


Plastic instability of annular crystalline membrane in circular confinementHonghui Sun and Zhenwei Yao ^{*}*School of Physics and Astronomy, and Institute of Natural Sciences, Shanghai Jiao Tong University, Shanghai 200240, China*

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Understanding the mechanical instabilities of two-dimensional membranes has strong connection to the subjects of structure instabilities, morphology control, and materials failures. In this work, we investigate the plastic mechanism developed in the annular crystalline membrane system for adapting to the shrinking space, which is caused by the controllable gradual expansion of the inner boundary. In the process of plastic deformation, we find the continuous generation of dislocations at the inner boundary and their collective migration to the outer boundary; this neat dynamic scenario of dislocation current captures the complicated reorganization process of the particles. We also reveal the characteristic vortex structure arising from the interplay of topological defects and the displacement field. These results may find applications in the precise control of structural instabilities in packings of particulate matter and covalently bonded systems.

DOI: [10.1103/PhysRevE.109.044802](https://doi.org/10.1103/PhysRevE.109.044802)**I. INTRODUCTION**

Mechanical instability of two-dimensional membranes in confined geometry is a common phenomenon in nature and industry [1–3]. The adaptation of the membrane to the confined space leads to a wealth of morphologies across length scales ranging from mesoscopic (e.g., fluid membranes and polymerized membranes) to macroscopic (e.g., paper, metal foils, tree leaves, and flower petals) that are closely related to biological applications and materials design [3–7]. Depending on the constituents composing the membrane and their interaction, the mechanical instability could be classified into elastic and plastic categories. Much has been learned about the nature of elastic instability by mechanical and statistical analysis of the emergent structures arising in elastic membranes [8–15]. An important illustrative example is the crumpling of a thin sheet within a confining container of specific shape or by hand [12,16,17]. A series of characteristic structures are developed in the self-adaptation process of the sheet, including developable cones (or d cones) [18–21], ridges [9,22,23], and folds [23–25]. Crumpled structures may serve as a basis for creating robust mechanical metamaterials for their desirable mechanical properties [17,26].

Plastic instability represents a distinct mechanism for restructuring the membrane [15,27–31]. In contrast to elastic instability, the plasticity phenomenon occurring in the regime of large deformation has not been explored thoroughly. Understanding the plastic instability, which cannot be avoided in real systems [23,32,33], has strong connection to the subjects of structure instabilities and materials failures [33,34]. The confluence of experimental and theoretical investigations on the model of the curved 2D crystal fabricated by packings of particles shows the profound role of topological defects in the disruption of crystalline order and suggests that topological

defects offer a unique perspective for understanding plasticity [35–38].

The goal of this work is to explore the plastic instability in the annular crystalline membrane system. The model consists of Lennard-Jones (L-J) particles in triangular lattice confined in an annulus. The schematic plot of the annular membrane model is shown in Fig. 1. The featured energy minimum structure in the L-J potential curve enables the breaking of the bonds and thus allows one to capture the plastic deformation process by analyzing the underlying topological defect structure [39]. The deformation of the annular membrane is driven by the controllable gradual expansion of the inner boundary; the outer boundary is anchored. Note that the elastic deformations of the annular membrane by shrinking or twisting the inner boundary have been investigated [40–43]. Upon the expansion, the annular geometry of the membrane naturally leads to focused stress near the inner boundary, which could trigger mechanical instabilities of different kinds depending on the interaction of the particles composing the membrane. In connection to applications, the subject of the plasticity instability of the annular membrane system is related to a series of biomechanical and mechanical engineering problems, such as the healing of skin wounds [44] and the mechanical effect of circular holes introduced in clinical procedures [45].

The main results of this work are presented below. Based on the L-J lattice model, we first resort to the combination of analytical elasticity theory and numerical simulations to explore the elastic regime of small deformation and to confirm the reliability of the computational approach, which is used to explore the regime of plastic deformation. In the large deformation regime of interest, the reorganization of the particles is analyzed from the perspective of topological defect. Upon the gradual expansion of the inner boundary, we observe the collective migration of the continuously generated dislocations at the inner boundary to the outer boundary. The reorganization process of the particles is captured by the dynamic scenario of the dislocation current. We also identify

