# Deceleration of fast ion rarefied beam due to Cherenkov interaction with ion-acoustic waves

A. A. Shelkovoy 1 and S. A. Uryupin 1,2,\* <sup>1</sup>Moscow Engineering Physics Institute, Moscow 115409, Russia <sup>2</sup>P. N. Lebedev Physical Institute of the Russian Academy of Sciences, Moscow 117924, Russia

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The deceleration of a low-density beam of ions in plasma with developed ion-acoustic turbulence arising in strong electric field is described. The time and length of beam deceleration along and across the anisotropy axis of the wave number distribution of ion-acoustic waves are found. It is shown to what extent an increase in the strength of the electric field that generates turbulence is accompanied by a decrease in the time and length of braking. As the beam propagates along the anisotropy axis, its velocity decreases to approximately the velocity of ion sound, and the direction of propagation does not change. When the beam is decelerated with an initial velocity across the anisotropy axis, a velocity component appears along the anisotropy axis during deceleration, which results in the beam deflection from the initial direction. In this case, the modulus of the beam velocity at the end of deceleration is close to the ion sound velocity.

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#### I. INTRODUCTION

Ion beams are often used in studies of plasma properties in installations created as part of the controlled thermonuclear fusion (see, for example, [1-4]). In these studies it is important to know the patterns of ion beams penetration into a plasma. Such patterns are well studied under the conditions when the deceleration of ions occurs due to Coulomb interaction of beam ions with plasma particles [5-7]. The existence of an appropriate theory allows us to adequately describe the beam deceleration if the plasma state is close to thermodynamical equilibrium. In the installations with magnetic plasma confinement, states are often realized in which the effective temperatures of electrons and ions differ very much. Ion-acoustic waves can exist in nonisothermal plasma [8,9]. At the same time, under the effect of an electric field heating plasma, electrons can move relative to ions with a velocity greater than the velocity of ion sound. In such strong electric fields ion-acoustic turbulence (IAT) is developing. In these conditions the effective frequencies of particle collisions are determined not by ordinary collisions, but by their Cherenkov interaction with ion-acoustic waves. Therefore, in the plasma with developed IAT, we should expect the implementation of qualitatively new patterns of ion beam deceleration. This article is devoted to the study of the deceleration of a rarefied ion beam in a plasma with a developed IAT, which arises under the influence of an electric field E.

Below the considered deceleration in the plasma of the ion beam propagating along and across the anisotropy axis of the IAT. It is shown how the beam velocity decreases over time or with distance increasing. In the case of beam propagation along the anisotropy axis of turbulence, ion deceleration is possible only along the vector  $-e\mathbf{E}$ , where e is the electron charge. Braking due to the Cherenkov interaction of the beam ions with ion-acoustic waves stops when the velocity of the beam ions decreases to the velocity of ion sound. Further deceleration, which occurs due to ordinary collisions of particles, is not considered. When the beam propagates across the anisotropy axis of IAT, simultaneously with a decrease in the transverse component of the velocity  $u_{\perp}$ , the longitudinal component of the velocity  $u_z$  changes along the vector  $-e\mathbf{E}$ . The features of the change in the  $u_z$  component depend on the magnitude of the field generating turbulence. In the weak field, the  $u_z$  component first increases, reaches a maximum, and then decreases. In this case, the beam deviates from the original direction by an angle greater than  $\pi/4$ . In the strong field, the  $u_z$  component first takes negative values, then changes its sign, reaches a maximum, and then decreases. As a result, the beam deflects by an angle less than  $\pi/4$  from the initial direction. Explicit dependencies of the deceleration time and length on the plasma and beam parameters are established. It is shown to what extent an increase in the strength of the field generating turbulence is accompanied by a decrease in the deceleration time and length. Numerical calculations are performed in the case of deuterium ion beam penetration into hydrogen plasma. However, the results obtained can also be used to describe the deceleration of beams of another species of ions. To do this, it is enough to replace the ratio of charge q to mass m for deuterium with the ratio  $q_b/m_b$  for ions of b species. At the same time, the deceleration time and length will change in  $(qm_b/e_bm)^2$  times, and the results of numerical calculations given in the text in the form of graphs and tables will remain unchanged.

