Inhibiting the self-discharging process of quantum batteries in non-Markovian noises

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One of the main challenges in developing high-performance quantum batteries is the self-discharging process, where energy is dissipated from a quantum battery into the environment. In this work, we investigate the influence of non-Markovian noises on the performance of a quantum battery. Our results demonstrate that adding auxiliary qubits to a quantum battery system can effectively suppress the self-discharging process, leading to an improvement in both the steady-state energy and extractable work. We reveal that the physical mechanism inhibiting the self-discharging process is the formation of system-environment bound states, rather than an increase in non-Markovianity. Our results could be of both theoretical and experimental interest in exploring the ability of quantum batteries to maintain long stored energy in the environment.

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I. INTRODUCTION

With the development of technology, batteries are becoming increasingly miniaturized [1,2]. When the size of a battery approaches that of a molecule or an atom, the quantum effect begins an important role. This has stimulated research on quantum batteries [3–10]. A quantum battery is a quantum system with multiple energy levels, designed for energy storage and extraction. Compared to classical batteries, the use of quantum resources yields enhanced performance in terms of energy storage density [11], extractable work [12–14], and charging power [15–20].

To achieve optimal quantum batteries, attention should be paid not only to quantum batteries with high charging performance [12,15,16,20–51] but also to quantum batteries with robust energy storage performance [52–56]. This ensures that the energy can be stored in the quantum battery for a prolonged period of time. Currently, the main obstacle limiting optimal energy storage performance is the self-discharging process [54], which refers to the dissipation of energy from the quantum battery to the environment. Recently, many methods to suppress the self-discharging process have been proposed. For instance, in Ref. [54], storing energy using quantum coherence has been found. Compared to a Markovian environment where energy flows unidirectionally from the system to the environment, the non-Markovian environment allows for a bidirectional energy flow between the system and the environment, leading to better energy storage performance [56]. However, previous research [22,43,56] has typically relied on creating strong coupling between the system and the environment to induce non-Markovian behavior, an approach that is currently feasible only in a few experimental platforms and remains limited to specific physical systems such as superconducting quantum circuits [57] or trapped ion systems [58]. Therefore, it is crucial and meaningful to develop an environmental non-Markovian scheme that can be experimentally widely implemented to enhance the energy storage performance of the quantum battery.

To do this, in this work, we propose a scheme that is easy to experimentally realize and improves the energy storage of quantum batteries, i.e., inserting auxiliary qubits into the quantum battery system can greatly suppress the self-discharging process of quantum batteries. Here, we do not take into account the interactions between qubits. This is experimentally achievable [57,59]. For example, in a superconducting circuit system, there can be no interaction between qubits and qubits by manipulating the capacitance [57]. In cavity QED systems, the dipole-dipole interaction can be ignored when the distance between qubits is much larger than the wavelength of the field [59]. We reveal that the reason behind suppressing the self-discharging process is the faster formation of bound states between the system and the environment due to the increase in auxiliary qubits, which is independent of the increase in non-Markovianity. Our result suggests that one can suppress the self-discharging process of open quantum batteries via engineering the formation of the bound state.

II. QUANTUM BATTERY

How to suppress the self-discharging process of quantum batteries and make them capable of retaining energy in the environment for a long time has been a hot topic of research. In this work, we focus on the self-discharging process of quantum batteries in finite time t. In this case, the performance of the battery can be studied in terms of the internal energy and the ergotropy. The internal energy of the quantum battery

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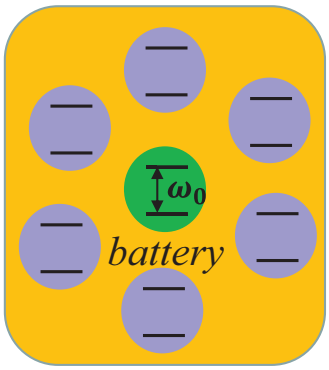

FIG. 1. Schematic diagram of a model of the quantum battery and auxiliary qubits in a common reservoir environment.

at t is defined as [16,33]

$$E_B(t) = \mathrm{Tr}\,[H_B \rho_B(t)], \tag{1}$$

where $\rho_B(t)$ is the reduced density matrix of the quantum battery at time t.