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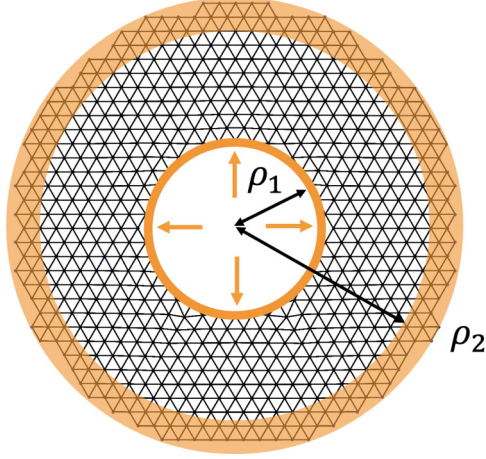


FIG. 1. Schematic plot of the 2D annular crystalline membrane. The model consists of Lennard-Jones particles in triangular lattice confined in the annulus. The 2D deformation of the annular membrane is driven by the gradual expansion of the inner boundary, whose radius is denoted as ρ_1 and ρ_1' in the initial stress-free state and the deformed state, respectively. To avoid the introduction of topological defects at the outer boundary, the anchored particles within the outer annulus (in orange) are in triangular lattice that is commensurate with the lattice of the annular membrane.

the irregular spots in the displacement field that allow one to pinpoint the locations of the ensuing topological defects, and we reveal the characteristic vortex structure arising from the interplay of topological defects and the displacement field. These results may have potential practical consequences in the precise engineering of structural instabilities in packings of particulate matter and covalently bonded systems.

II. MODEL AND METHOD

We resort to the L-J lattice models to explore the two-dimensional (2D) plastic deformation of annular crystalline membranes. The schematic plot of the model system is shown in Fig. 1. The L-J lattice model consists of particles in triangular lattice under the L-J potential:

$$V(r) = 4\epsilon_0 \left[\left(\frac{\sigma_0}{r} \right)^{12} - \left(\frac{\sigma_0}{r} \right)^6 \right], \quad (1)$$

where r is the distance between two particles, and the parameters σ_0 and ϵ_0 are related to the length scale and the energy scale of the L-J potential. The potential energy has the lowest value $-\epsilon_0$ at the balance distance of $\ell_0 = 2^{1/6}\sigma_0$. In this work, ℓ_0 is set to be the unit of length. The L-J potential is featured with the energy minimum structure in the potential curve, which enables the breaking of the bonds and thus allows one to capture the physical process of the plastic deformation [39]. In simulations, a cutoff length of $r_c = 3\ell_0$ is introduced in the L-J potential.

The crystalline membrane consisting of L-J particles is confined in an annulus of inner radius ρ_1 and outer radius ρ_2 . In the initial configuration, the lattice spacing is set to be the balance length ℓ_0 of the L-J potential. The deformation of the annular membrane is driven by gradually expanding the inner boundary. The outer boundary consists of a few layers of

regularly packed particles, as highlighted in Fig. 1. The lattice of these boundary particles is compatible with that of the membrane to avoid the introduction of topological defects that may lead to uncontrollable displacement of the particles near the outer boundary in the expansion process. In simulations, to expand the inner boundary, we introduce a circle as a geometric constraint and gradually increase the radius of this circle to push the particles outward. The displacement of the particles is confined on the plane. The amount of the expansion rate h is set to be as small as $0.2\ell_0$ to fulfill the quasistatic condition. The radius of the inner boundary is increased by h in each expansion. The new radius of the inner boundary is denoted as ρ_1' . After each expansion, the system is mechanically relaxed. By the standard steepest descend method, the local lowest-energy particle configuration is identified at the resolution of the step size $s = 10^{-4}\ell_0$ [46,47].

III. RESULTS AND DISCUSSION

This section consists of two subsections. In Sec. III A, we first analyze the small deformation of the annular membrane by the combination of analytical elasticity theory and numerical simulations. The agreement of the numerical and theoretical results shows the reliability of the computational approach, which is used to explore the regime of plastic deformation. In Sec. III B, we explore the phenomenon of plastic instability in the regime of large deformation. The reorganization of the particles upon the gradual expansion of the inner boundary is analyzed from the perspective of topological defect. We reveal the characteristic dynamic structures of the dislocation current and the concurrent vortices in the plastic deformation process.