### II. GENERAL EQUATIONS

Consider a nonisothermal plasma, in which under the effect of a strong electric field  $\mathbf{E} = (0, 0, \mathbf{E})$  a quasistationary

<sup>\*</sup>uryupinsa@lebedev.ru

distribution of ion-acoustic waves  $N(\mathbf{k})$  is created, where  $N(\mathbf{k})$  is wave number density in the wave number space  $\mathbf{k}$ . Let us study the effect of an ion beam with velocities greater than the velocities of ion-acoustic waves on such a plasma. The ion beam density is considered small compared to the plasma density. That permits neglect ion beam effect on the  $N(\mathbf{k})$ . To describe propagation of an ion beam in the plasma we use a kinetic equation for the ion distribution function of the beam  $f = f(\mathbf{v}, \mathbf{r}, t)$ :

$$\frac{\partial f}{\partial t} + \left(\mathbf{v}\frac{\partial f}{\partial \mathbf{r}}\right) + \frac{q}{m}\mathbf{E}\frac{\partial f}{\partial \mathbf{v}} = \frac{\partial}{\partial v_{\alpha}}D_{\alpha\beta}(\mathbf{v})\frac{\partial}{\partial v_{\beta}}f, \quad (1)$$

where q and m are the charge and mass of beam ions. Small influence of relatively rare collisions of the beam ions with plasma particles is neglected. In (1)  $D_{\alpha\beta}(\mathbf{v})$  is the quasilinear diffusion tensor describing interaction of beam ions with ion-acoustic waves:

$$D_{\alpha\beta}(\mathbf{v}) = \frac{q^2}{2\pi m^2} \int d\mathbf{k} \frac{k_{\alpha}k_{\beta}}{k^2} \frac{\omega_s^3}{\omega_{Li}^2} N(\mathbf{k}) \delta(\omega_s - \mathbf{k}\mathbf{v}). \quad (2)$$

Here  $\omega_s = kv_s/\sqrt{1 + k^2 r_{De}^2}$  is the frequency of ion-acoustic waves,  $v_s = \omega_{Li} r_{De}$  is the velocity of ion sound,  $r_{De}$  is the Debye electron radius, and  $\omega_{Li}$  is the plasma ions Langmuir frequency. We also assume that the velocity spread of the beam ions can be neglected, and their distribution function has the form

$$f(\mathbf{v}, \mathbf{r}, t) = n(\mathbf{r}, t)\delta(\mathbf{v} - \mathbf{u}(\mathbf{r}, t)), \tag{3}$$

where  $n = n(\mathbf{r}, t)$  is the density of the ion beam, and  $\mathbf{u} = \mathbf{u}(\mathbf{r}, t)$  is their velocity. If the thermal velocity of the beam ions is much less (more than three times) than the velocity of the directed motion, then taking into account their thermal motion does not have a noticeable effect on further consideration. If the thermal velocity is greater than or comparable to  $\mathbf{u}$ , then there is no point in talking about the beam. This last case is not considered further. Taking into account the explicit form of  $f(\mathbf{v}, \mathbf{r}, t)$  (3) and integrating over velocities of Eq. (1), we obtain the continuity equation

$$\frac{\partial n}{\partial t} + \operatorname{div}(n\mathbf{u}) = 0. \tag{4}$$

Further, multiplying Eq. (1) by  $v_{\alpha}$  and integrating over velocities, taking into account the continuity equation (4) and the explicit form of the beam ion distribution (3), one gets

$$\frac{\partial}{\partial t}u_{\alpha} + \left(\mathbf{u}\frac{\partial}{\partial \mathbf{r}}\right)u_{\alpha} = \frac{q}{m}E_{\alpha} + \frac{\partial}{\partial u_{\beta}}D_{\alpha\beta}(\mathbf{u}). \tag{5}$$

