chen, and C. Gu, Chaos Solitons Fractals **156**, 111862 (2022).
[17] F. C. Santos, M. D. Santos, and J. M. Pacheco, Nature (London) **454**, 213 (2008).
[18] F. Battiston, M. Perc, and V. Latora, New J. Phys. **19**, 073017 (2017).
[19] K. Xie, X. Liu, H. Wang, and Y. Jiang, Chaos Solitons Fractals **172**, 113562 (2023).
[20] M. Fu, W. Guo, L. Cheng, S. Huang, and D. Chen, Physica A **525**, 1323 (2019).
[21] A. Szolnoki and M. Perc, Europhys. Lett. **92**, 38003 (2010).
[22] M. Perc, J. Gómez-Gardenes, A. Szolnoki, L. M. Floría, and Y. Moreno, J. R. Soc. Interface. **10**, 20120997 (2013).
[23] X. Chen and A. Szolnoki, Sci. Rep. **6**, 32802 (2016).
[24] F. Battiston, G. Cencetti, I. Iacopini, V. Latora, M. Lucas, A. Patania, J.-G. Young, and G. Petri, Phys. Rep. **874**, 1 (2020).
[25] S. Boccaletti, P. De Lellis, C. Del Genio, K. Alfaro-Bittner, R. Criado, S. Jalan, and M. Romance, Phys. Rep. **1018**, 1 (2023).
[26] P. Buesser, M. Tomassini, and A. Antonioni, Phys. Rev. E **88**, 042806 (2013).
[27] A. Antonioni, M. Tomassini, and P. Buesser, J. Theor. Biol. **344**, 40 (2014).
[28] M. Tomassini and A. Antonioni, J. Theor. Biol. **364**, 154 (2015).
[29] S. Xiao, L. Zhang, H. Li, Q. Dai, and J. Yang, Eur. Phys. J. B **95**, 67 (2022).
[30] Y. Li, J. Zhang, and M. Perc, Appl. Math. Comput. **320**, 437 (2018).
[31] Q. Wang, H. Wang, Z. Zhang, Y. Li, Y. Liu, and M. Perc, Physica A **502**, 570 (2018).
[32] M. Tomassini and A. Antonioni, J. Phys. **2**, 045013 (2021).
[33] M. Tomassini and A. Antonioni, Front. Phys. **8**, 58 (2020).
[34] T. Sasaki and S. Uchida, Biol. Lett. **10**, 20130903 (2014).
[35] L. Nockur, S. Pfattheicher, and J. Keller, J. Exptl. Soc. Psychol. **93**, 104096 (2021).
[36] N. Van Miltenburg, V. Buskens, D. Barrera, and W. Raub, Intl. J. Commons **8**, 47 (2014).