A. Elasticity analysis in the regime of small deformation

We first resort to analytical continuum elasticity theory to analyze the 2D elastic deformation of the annular membrane system under the radial expansion of the inner circular boundary [40,48]. For an isotropic membrane in mechanical equilibrium, the displacement field \vec{u} is governed by the following balance equation in the absence of body force:

$$\nabla \nabla \cdot \vec{u} - \frac{1}{2}(1 - \sigma)\nabla \times (\nabla \times \vec{u}) = 0, \quad (2)$$

where σ is the Poisson's ratio. To discuss the case of radial expansion of the inner boundary, we work in polar coordinates (ρ, φ) . The inner circular boundary is expanded by an amount of h . The boundary conditions are $u_\rho(\rho = \rho_1) = h$, $u_\varphi(\rho = \rho_1) = 0$, $u_\rho(\rho = \rho_2) = 0$, and $u_\varphi(\rho = \rho_2) = 0$.

Consider the axisymmetric solution

$$\vec{u}(\rho) = u_\rho(\rho)\hat{e}_\rho, \quad (3)$$

where \hat{e}_ρ is a unit vector along the radial direction. By inserting Eq. (3) into Eq. (2), the balance equation becomes

$$\partial_\rho \left[\frac{1}{\rho} \partial_\rho (\rho u_\rho) \right] = 0. \quad (4)$$

Note that the effect of the Poisson ratio vanishes, because $\nabla \times \vec{u} = 0$ for the axisymmetric solution in Eq. (3). By substituting the boundary conditions into Eq. (3), we obtain the following

solution:

$$u_\rho = \frac{h\rho_1}{\rho_2^2 - \rho_1^2} \frac{\rho_2^2 - \rho^2}{\rho}, \quad (5)$$

where $\rho \in [\rho_1, \rho_2]$. One may check that $u_\rho(\rho_1) = h$ and $u_\rho(\rho_2) = 0$ in Eq. (5). From Eq. (5), we derive for the strain along the radial and azimuthal directions:

$$u_{\rho\rho} = -\frac{h\rho_1}{\rho_2^2 - \rho_1^2} \left(1 + \frac{\rho_2^2}{\rho^2}\right),$$

$$u_{\varphi\varphi} = \frac{h\rho_1}{\rho_2^2 - \rho_1^2} \left(\frac{\rho_2^2}{\rho^2} - 1\right). \quad (6)$$

Now, we analyze the strain field in the annular membrane upon the expansion of the inner boundary based on Eq. (6). Overall, the magnitudes of both $u_{\rho\rho}$ and $u_{\varphi\varphi}$ decrease in the form of $\sim \rho^{-2}$ when approaching the outer boundary. The radial expansion of the circular boundary originally at $\rho = \rho_1$ leads to radial compression (negative $u_{\rho\rho}$) and azimuthal stretching (positive $u_{\varphi\varphi}$). The signs of $u_{\rho\rho}$ and $u_{\varphi\varphi}$ are always opposite over the entire annular membrane ($\rho \in [\rho_1, \rho_2]$). This observation has implications in the stability of the membrane. In elastic membrane theory, it has been proved that an equilibrium configuration cannot be stable or neutrally stable unless the principal stresses are everywhere non-negative [49]. By this criterion, pulling the inner boundary inward may lead to wrinkles in the region of $\rho/\rho_2 < \rho^*$; $\rho^* = \sqrt{(1-\sigma)/(1+\sigma)}$ ($\rho_1 > \rho^*$) [40,41].

We proceed to perform numerical simulations to study the elastic deformation of the L-J lattice model, and we compare the numerical and analytical results. The goal is to check the reliability of the computational approach, which will be used to explore the regime of plastic deformation and also to analyze the region of validity of the linear elasticity theory for the L-J lattice model.

In order to compare with the theoretical results, we first count the total number of particles $N(\rho')$ within the circle of radius ρ' in the deformed annular lattice in mechanical equilibrium; the prime symbol in ρ' is to indicate that the variable ρ' is defined over the deformed membrane. ρ' ranges from $\rho_1 + h$ to ρ_2 . In the following, we derive $N(\rho')$ from Eq. (6). First of all, the area element dA in the undeformed lattice becomes dA' in the deformation. The relation of dA' and dA is

$$\frac{dA'}{dA} = (1 + u_{\rho\rho})(1 + u_{\varphi\varphi}). \quad (7)$$

Note that in the regime of linear elasticity, Eq. (7) is written as the trace of the strain tensor:

$$\frac{dA'}{dA} \approx 1 + u_{\rho\rho} + u_{\varphi\varphi}. \quad (8)$$

According to Eq. (7), the particle density defined on the undeformed lattice is changed from $\lambda(\rho)$ to $\lambda'(\rho)$, where $\rho \in [\rho_1, \rho_2]$. λ' and λ are related by

$$\frac{\lambda'}{\lambda} = \frac{dA}{dA'} = [(1 + u_{\rho\rho})(1 + u_{\varphi\varphi})]^{-1}. \quad (9)$$

Equation (9) clearly shows that stretching the lattice (positive $u_{\rho\rho}$ and $u_{\varphi\varphi}$) leads to the reduction of the particle density.