The density distribution of the number of ion-acoustic waves arising under the effect of the electric field  $\mathbf{E}$  is axially symmetric  $N(\mathbf{k}) = N(k, \cos\theta_k)$ , where  $\theta_k$  is the angle between wave vector  $\mathbf{k}$  and vector  $-e\mathbf{E}$ , and -e is the electron charge. Taking this into account and directing the z axis along the vector  $-e\mathbf{E}$ , we write the equation (5) for the components of the velocity vector  $\mathbf{u} = (u_\perp, u_z)$  along and across the symmetry axis:

$$\frac{\partial}{\partial t}u_{\gamma} + \left(\mathbf{u}\frac{\partial}{\partial \mathbf{r}}\right)u_{\gamma} = \frac{q}{m}E_{\gamma} + \frac{\partial}{\partial u_{\gamma}}D(u_{\perp}, u_{z}), \tag{6}$$

where  $\gamma = \perp$ , z, and diffusion coefficient has the form

$$D(u_{\perp}, u_{z}) = \frac{q^{2}}{\pi m^{2}} \int_{k_{\min}}^{k_{\max}} dkk \frac{\omega_{s}^{3}}{\omega_{Li}^{2}} \int_{0}^{1} dx N(k, x) U(u_{\perp}, u_{z}, kr_{De}, x),$$
(7)

$$U(u_{\perp}, u_z, kr_{De}, x) = \left[ u_{\perp}^2 (1 - x^2) - \left( \frac{v_s}{\sqrt{1 + k^2 r_{De}^2}} - x u_z \right)^2 \right]^{-1/2} \eta \left[ u_{\perp}^2 (1 - x^2) - \left( \frac{v_s}{\sqrt{1 + k^2 r_{De}^2}} - x u_z \right)^2 \right].$$
 (8)

Here  $\eta(y)$  is a Heaviside step function, and  $x = \cos \theta_k$ ,  $k_{\min}$  and  $k_{\max}$  are the boundaries of the turbulence region in the space of wave numbers. When solving a system of equations for  $u_{\perp}$  and  $u_z$  [see (6)], an explicit form of the function N(k,x) will be needed. The consequences of the IAT theory used below are mainly presented in the review [10]. According to [10], the basis of the IAT theory is the kinetic equations for electrons and ions, which takes into account the Cherenkov interaction of particles with ion-acoustic waves, and the equation for  $N(\mathbf{k})$ . In this case, the equation for  $N(\mathbf{k})$  takes into account the Cherenkov interaction of waves with particles and induced scattering of waves by ions. Approximate methods for constructing an analytical solution of a coupled system of equations for waves and particles are presented in [10]. Let us present the necessary results of the theory from [10]. Approximately, the function N(k,x) can be represented as a product of two functions  $N(k,x) = N(k)\Phi(x)$ . In this case, the wave number modulo distribution has the form [10] [see formula (2.117)]

$$N(k) = \sqrt{2\pi} \pi \frac{r_{De}^2}{r_{Di}^2} \frac{n_i m_i v_s}{k^4} \frac{\omega_{Li}}{\omega_{Le}} \left(\frac{\omega_s}{k v_s}\right)^3 \left[ \ln \frac{\omega_{Li}}{\omega_s} - \frac{1}{2} \left(\frac{\omega_s}{k v_s}\right)^2 - \frac{1}{4} \left(\frac{\omega_s}{k v_s}\right)^4 \right], \tag{9}$$

where  $m_i$  and  $n_i$  are the mass and the density of plasma ions,  $r_{Di}$  is the Debye plasma ions radius, and  $\omega_{Le}$  is the electron Langmuir frequency. In the region  $kr_{De} \ll 1$ , formula (9) gives the Kadomtsev-Petviashvili spectrum, and at  $kr_{De} \gg 1$  it reproduces the Galeev-Sagdeev scaling. In turn, the distribution over angles  $x = \cos \theta_k$  depends on the magnitude of the field strength E. In a relatively weak field (see [11] or formula (2.210) from [10]. The slight difference between (10) and (2.210) is due to the

derivation of (10) with greater accuracy)

$$\Phi(x) = \frac{4E}{3\pi E_N (1+\delta)x} \frac{d}{dx} \frac{x^4}{(1-x+\epsilon)^{1-\alpha}},$$

$$E < E_N (1+\delta)^2,$$
(10)