Then the second law of thermodynamics tells us that not all energy can be extracted from a quantum battery. The amount of energy a quantum battery can extract can be measured with the ergotropy [12–14], i.e.,

$$W_B(t) = \mathrm{Tr}\,[\rho_B(t) H_B] - \mathrm{Tr}(\sigma_{\rho_B} H_B), \tag{2}$$

where σ_{ρ_B} is the passive counterpart of $\rho_B(t)$. Passive states are defined as those states that do not allow for work extraction in a cyclic (unitary) process [60,61]. Ergotropy represents the maximum energy that can be extracted by a quantum battery at the end of the charging process under cyclic unitary operation.

To obtain the conditions under which a quantum battery can store energy for a long time under the influence of the environment, the larger $E_B(t)$ and $W_B(t)$ are required.

III. SELF-DISCHARGING PROCESS OF A QUANTUM BATTERY IN THE PRESENCE OF ADDITIONAL QUBITS

We consider the self-discharging process of a quantum battery (i.e., a two-level system) in the presence of multiple auxiliary qubits, as shown in Fig. 1. The total system Hamiltonian is

$$H = \sum_{l=1}^{N} \omega_0 \sigma_l^+ \sigma_l^- + \sum_k \omega_k b_k^+ b_k + \sum_{l=1}^{N} \sum_k (g_k^* b_k^\dagger \sigma_l^- + g_k b_k \sigma_l^+), \tag{3}$$

where σ_l^+ (σ_l^-) are the raising (lowering) operators of the lth qubit. The coupling strength between the two-level systems and their environment is denoted by g_k. In the interaction picture, the Hamiltonian H of the total system can be written as

$$H_{int} = \sum_{l=1}^{N} \sum_k (g_k^* b_k^\dagger \sigma_l^- e^{i\Delta_k t} + g_k b_k \sigma_l^+ e^{-i\Delta_k t}), \tag{4}$$

where $\Delta_k = \omega_k - \omega_0$.

Suppose the initial state of the quantum battery (i.e., the lth qubit) is of the form $\alpha(0)|0\rangle_B + \beta_l(0)|1\rangle_B$ and the states of the additional qubits and the reservoir respectively read $|0\rangle^{\otimes N-1}$ and $|\bar{\mathbf{0}}\rangle_R$ with $|0\rangle^{\otimes N-1} = |0,0,0,\ldots,0\rangle$ and $|\bar{\mathbf{0}}\rangle_R = \prod_k |0_k\rangle$. Then the initial state of the total system $|\phi(0)\rangle = (\alpha(0)|0\rangle_B + \beta_l(0)|1\rangle_B) \otimes |0\rangle^{\otimes N-1} \otimes |\bar{\mathbf{0}}\rangle_R$ evolves after time $t > 0$ into the state $|\phi(t)\rangle = \alpha(0)|0\rangle_B|0\rangle^{\otimes N-1}|\bar{\mathbf{0}}\rangle_R + \sum_{l=1}^{N} \beta_l(t)|0\rangle_{l\mathrm{th}\equiv 1}^{\otimes N}|\bar{\mathbf{0}}\rangle_R + |0\rangle^{\otimes N} \otimes C_k(t)|\mathbf{1_k}\rangle_R$, where $|\mathbf{1_k}\rangle_R \equiv |0\cdots 1_k \cdots 0\rangle_R$ means that there is one excitation in the kth mode of the reservoir, $|0\rangle_{l\mathrm{th}\equiv 1}^{\otimes N}$ denotes that the lth qubit is excited $|1\rangle$, and the rest of the qubits are in the ground state $|0\rangle$. According to the Schrödinger equation in the interaction picture, the time evolution of the total system is determined by the following differential equations:

$$\begin{aligned} d\beta_l(t)/dt &= -i \sum_k g_k^* e^{-i\Delta_k t} C_k(t), \\ dC_k(t)/dt &= -i g_k e^{i\Delta_k t} \sum_{l'=1}^{N} \beta_{l'}(t), \end{aligned} \tag{5}$$

where $l = 1, 2, \ldots, N$. Integrating $\dot{C}_k(t)$ with the initial condition $C_k(0) = 0$ and inserting the solution into $\dot{\beta}_l(t)$, one obtains the integro-differential equation for the amplitude $\beta_l(t)$:

$$\dot{\beta}_l(t) = -\int_0^t \sum_k |g_k|^2 \sum_{l'=1}^{N} \beta_{l'}(t') e^{-i\Delta_k(t-t')} dt'. \tag{6}$$