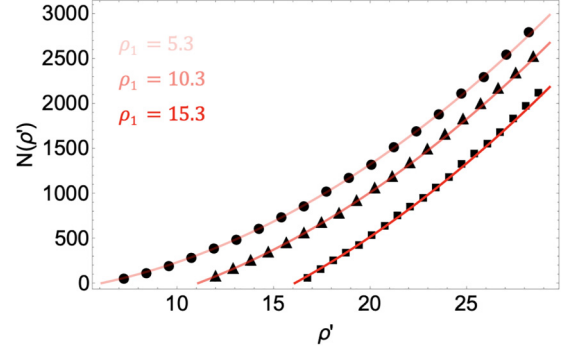


FIG. 2. Cumulative particle distributions over the deformed annular membrane at varying initial radius of the inner boundary. $N(\rho')$ is the total number of particles within the circle of radius ρ' in the deformed lattice; $\rho' \in [\rho_1 + h, \rho_2]$. The expansion rate $h = 0.8$. The simulation data (dots) could be well fitted by the theoretical results (red curves). The radius of the outer boundary $\rho_2 = 29.4$.

The number of particles within the circle of radius ρ in the unformed lattice is as follows:

$$N(\rho) = \int_{\rho_1}^{\rho} \lambda(\rho) d\rho. \quad (10)$$

Now, we count the total number of particles $N(\rho')$ within the circle of radius ρ' from the deformed annular lattice in mechanical equilibrium. To this end, we consider the movement of a particle from ρ to ρ' in the deformation; ρ and ρ' are connected by

$$\rho' = \rho + u_\rho. \quad (11)$$

The displacement vector u_ρ is defined on the undeformed lattice. The expression for u_ρ is given in Eq. (5). The plots of $N(\rho')$ at varying initial radius of the inner circular boundary (ρ_1) are presented in Fig. 2. The simulation data (dots of different shapes) could be well fitted by the theoretical results (solid red curves). Simulations show that the results of simulation and linear elasticity theory agree well up to $h/(\rho_2 - \rho_1) \approx 10\%$ for the case of $\rho_1 = 15.3$ and $\rho_2 = 29.4$.

B. Plastic instability in the regime of large deformation

We resort to the computational approach to explore the regime of large deformation, where the L-J lattice model exhibits plastic deformations. The reorganization of the particles in the strong expansion of the inner boundary is analyzed from the perspective of topological defects.

Figure 3 shows the plastic deformations of the annular lattice with the expansion of the inner circular boundary. The plastic deformation is characterized by the emergence of disclinations, which are indicated by colored dots in Fig. 3. Disclinations are identified by the standard Delaunay triangulation [36]. The five- and seven-fold disclinations are represented by the red and blue dots. A pair of five- and seven-fold disclinations constitute a dislocation. To track the deformation process, we also plot the displacement field associated with the expansion of the inner boundary from $\rho'_i = \rho_i$ to $\rho'_f = \rho_f$; the values of ρ_i and ρ_f are given in the square brackets below each figure.