where  $\varepsilon = -K \ln K$ ,  $\alpha = -\ln 2/\ln K$ , and  $K = 4E/3\pi E_N(1+\delta)^2 \ln 2$  are small parameters,  $\delta$  is the ratio of the damping rate of ion-acoustic waves on hot resonant ions to the damping rate on electrons (see [12]), and the field strength  $E_N$  is determined by the plasma parameters and has the form

$$E_N = \frac{m_e v_s \omega_{Li}}{6\pi e} \frac{r_{De}^2}{r_{Di}^2},\tag{11}$$

where  $m_e$  is the electron mass. In a stronger field (see [10,13], or formula (2.128) from [10])

$$\Phi(x) = \frac{2}{\pi x^2} \sqrt{\frac{E}{E_N}} \frac{d}{dx} \int_0^x \frac{t^5 dt}{\sqrt{x^2 - t^2}} Q(t), \quad E > E_N (1 + \delta)^2,$$
(12)

$$Q(t) = \left[ 0.26 - 0.19t^2 + 0.31t^4 + t^2\sqrt{1 - t^2}(0.09 - 0.31t^2) \ln \frac{1 + \sqrt{1 - t^2}}{t} \right]^{-1}.$$
 (13)

The functions (9), (10), and (12) completely determine the diffusion coefficient (7), which, taking into account the explicit form of N(k) (9), can be represented as

$$D(u_{\perp}, u_z) = v_f v_s^3 \int_{z_{\min}}^{z_{\max}} \frac{dz}{(1+z^2)^3} \left[ \ln \frac{\sqrt{1+z^2}}{z} - \frac{0.5}{1+z^2} - \frac{0.25}{(1+z^2)^2} \right] \int_0^1 dx \Phi(x) U(u_{\perp}, u_z, z, x), \tag{14}$$

where  $z_{\min} = k_{\min} r_{De} \ll 1$  and  $z_{\max} = k_{\max} r_{De} \gg 1$ , and  $v_f$  is the frequency characterizing deceleration of the ion beam:

$$v_f = \frac{\omega_{Li}}{\sqrt{8\pi}} \frac{r_{De}^2}{r_{Di}^2} \frac{\omega_{Li}}{\omega_{Le}} \left(\frac{qm_i}{e_i m}\right)^2. \tag{15}$$

The expression for the diffusion coefficient (14) and the system of equations (6) for the velocity components  $u_{\perp}$  and  $u_z$  allow us to consider the features of ion beam deceleration depending on the level of turbulence and the orientation of the ion beam relative to turbulence symmetry axis.

The proposed consideration is valid in the case of lowdensity beam deceleration. We indicate a restriction on the beam density. When the beam velocity u is greater than  $v_s$ , the beam ions provide an additional contribution to the increment of ion-acoustic instability and could affect the IAT. For example, in the case of the ion beam with a small thermal velocity spread, when

$$f(\mathbf{v}) = \frac{n}{(2\pi)^{3/2} v_b^3} \exp\left[-\frac{(\mathbf{v} - \mathbf{u})^2}{2v_b^2}\right],\tag{16}$$

where  $v_b = \sqrt{\kappa T_b/m}$ ,  $\kappa$  is the Boltzmann constant and  $T_b$  is the effective temperature of the beam, the contribution to the instability increment is equal to

$$\gamma_b = \sqrt{\frac{\pi}{8}} \frac{\omega_{Lb}^2}{\omega_{Li}^2} \left(\frac{\omega_s}{k v_b}\right)^3 (\omega_s - \mathbf{k} \mathbf{u}) \exp\left[-\frac{(\omega_s - \mathbf{k} \mathbf{u})^2}{2k^2 v_b^2}\right], \quad (17)$$

where  $\omega_{Lb}$  is the beam ions Langmuir frequency. This formula takes place without taking into account the effect of IAT on the increment. IAT is decreasing the increment only. At the same

time,  $\gamma_b$  does not exceed

$$\gamma_m = \sqrt{\frac{\pi}{8}} \frac{\omega_{Lb}^2}{\omega_{Li}^2} \omega_s \left(\frac{\omega_s}{k v_b}\right)^2. \tag{18}$$