$\sum_k |g_k|^2 e^{-i\Delta_k(t-t')}$ can be recognized as the correlation function $f(t-t')$ of the reservoir R. Here we assume the reservoir R has a Lorentzian spectrum $J(\omega) = \Gamma\lambda^2/\{2\pi[(\omega-\omega_0)^2 + \lambda^2]\}$, where $\tau = \lambda^{-1}$ is the correlation time (memory time) of the reservoir environment; λ and Γ refer to the spectrum width of the Lorentzian spectrum and the qubit-reservoir coupling strength, respectively. The correlation function $f(t-t') = \sum_k |g_k|^2 e^{-i\Delta_k(t-t')} = \Gamma\lambda\, e^{-\lambda|t-t'|}/2$ can be given. Typically, in a weak-coupling regime ($\lambda > 2\Gamma$), the dynamics behavior of the system is Markovian and irreversible decay occurs. For a strong-coupling regime ($\lambda < 2\Gamma$), non-Markovian dynamics occur accompanied by an oscillatory reversible decay. Then $\beta_l(t)$ is solved by the Laplace transformation with the solution given by

$$\beta_l(t) = (N-1)\beta_l(0)/N + \beta_l(0) e^{-\frac{1}{2}\lambda t}\left(\cosh\frac{1}{2}\Omega t + \frac{\lambda}{\Omega}\sinh\frac{1}{2}\Omega t\right)\Big/ N, \tag{7}$$

where $\Omega = \sqrt{\lambda^2 - 2\Gamma\lambda N}$. The reduced density matrix of the lth qubit in the system's basis $\{|1\rangle, |0\rangle\}$ can be expressed as

$$\rho(t) = \begin{pmatrix} \rho_{11}(0)|\beta_l(t)|^2 & \rho_{10}(0)\beta_l(t) \\ \rho_{01}(0)\beta_l^*(t) & \rho_{00}(0) + \rho_{11}(0)(1 - |\beta_l(t)|^2) \end{pmatrix}, \tag{8}$$

where $\rho_{11}(0) = |\beta_l(0)|^2$, $\rho_{00}(0) = |\alpha(0)|^2$, and $\rho_{10}(0) = \rho_{01}^*(0) = \beta_l(0)\alpha^*(0)$. According to the procedure in

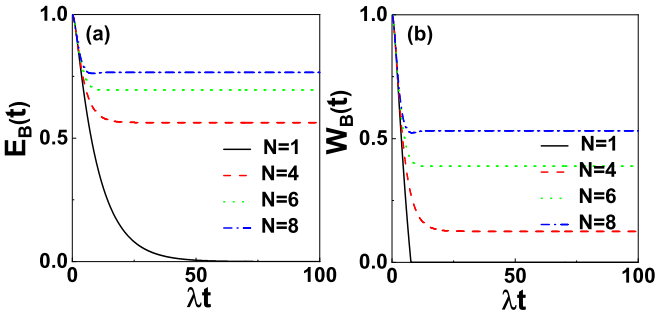

FIG. 2. (a), (b) Internal energy $E_B(t)$ and the ergotropy $W_B(t)$ of the quantum battery as a function of the dimensionless quantity λt for different numbers N of auxiliary qubits in the weak qubit-environment coupling mechanism. The parameter is (a), (b) $\Gamma/\lambda = 0.1$. The internal energy $E_B(t)$ and the ergotropy $W_B(t)$ of a quantum battery are in units of ω_0. The quantum battery is initially in an excited state.