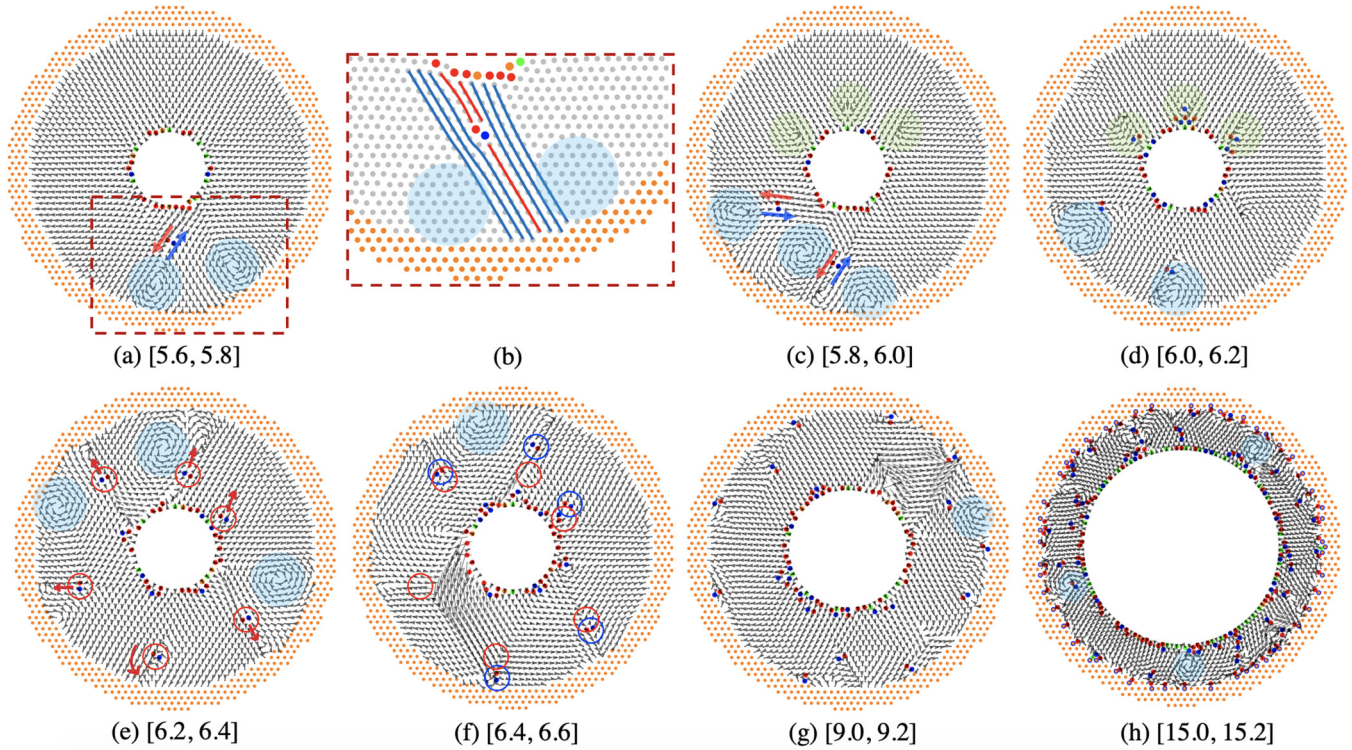


FIG. 3. The pattern of dislocation current is revealed in the 2D plastic deformation of the annular crystalline membrane upon continuous expansion of the inner boundary. The displacement vectors of the particles as the radius of the inner boundary is increased from ρ_i to ρ_f are represented by the arrows; the values of ρ_i and ρ_f are given in the square brackets below each figure. (a) The red and blue arrows indicate the shearing deformation, leading to the emergence of the dislocation and the vortices (as indicated by the blue disks). The dislocation is composed of a pair of five- and seven-fold disclinations as represented by the red and blue dots. The boxed region in panel (a) is enlarged in panel (b) for showing the reorganization of the crystal lattice by the dislocation. With the expansion of the inner boundary, we observe the collective migration of the dislocations from the inner to the outer boundary, as shown in panels (e) and (f). In panel (f), the previous locations of the dislocations are indicated by red circles for visual convenience. The system is stress-free in the initial state, where the value of the radius of the inner boundary $\rho_1 = 5$ and $\rho_2 = 24.4$. The annular L-J lattice consists of 1574 particles.

As a signal for the occurrence of the plastic deformation, the first dislocation appears when the radius of the inner boundary is increased from $\rho'_1 = 5$ to $\rho'_1 = 5.6$, as shown in Fig. 3(a). The zoomed-in particle configuration near the dislocation is presented Fig. 3(b). The particle arrays indicated by the blue and red lines show the effect of the dislocation on the reorganization of the particles upon the expansion. Specifically, in the presence of the dislocation, an extra particle array along the red line is introduced in the interior side of the annular lattice. The appearance of this inserted particle array terminated at the inner boundary reflects the adaptation of the particle configuration to the increased perimeter of the inner boundary upon expansion. The perspective of topological defect allows one to capture this neat self-adaption process.