Obviously, if  $\gamma_m \ll \nu_f$ , then before the beam affects the IAT, its velocity will decrease to  $v_s$  and there will be no affect. Taking  $\omega_s \approx k v_s \approx \omega_{Li}$  for evaluation, we rewrite the inequality  $\gamma_m \ll \nu_f$  as

$$\frac{n}{n_i} \ll \frac{1}{\pi} \frac{\omega_{Li}}{\omega_{Le}} \frac{T_b}{T_i} \frac{m_i^2}{m^2}.$$
 (19)

This inequality imposes a restriction on the beam density at which the theory is definitely applicable. Since the thermal motion of ions was not taken into account when considering beam deceleration, this inequality makes sense provided that  $\kappa T_b \ll mu^2$ .

## III. ION BEAM DECELERATION

Consider the propagation of deuterium ion beam in hydrogen plasma. Assume that the densities of electrons  $n_e$  and ions  $n_i$  are equal to  $10^{13}$  cm<sup>-3</sup>, and their temperatures differ by six times:  $T_e = 300$  eV,  $T_i = 50$  eV. Under these conditions we have the velocity of sound  $v_s = 1.7 \cdot 10^7$  cm/s, parameter  $\delta = 6$  [12], characteristic collision frequency  $v_f = 2.9 \cdot 10^7$  s<sup>-1</sup>, and field strength  $E_N = 13$  V/cm. The frequency  $v_f$  determines effective deceleration time  $\tau_{\rm eff}$ , which depends on the electric field strength:  $\tau_{\rm eff} = v_f^{-1}(E_N/E) \max\{1 + \delta, \sqrt{E/E_N}\}$ . In this case, deceleration distance can be estimated as  $\sim v_s \tau_{\rm eff}$ .

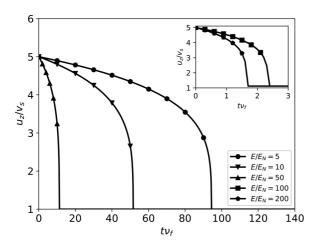


FIG. 1. Change in beam velocity  $u_z$  in time at  $u_z(0) = 5v_s$ .

For  $\gamma=z$ , one of the equations (6) contains electric field, which generates turbulence. This field also affects the ion beam velocity. However, this influence is relatively small. In particular, for  $u\sim v_s$ ,  $E/E_N\sim (1+\delta)^2$  and  $(qm_i/e_im)\sim 1$ , the corresponding term in Eq. (6) can be neglected because  $(1+\delta)\ll \omega_{Le}/\omega_{Li}$  [12]. In the numerical solution of Eq. (6), the term  $qE_\gamma/m$  is omitted. The relatively weak influence of the electric field on the motion of the beam ions is due to the fact that the effect of the effective friction force arising due to the Cherenkov interaction of ions with ion-acoustic waves is almost an order of magnitude stronger in the conditions under consideration.

Consider two configurations of beam propagation in the plasma. First, assume that the initial velocity of the ion beam is directed along the vector  $-e\mathbf{E}$  and is equal to  $5v_s$  or  $3v_s$ . Note that if the initial velocity is directed against the vector  $-e\mathbf{E}$ , then there is no deceleration due to the Cherenkov interaction with ion-acoustic waves, since there are no ion-acoustic waves having a component of the wave vector directed against the vector  $-e\mathbf{E}$ . Due to axial symmetry of the distribution of ion-acoustic waves in the case under consideration, the transverse velocity component does not arise. The numerical solution of the approximate equation (6) is shown in Fig. 1 and Fig. 2. Figure 1 shows dependencies of  $u_z/v_s$  on  $v_f t$  obtained for several values of  $E/E_N$ . The curves in Fig. 1 illustrate the decrease in time of the beam velocity at the selected point in space and are obtained in disregard of the term  $\mathbf{u}(\partial/\partial\mathbf{r})u_z$ in Eq. (6) for the velocity  $u_7$ . In another case, the curves in Fig. 2 show how the beam velocity changes in space provided that the deceleration process has been established and the time derivative  $\partial u_z/\partial t$  can be neglected. In addition, in Table I numerical values of  $t_f$  time and  $z_f$  distance of deceleration are given. The deceleration stops when  $u_z$  is close to  $v_s$ . The absence of significant deceleration at  $u_z$  lower  $v_s$  is associated with an abnormally rapid decrease in the density of the number of ion-acoustic waves with phase velocity lower  $v_s$ . According to Fig. 1, Fig. 2, and Table I, the stronger the electric field, i.e., the higher the turbulent level, the shorter the time and distance of the ion beam deceleration. It can be seen from Fig. 1, Fig. 2, and Table I that as the beam velocity approaches the velocity of sound, the braking process accelerates sharply. Therefore, the time and length of the beam