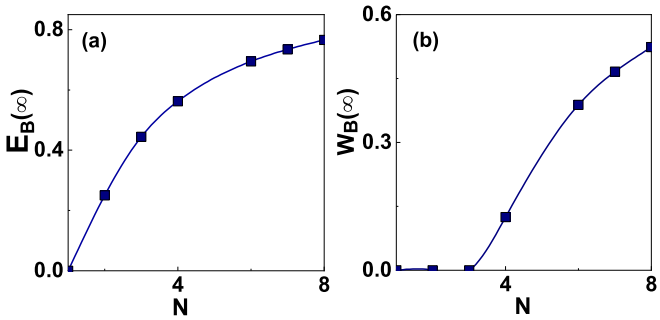

FIG. 3. Steady-state energy $E_B(\infty)$ and extractable work $W_B(\infty)$ of a quantum battery vary with the number of auxiliary qubits in the weak qubit-environment coupling mechanism. The quantum battery steady-state energy $E_B(\infty)$ and ergotropy $W_B(\infty)$ are in units of ω_0. The quantum battery is initially in an excited state.

Appendix A, the passive state can be written as $\sigma_{\rho_B} = \eta_1|1\rangle\langle 1| + \eta_0|0\rangle\langle 0|$, where η_0 and $\eta_1(\eta_0 > \eta_1)$ are the eigenvalues of $\rho(t)$. The specific expressions of eigenvalues can be found in Appendix A.

From Eq. (A4) in Appendix A, the impact of additional qubits on the self-discharging process of a quantum battery does not depend on whether they are initially in the excited state or coherent state. For convenience, we consider the first qubit as a quantum battery and assume it is initially in the excited state. At this time, Eq. (A4) can be written as

$$E_B(t) = \omega_0|\beta_1(t)|^2,$$
$$W_B(t) = \omega_0[2|\beta_1(t)|^2 - 1]\Theta[|\beta_1(t)|^2 - 1/2]. \quad (9)$$

Here $\Theta(x - x_0)$ is the Heaviside function. In the following, based on Eq. (9), we analyze the effect of auxiliary qubits on the self-discharging process of the quantum battery in a weak or strong qubit-environment coupling mechanism.

A. Weak-coupling regime

When the quantum battery is weakly coupled to the reservoir, the energy of the quantum battery decays irreversibly into the environment, resulting in poor energy storage performance. Here, we study the impact of adding auxiliary qubits to the quantum battery system under the weak-coupling regime on the self-discharging process. We plot the variation of the internal energy $E_B(t)$ and the ergotropy $W_B(t)$ with λt, as shown in Figs. 2(a) and 2(b). We find that under long-term evolution, a larger number of auxiliary qubits N leads to higher internal energy $E_B(t)$ and extractable work $W_B(t)$. This means that the increase in auxiliary qubits has a resisting effect on the self-discharging process of a quantum battery in the environment. Therefore, in the case of weak coupling between the system and the environment, in order for a quantum battery to store energy in the environment for a long time, more auxiliary qubits are required.

To get a full picture of the effect of the number N of auxiliary qubits on the final energy of the quantum battery in the environment, the variations of the steady-state energy $E_B(\infty)$ and extractable work $W_B(\infty)$ of a quantum battery with N are plotted in Figs. 3(a) and 3(b). We find that the steady-state energy $E_B(\infty)$ increases with the number of auxiliary qubits N. This is because, in the long-time evolution limit, the steady-state energy $E_B(\infty) = \omega_0(N-1)^2/N^2$ is a monotonically increasing function of N. As for the extractable work in Fig. 3(b), when the number of auxiliary qubits N exceeds $N_{cr} = 3$ (N_{cr} is a critical number of qubits), the extractable work increases monotonically with the number of auxiliary qubits. While $N \leqslant N_{cr} = 3$, no useful energy can be extracted from the quantum battery. To explain the above phenomena, we start from the expression of the steady-state extractable work. According to Eq. (10), the steady-state extractable work can be written as $W_B(\infty) = \omega_0[2(N-1)^2/N^2 - 1]\Theta[(N-1)^2/N^2 - 1/2]$. When $(N-1)^2/N^2 - 1/2 > 0$, $W_B(\infty) > 0$. By solving for $(N-1)^2/N^2 - 1/2 > 0$, $N > 2 + \sqrt{2}$ or $N < 2 - \sqrt{2}$ is obtained. So there is no extractable work for $N = 2$ and $N = 3$. According to the expression $W_B(\infty)$ of the extractable work in the steady state, $W_B(\infty)$ is only related to the number N of qubits. Thus energy cannot be extracted from the battery in the steady state for $N = 1, 2, 3$ without doing anything to the system. Then to analyze how $W_B(\infty)$ varies with N, $dW_B(\infty)/dN = 4/N^2 - 4/N^3$ $(N > 3)$. Clearly, for the case of $N > 3$, $W_B(\infty)$ is monotonically increasing with N. So, in the weak system-environment coupling mechanism, to achieve the excellent energy storage performance of a quantum battery, a larger number of auxiliary qubits are needed.