The question of why the dislocation appears upon the expansion of the inner boundary is discussed in the preceding discussion. We further inquire how the dislocation arises. To address this question, we carefully analyze the displacement field associated with the incremental expansion of the inner circular boundary, as shown in Fig. 3(a). Near the dislocation, we find antiparallel arrays of the displacement vectors (indicated by the pair of red and blue arrows) that are perpendicular to the line connecting the five- and seven-fold disclinations composing the dislocation. Such a pattern of the displacement field is an indicator of the shearing deformation. As such,

the appearance of the dislocation is caused by the shearing deformation of particle arrays. Note that the crystal instability phenomenon is conventionally analyzed in terms of fluctuations of varying wavelength in the approach of statistical field theory [50,51]. Here, the computational approach enables the analysis of the microscopic shearing process, which reveals the detailed information on the crystal disruption process.

We further observe that the shearing deformation leads to the emergence of a pair of clockwise and counterclockwise vortices as highlighted by the blue disks in Fig. 3(a). In other words, the reorganization of the particles upon the expansion involves the collective movement of the particles, where the coherent vortex structure emerges in the displacement field. While the shear-driven emergence of vortices is common in fluids, the observed vortex structure in our solid mechanical system and its crucial role in reorganizing the particles upon stress are remarkable. Figure 3(b) shows that in the regions of the vortices indicated by the blue disks, the crystalline order is well preserved. Here, we point out that, due to the matched lattices of the particle array in the outer boundary and in the crystalline membrane, the entire annular region near the outer boundary is free of defects in the expansion process until ρ'_1 is increased from the initial value of 5 to about 6.6.

Further expanding the inner boundary leads to the emergence of the second dislocation, as shown in Fig. 3(c).

Simultaneously, an extra vortex (the left one among the three indicated by blue disks) is observed. We further notice the irregular distribution of the displacement vectors in the regions indicated by green disks, where the crystalline order is still well preserved. Upon further slight expansion as shown in Fig. 3(d), either dislocations or compound defects appear in these irregular spots in the displacement field. Therefore, the irregular spots in the displacement field serve as precursors for the ensuing appearance of topological defects in the lattice. Specifically, identifying these irregular spots allows us to pinpoint the location of the occurring topological defects. Here, it is shown that scrutiny of the displacement field yields useful insights into the plastic deformation.

Here, it is of interest to discuss the dynamics of the vortices in the displacement field with the expansion of the inner boundary. We observe the generation of vortices accompanying the shearing deformation, as shown in Figs. 3(a) and 3(b). Comparison of Figs. 3(c) and 3(d) also shows the annihilation of a vortex. Furthermore, from Figs. 3(a) to 3(d), we see that a gentle expansion by a fraction of one lattice spacing leads to a large displacement of the vortex center at the order of ten lattice spacings, indicating the fast dynamics of the vortices.

Under stronger expansion, we observe a proliferation of dislocations as shown in the lower panels in Fig. 3. In the presence of these dislocations, the entire displacement field is further distorted; more vortices and irregular spots arise. The microscopic scenario of the plastic deformation of the annular lattice exhibits complicated interplay of topological defects and displacement field.

Is there any pattern underlying the plastic deformation process? To address this question, we track the movement of the dislocations by comparing the particle configurations in the gradual expansion of the inner boundary. Simulations show that the circled dislocations in Fig. 3(e) move to the locations in the blue circles in Fig. 3(f), where the previous locations of the dislocations are indicated by red circles for visual convenience. We also observe the rotation of the dislocation, as shown in the lowest circled dislocation in Fig. 3(e). The expansion-driven radial migration of dislocations towards the outer boundary continues under stronger expansion. As such, the migrating dislocations form a radial current across the annular membrane. The inner boundary that is filled with defects serves as the source, and the outer boundary plays the role of the sink in the current of the dislocations. Upon their arrival at the outer boundary, the dislocations are anchored therein, leading to the accumulation of dislocations at the outer boundary in the expansion of the inner boundary. The plastic deformation process in Fig. 3 is recorded in a movie presented in the Supplemental Material [52]. Note that the dynamic scenario of the current of dislocations is invariant under the finer expansion rates of $h = 0.1\ell_0$ and $h = 0.2\ell_0$ for the case in Fig. 3.

The accumulation of topological defects at the outer boundary is quantitatively analyzed. In Fig. 4(a), we show the variation of the number of α -fold disclinations (N_α) near the outer boundary in the expansion of the inner boundary. The radius of the inner boundary $\rho_1 = 5$ in the initial stress-free state. The defects within the region of 1.5 lattice spacings away from the outer boundary are counted. The four-, five-, seven-, and eight-fold disclinations are indicated by orange, red, blue, and yellow dots, respectively.