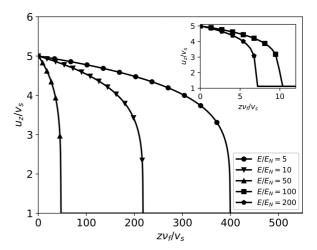


FIG. 2. Change in beam velocity  $u_z$  in space at  $u_z(0) = 5v_s$ .

deceleration at an initial velocity of  $3v_s$  is more than an order of magnitude less than at an initial velocity of  $5v_s$ . When the beam is decelerated along the vector  $-e\mathbf{E}$ , the ions move along a straight line. Moreover, according to the continuity equation  $nu_z = const$ , that is, the beam density increases as it decelerates  $n \propto 1/u_7$ . The second case corresponds to the situation when the initial velocity of the ion beam is directed across the axis of anisotropy of turbulence and is equal to  $5v_s$  or  $3v_s$ . In this case, simultaneously with the decrease of the transverse component of the velocity  $u_{\perp}$ , there is a change in the longitudinal component  $u_z$ , which was absent in the initial state. The change in  $u_z$  is due to the fact that, simultaneously with a decrease in the transverse component of the velocity, the beam deflects along the anisotropy axis of turbulence. Patterns of the deviation depend on the type of the function  $\Phi(x)$ —distribution of ion-acoustic waves over wave vector angles. In a relatively weak electric field, when  $E < E_N(1+\delta)^2$  and distribution (10) takes place, an increase in the velocity  $u_z$  in the direction of the vector  $-e\mathbf{E}$  is replaced by a subsequent decrease. In this case, the final value of  $u_{zf}$ is greater than  $u_{\perp f}$  (see Table II). In a stronger electric field when  $E > E_N(1 + \delta)^2$  and the distribution has the form (12), the behavior of  $u_z$  is different. First, the velocity  $u_z$  increases against the direction of the vector  $-e\mathbf{E}$ , i.e., it takes negative values (see Fig. 4 and Fig. 6). Then  $u_z$  reaches minimum and begins to increase, as in a weaker field. As a result, the final value of  $u_{zf}$  is less than  $u_{\perp f}$  (see Table II). This behavior is most clearly manifested when the beam is decelerated with

TABLE I. Times and lengths of beam deceleration along the electric field.

|         | $u_z(0) =$                | $=5v_s$    | $u_z(0) = 3v_s$           |            |  |  |
|---------|---------------------------|------------|---------------------------|------------|--|--|
| $E/E_N$ | $t_f (10^{-6} \text{ s})$ | $z_f$ (cm) | $t_f (10^{-7} \text{ s})$ | $z_f$ (cm) |  |  |
| 5       | 3.26                      | 234        | 1.91                      | 8.36       |  |  |
| 10      | 1.78                      | 128        | 1.09                      | 4.76       |  |  |
| 50      | 0.39                      | 27.7       | 0.28                      | 1.22       |  |  |
| 100     | 0.08                      | 6.07       | 0.05                      | 0.22       |  |  |
| 200     | 0.06                      | 4.21       | 0.04                      | 0.15       |  |  |

TABLE II. Times and lengths of beam deceleration across the electric field, as well as velocities upon termination braking across the electric field.