B. Strong-coupling regime

We also verify whether increasing the number of auxiliary qubits N under a strong system-environment coupling regime can suppress the self-discharging process of quantum batteries in the environment. The variation of the internal energy $E_B(t)$ and extractable work $W_B(t)$ with λt for different values of N is plotted in Fig. 4. As shown, the greater the number of auxiliary qubits, the greater the internal energy $E_B(t)$ and the greater the extractable work $W_B(t)$. In contrast to weak coupling, in strong-coupling mechanisms, the internal energy and extractable work of a quantum battery fluctuate over time. The cause of this phenomenon may lie in the memory effect of the environment that appears under strong coupling. Then,

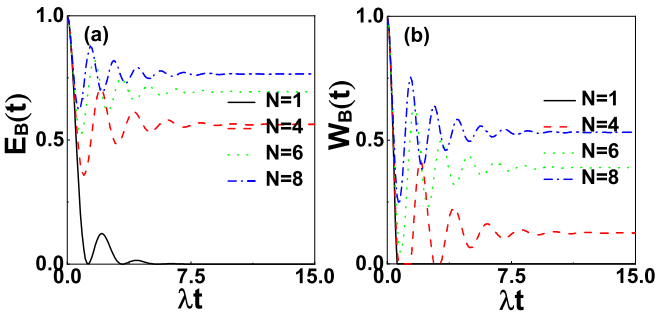

FIG. 4. (a), (b) Internal energy $E_B(t)$ and the ergotropy $W_B(t)$ of the quantum battery as a function of the dimensionless quantity λt for different numbers N of auxiliary qubits in the strong qubit-environment coupling mechanism. The parameter is (a), (b) $\Gamma/\lambda = 5$. The internal energy $E_B(t)$ and the ergotropy $W_B(t)$ of a quantum battery are in units of ω_0. The quantum battery is initially in an excited state.

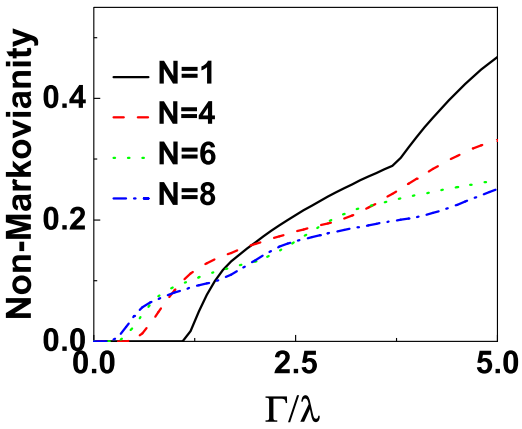

FIG. 5. Non-Markovianity as a function of the coupling strength Γ/λ.

according to Eq. (9), the steady-state energy and extractable work have the same expressions under the weak- or strong-coupling regime for long time evolution, so they vary with N in the same way as in Fig. 3. Combined with the above analysis, in the weak or strong system-environment coupling mechanism, more auxiliary qubits are required to obtain a quantum battery with a long time energy storage capacity in the environment.

IV. NON-MARKOVIAN DYNAMICS CONTROL

Now one may wonder why increasing the number of auxiliary qubits can improve the energy storage performance of quantum batteries in the environment. According to Refs. [43,56], environmental non-Markovianity can inhibit the self-discharging process of a quantum battery. Therefore, we first checked whether the addition of auxiliary qubits could increase the environmental non-Markovianity and thus suppress the self-discharging process. Before testing our ideas, we recall the quantitative expressions for assessing the non-Markovian effect. The non-Markovian effect can be quantified by the non-Markovianity [62]

$$\mathcal{N} = \max_{\rho_1(0),\rho_2(0)} \int_{\sigma>0} \sigma[t, \rho_1(0), \rho_2(0)]dt, \tag{10}$$