|         | $u_{\perp}(0) = 5v_s$     |            |            |                   | $u_{\perp}(0) = 3v_s$ |                           |            |            |                   |              |
|---------|---------------------------|------------|------------|-------------------|-----------------------|---------------------------|------------|------------|-------------------|--------------|
| $E/E_N$ | $t_f (10^{-7} \text{ s})$ | $z_f$ (cm) | $r_f$ (cm) | $u_{\perp f}/v_s$ | $u_{zf}/v_s$          | $t_f (10^{-7} \text{ s})$ | $z_f$ (cm) | $r_f$ (cm) | $u_{\perp f}/v_s$ | $u_{zf}/v_s$ |
| 5       | 6.58                      | 7.57       | 42.9       | 0.431             | 0.902                 | 1.59                      | 1.81       | 6.45       | 0.431             | 0.902        |
| 10      | 5.26                      | 5.45       | 34.2       | 0.490             | 0.872                 | 1.25                      | 1.26       | 5.06       | 0.491             | 0.871        |
| 50      | 0.977                     | 0.028      | 6.29       | 0.811             | 0.585                 | 0.202                     | 0.047      | 0.797      | 0.811             | 0.585        |
| 100     | 0.691                     | 0.020      | 4.45       | 0.811             | 0.585                 | 0.143                     | 0.035      | 0.565      | 0.811             | 0.585        |
| 200     | 0.489                     | 0.013      | 3.15       | 0.812             | 0.584                 | 0.101                     | 0.023      | 0.401      | 0.811             | 0.585        |

an initial velocity of  $5v_s$ . At the initial velocity of  $3v_s$ , the initial decrease of  $u_7$  to negative values is weaker. For this reason, the final length of deceleration along the anisotropy axis is greater than at an initial velocity of  $5v_s$  (see Table II). In all cases, the braking process stops when the beam velocity modulus is almost the same as the velocity of sound. At the same time, the magnitude of both velocity components is less than  $v_s$ . In the weak field, the component  $u_{zf}$  is slightly larger than  $u_{\perp f}$ . That is, at the end of braking, the beam propagation direction is inclined to the anisotropy axis at an angle less than  $\pi/4$ . On the contrary, in a strong field,  $u_{zf}$  is less than  $u_{\perp f}$ , and the beam inclination angle to the anisotropy axis is greater than  $\pi/4$  (see Table II). Since the deceleration of the beam across the symmetry axis of the IAT is considered under the assumption that there is no inhomogeneity along the symmetry axis, then from the continuity equation it follows  $nu_{\perp}$  = const. In this case, the beam density increases along its curved trajectory according to the law  $n \propto 1/u_{\perp}$ .

The described patterns of beam velocity changing are shown in Fig. 3–Fig. 6, where the numerical solution to Eq. (6) is shown. Figure 3 and Fig. 4 illustrate the change of the velocity components  $u_{\perp}$  and  $u_z$  in time, and Fig. 5 and Fig. 6 in space. Comparison of curves for  $u_{\perp}$  and  $u_z$  allows us to see how much the beam velocity will deviate from the initial direction during deceleration. Table II shows the characteristic values of  $t_f$  times and  $r_{\perp f}$ ,  $z_f$  deceleration distances depending on the generating turbulence field strength. In addition, Table II shows the values of  $u_{\perp f}$  and  $u_{zf}$  when braking stops.

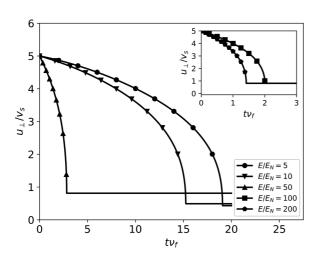


FIG. 3. Change in beam velocity  $u_{\perp}$  in time in the case of  $u_{\perp}(0) = 5v_s$ .

Comparing Fig. 1 and Fig. 2 with Fig. 3 and Fig. 5, as well as data from Tables I and II, it can be seen that when the beam propagates across the anisotropy axis, the beam deceleration lengths and times are less than when the beam propagates along the vector  $-e\mathbf{E}$ . More effective deceleration in this case arises due to the fact that in accordance with the Cherenkov interaction condition  $\omega_s = \mathbf{k}\mathbf{u}$ , ions with a velocity greater than  $v_s$  interact with waves having a small component of the wave vector  $\mathbf{k}$  in the direction of the vector  $\mathbf{u}$ . At the same time, according to formulas (10) and (12), ion-acoustic waves are mainly concentrated in the directions close to the direction of the vector  $-e\mathbf{E}$ . As with the longitudinal propagation of the beam, the deceleration time and length decrease with an increase in the generating turbulence field strength. This statement is clearly illustrated in Fig. 3-Fig. 6 and the data in Table II. Calculations of deceleration are performed for a deuterium ion beam. However, they are also suitable in the case of deceleration of a rarefied ion beam with a different charge  $e_b$  and mass  $m_b$ . To obtain relevant results, it is enough to change the effective frequency  $v_f$  (15) in  $(e_b m/q m_b)^2$  times. At the same time in  $(qm_b/e_bm)^2$  times the deceleration time and length will change.

## IV. CONCLUSION

The results of calculations of the time and length of deceleration of the monoenergetic ion beam in plasma with developed IAT generated by a strong electric field are presented above. IAT makes it possible to decelerate beam ions

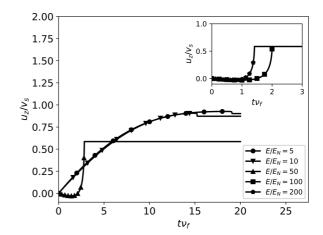


FIG. 4. Change in beam velocity  $u_z$  in time in the case of  $u_{\perp}(0) = 5v_s$ .

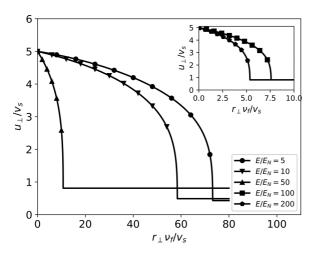


FIG. 5. Change in beam velocity  $u_{\perp}$  in space in the case of  $u_{\perp}(0) = 5v_s$ .

to a velocity close to the sound velocity. In the calculations, plasma parameters close to those found in experiments were selected [3,14–19]. The calculations are based on analytical expressions for the distribution of ion-acoustic waves over frequencies (9) and directions of the wave vector (10), (12). At the same time, the use of approximate formulas (10), (12) may lead to a noticeable error at the boundary of their applicability at  $E \sim E_N(1+\delta)^2$ . On the contrary, with E smaller or larger  $E_N(1+\delta)^2$ , the errors are relatively small. In addition, plasma heating installations have a magnetic field, which is usually directed along the electric field E. Such a field does not affect the deceleration of ion beams along the anisotropy axis of turbulence. At the same time, when the beam is decelerated across E, that is, across B, the effect of magnetic field on the beam ions can be significant if  $\Omega t_f > 1$ , where  $\Omega = qB/mc$ is the cyclotron frequency, and c is the speed of light. For

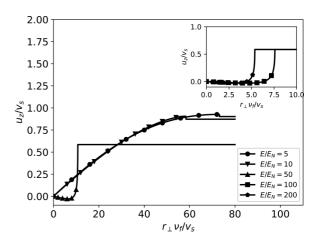


FIG. 6. Change in beam velocity  $u_z$  in space in the case of  $u_{\perp}(0) = 5v_s$ .

example, in the magnetic field with a strength of 10<sup>3</sup> G, the cyclotron frequency of deuterium ions is  $\Omega = 5 \cdot 10^6 \, \text{s}^{-1}$ . This means that the above-described patterns of beam deceleration across the magnetic field will mostly remain at  $E/E_N > 50$ , and for lower magnetic field at lower  $E/E_N$ . In the strong magnetic field, when  $\Omega t_f > 1$ , the theory of ion deceleration across the magnetic field should be supplemented by taking into account changes in their trajectory in the magnetic field. Another limitation of the theory is due to the assumption of a low density of beam ions. At the same time, situations are possible when the beam ion density is comparable to the plasma density (see, for example, [20] and [21]). Under such conditions, it is necessary to take into account the effect of beam ions on the turbulence itself. Despite these shortcomings of the theory, it can be useful for the interpretation and planning of experiments, since there are no analogs yet.

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