in which $\sigma[t, \rho_1(0), \rho_2(0)] = dD[\rho_1(t), \rho_2(t)]/dt$ is the rate of change of the trace distance given by $D[\rho_1(t), \rho_2(t)] = \mathrm{Tr}|\rho_1(t) - \rho_2(t)|/2$, where $|A| = \sqrt{A^+A}$. $\sigma[t, \rho_1(0), \rho_2(0)] \leqslant 0$ corresponds to all dynamical semigroups and all time-dependent Markovian processes. While a process is non-Markovian if there exists a pair of initial states and at certain time t such that $\sigma[t, \rho_1(0), \rho_2(0)] > 0$. We should take the maximum overall initial states $\rho_{1,2}(0)$ to calculate the non-Markovianity. For our qubits system, it has been proven that the optimal pair of initial states to maximize non-Markovianity are $\rho_{1,2}(0) = |\pm\rangle\langle\pm|$ with $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$ [63], which means $\sigma[t, \rho_1(0), \rho_2(0)] = d|\beta_1(t)|/dt$.

Then, the variation of the non-Markovianity with the coupling strength Γ/λ is plotted in Fig. 5. It is worth noting that, by fixing the number of qubits, a remarkable dynamical crossover from Markovian behavior to non-Markovian behavior can occur at a certain critical coupling strength Γ_c/λ. When $\Gamma/\lambda < \Gamma_c/\lambda$, the dynamics process abides by Markovian behavior and then the non-Markovianity increases monotonically with increasing Γ/λ. The critical coupling strength Γ_c/λ is mainly determined by the number N. The larger the value of N is, the smaller the value of the critical coupling strength Γ_c/λ should be requested. Furthermore, the results in Fig. 5 also demonstrate that the number of auxiliary qubits has a different effect on the non-Markovianity for different coupling strengths Γ/λ. In particular, when the fixed coupling strength $\Gamma/\lambda = 5$ (i.e., in the strong system-environment coupling regime), a smaller number of auxiliary qubits excites larger non-Markovianity. Combining Fig. 3 and Fig. 5, we find that, for a fixed number of auxiliary qubits, the strength of non-Markovianity does not determine the magnitude of the steady-state energy $E_B(\infty)$ and extractable work $W_B(\infty)$ of a quantum battery in the environment. That is to say, the enhancement of non-Markovianity is not the physical reason for improving the energy storage performance of a quantum battery in the environment.

V. FORMATION OF SYSTEM-ENVIRONMENT BOUND STATES

To further explore why the addition of auxiliary qubits can improve the energy storage performance of a quantum battery, we attempt to explain it from the perspective of the formation of bound states between the system and the environment. The origin of this idea lies in the fact that the formation of bound states is very resistant to decoherence [64–69].

We start with the energy spectrum of the total system. The energy spectrum can be obtained by solving the eigenvalue equation $H|\phi(t)\rangle = E|\phi(t)\rangle$, where H is the Hamiltonian of the total system in the interaction picture. Then we can get the following $N+1$ equations: $\omega_k C_k(t) + \sum_{l=1}^{N} g_k^* e^{-i(\omega_0-\omega_k)t}\beta_l(t) = EC_k(t)$, $\omega_0\beta_1(t) + \sum_k g_k e^{i(\omega_0-\omega_k)t}C_k(t) = E\beta_1(t), \ldots, \omega_0\beta_N(t) + \sum_k g_k e^{i(\omega_0-\omega_k)t}C_k(t) = E\beta_N(t)$. By taking $C_k(t)$ from the first equation to the other equations and eliminating the amplitudes in these equations, the combination of these equations gives

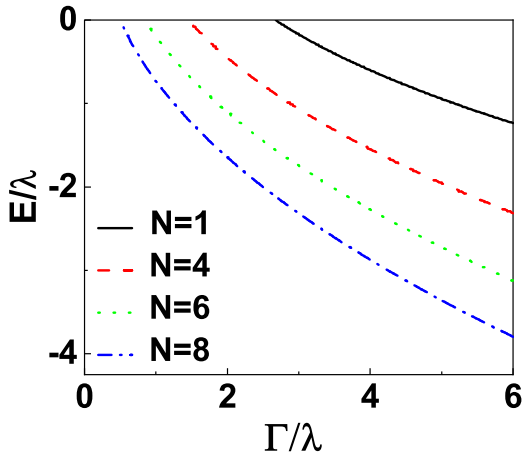

FIG. 6. Energy spectrum of the system reservoir as a function of the coupling strength Γ/λ.

this compact form

$$y(E) \equiv \omega_0 - N\int_0^\infty \frac{J(\omega)}{\omega - E}d\omega = E. \tag{11}$$

Refer to Appendix B for the detailed calculation process. Since $y(E)$ decreases monotonically with the increase of E in $E < 0$ and $\lim_{E\to-\infty} = \omega_0$, the condition for Eq. (11) to have at least one isolated root is $E < 0$. We call the eigenstate corresponding to this isolated eigenvalue E_{BS} the bound state. Then the existence of bound states in the total Hamiltonian spectrum depends on the fact that Eq. (11) must satisfy the condition $y(0) < 0$; otherwise, the formation of bound states is suppressed. To demonstrate whether the increase in auxiliary qubits is more conducive to the formation of bound states, we plot the variation of the system-environment energy spectrum with the coupling strength Γ/λ in Fig. 6. We show that an increase in the number of auxiliary qubits leads to a faster emergence of bound states (i.e., $E < 0$). In other words, stronger bound states are created by inserting more additional qubits into the reservoir. It is concluded that the increase in the number of auxiliary qubits leads to improved performance of quantum battery energy storage possibly associated with the faster formation of system-environment bound states.

To further understand the role of bound states in the self-discharging process of a quantum battery, we can expand the initial state $|\phi(0)\rangle$ based on the eigenstates of the total system, i.e., $|\phi(0)\rangle = d_{\rm BS}|\varphi_{\rm BS}\rangle + \sum_{j\in{\rm CB}} d^j_{\rm CB}|\varphi^j_{\rm CB}\rangle$, where $|\varphi_{\rm BS}\rangle$ is the potentially formed bound state and $|\varphi^j_{\rm CB}\rangle$ are the eigenstates in the continuous energy band. The time evolution of the total system at any given moment is written as $|\phi(t)\rangle = d_{\rm BS}e^{-iE_{\rm BS}t}|\varphi_{\rm BS}\rangle + \sum_{j\in{\rm CB}} d^j_{\rm CB}e^{-iE^j_{\rm CB}t}|\varphi^j_{\rm CB}\rangle$, where $E_{\rm BS}$ is the eigenenergy of the bound state. Due to the out-of-phase interference contributed by the continuum energy $E^j_{\rm CB}$, all of the initial populations in the summation of $|\phi(t)\rangle$ tend to disappear and only the excited state population in the bound state component exists in the long time limit. By this argument, only the bound state has a major role in the energy storage performance of a quantum battery in the environment. Then, in Fig. 6, the higher the number of auxiliary qubits placed, the easier the formation of bound states between the system and the environment. Therefore, the increase in the number of auxiliary qubits leads to faster formation of system-environment bound states, which is the fundamental reason for resisting the self-discharging process of a quantum battery in the environment.

VI. CONCLUSION

In conclusion, we have studied the self-discharging process of a two-level quantum battery in the presence of auxiliary qubits. It is found that the inclusion of auxiliary qubits can enhance the ability of a quantum battery to store energy for a long time in either strong or weak system-environment coupling regimes. We have revealed that this phenomenon is attributed to the formation of the whole system bound states, rather than an increase in environmental non-Markovianity. Our work opens an avenue to suppress the self-discharging process of a quantum battery via engineering the formation of the bound state.

Then it is worth noting that our model can be realized by ultracold atoms held by optical lattice [70,71] and by transmon qubits in a circuit QED system [72,73]. Currently, the system-environment bound state has been observed in circuit QED and ultracold-atom systems. For example, in photonic crystals [72], a single-photon bound state is formed when a bare qubit is tuned near the band edge. And this bound state can be used to control quantum transport within the band gap. For artificial emitter systems implemented using ultracold atoms in optical lattices [70], the authors detect a tunable bound state that contains evanescent matter waves by controlling vacuum coupling and the excitation energy. It also indicates that suppressing the self-discharging process of a quantum battery can be achieved in the most advanced quantum optical experimental technology by manipulating the system-environment bound state.

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APPENDIX A: DETAILS ON THE PASSIVE STATE

Denoting ρ as the density matrix of a quantum system, we describe it in the context of a Hamiltonian H and we articulate both matrices through their spectral decompositions:

$$\rho = \sum_n \eta_n|\eta_n\rangle\langle\eta_n|,$$
$$H = \sum_n e_n|e_n\rangle\langle e_n|. \tag{A1}$$

Here, $\{|\eta_n\rangle\}_n$ and $\{|e_n\rangle\}_n$ denote the eigenvectors of ρ and H, respectively, and $\eta_0 \geqslant \eta_1 \geqslant \cdots$ and $e_0 \leqslant e_1 \leqslant \cdots$ are the associated eigenvalues, which have been properly ordered. The passive state is defined as the following density matrix [60,61]:

$$\sigma_{\rho_B} = \sum_n \eta_n |e_n\rangle\langle e_n|. \tag{A2}$$

Then, for our system, the eigenvalues of ρ are $\eta_1 = (1-m)/2$ and $\eta_0 = (1+m)/2$, where $m = \sqrt{1-4\rho_{11}^2(0)\beta_l(t)^2 + 4\rho_{11}^2(0)\beta_l(t)^4}$. The passive state can be written as

$$\sigma_{\rho_B} = (1-m)/2|1\rangle\langle 1| + (1+m)/2|0\rangle\langle 0|. \tag{A3}$$

According to Eqs. (1) and (2), the internal energy $E_B(t)$ and the ergotropy $W_B(t)$ during self-discharging can be given by

$$\begin{aligned} E_B(t) &= \omega_0 \rho_{11}(0)|\beta_l(t)|^2, \\ W_B(t) &= \omega_0 \rho_{11}(0)|\beta_l(t)|^2 - (1-m)\omega_0/2. \end{aligned} \tag{A4}$$

APPENDIX B: DETAILS ON THE FORMATION OF SYSTEM-ENVIRONMENT BOUND STATES

The energy spectrum for the entire system, as previously discussed, can be derived by addressing the eigenvalue equation $H|\phi(t)\rangle = E|\phi(t)\rangle$, where $H = \sum_{l=1}^N \omega_0 \sigma_l^+ \sigma_l^- + \sum_k \omega_k b_k^+ b_k + H_{\text{int}}$ [H_{int} is given in Eq. (4)] is the Hamiltonian of the total system in the interaction picture and $|\phi(t)\rangle$ has been given below Eq. (4). According to the eigenvalue equation, a series of equations can be obtained, i.e.,

$$\begin{aligned} &\omega_k C_k(t) + \sum_{l=1}^N g_k^* e^{-i(\omega_0-\omega_k)t}\beta_l(t) = E C_k(t), \\ &\omega_0 \beta_1(t) + \sum_k g_k e^{i(\omega_0-\omega_k)t} C_k(t) = E\beta_1(t), \\ &\qquad\qquad \vdots \\ &\omega_0 \beta_N(t) + \sum_k g_k e^{i(\omega_0-\omega_k)t} C_k(t) = E\beta_N(t). \end{aligned} \tag{B1}$$

Acquiring $C_k(t)$ via the first equation and then substituting it into the subsequent equations results in the following integral equations set:

$$\begin{aligned} (\omega_0 - E)\beta_1(t) &= \int_0^\infty \frac{J(\omega)}{\omega - E} d\omega \sum_{l=1}^N \beta_l(t), \\ (\omega_0 - E)\beta_2(t) &= \int_0^\infty \frac{J(\omega)}{\omega - E} d\omega \sum_{l=1}^N \beta_l(t), \\ &\vdots \\ (\omega_0 - E)\beta_N(t) &= \int_0^\infty \frac{J(\omega)}{\omega - E} d\omega \sum_{l=1}^N \beta_l(t). \end{aligned} \tag{B2}$$

The derivation of the aforementioned equations utilized the correlation functions of the reservoir, with a detailed introduction provided below Eq. (6). Then by eliminating these amplitudes, these equations can be combined into a compact form:

$$\omega_0 - N\int_0^\infty \frac{J(\omega)}{\omega - E} d\omega = E. \tag{B3}$$

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