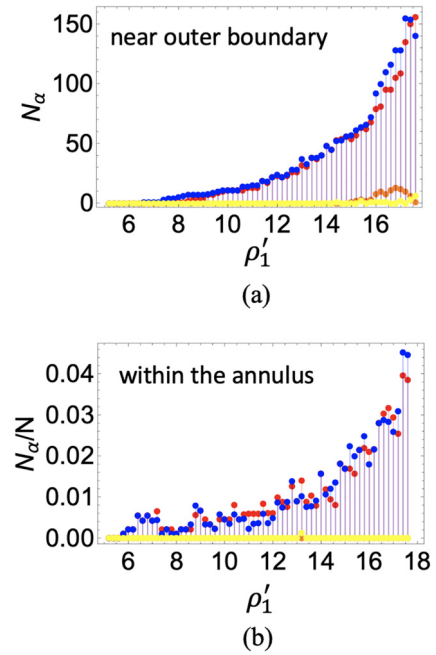


FIG. 4. Statistics of the emergent disclinations in the plastic deformation of the annular membrane driven by the expansion of the inner boundary. (a) Plot of the number of α -fold disclinations N_α accumulated near the outer boundary vs the inner radius ρ'_1 of the deformed membrane. The defects within the thin annular region of 1.5 lattice spacings away from the outer boundary are counted. (b) Plot of the relative number of α -fold disclinations within the annular membrane vs the inner radius ρ'_1 of the deformed membrane. The thin rims (1.5 lattice spacings) near both inner and outer boundaries are excluded. The four-, five-, seven-, and eight-fold disclinations are indicated by orange, red, blue, and yellow dots, respectively. In the initial stress-free state, the radius of the inner boundary $\rho_1 = 5$, $\rho_2 = 24.4$, and $N = 1574$.

red, blue, and yellow dots, respectively. Figure 4(a) shows that the five- and seven-fold disclinations dominate over the other types of disclinations. We see that the value of N_α becomes nonzero when ρ_1 exceeds about 6.6 for the case of $\rho_1 = 5$. In other words, the first dislocation arrives at the outer boundary when the amount of the expansion of the inner boundary exceeds some critical value $\delta\rho_c$; $\delta\rho_c = 1.6$ for the case of $\rho_1 = 5$ and $\rho_2 = 24.4$. Simulations show that the value of $\delta\rho_c$ is insensitive to the initial radii of the inner and outer boundaries. Specifically, the value of $\delta\rho_c$ is in the range of 1.4 ± 0.2 for $\rho_1 = \{5, 10, 15\}$ and $\rho_2 = \{20.4, 24.4, 28.4, 32.4\}$.

Here, we point out that the geometric incompatibility of the triangular lattice in the crystalline membrane and the circular inner boundary leads to the formation of disclinations within the annular region near the inner boundary. While the dislocations migrating to the outer boundary may be regarded as being pulled out of the crowd of these disclinations, the absence of isolated dislocations therein makes it difficult to describe their nucleation analytically. Especially, it is a challenge to analytically determine the critical value of ρ'_1 , above which the first batch of dislocations starts to move out of the crowded disclinations at the inner boundary. However, the common value of $\delta\rho_c$ in systems of varying geometry in the

preceding discussion gives an upper bound estimate for the critical value of ρ'_1 .

In Fig. 4(b), we also show the expansion-driven variation of the relative number of α -fold disclinations N_α within the annulus of the L-J lattice. Note that in the counting of the disclinations, the thin rims (1.5 lattice spacings) near both inner and outer boundaries are excluded. Similar to the case in Fig. 4(a), the five- and seven-fold disclinations (indicated by red and blue dots) dominate over the other types of disclinations. Figure 4(b) shows that the proportion of defects is only about 4% even when the value of ρ_1 is enlarged by over three times. The small amount of topological defects in the interior of the annular membrane suggests that the current of dislocations provides the mechanism for protecting the crystalline order. This result may have implication to the restoration of the crystalline order by creating a flow of disclinations in 2D packings of particles.

In the preceding discussions, we reveal the scenario of the current of dislocations underlying the expansion-driven plastic deformation of the annular lattice. It is of interest to inquire about the generality of this physical scenario. We perform further simulations for the elliptic system, where the lattice is confined between an inner circular boundary and an outer elliptic boundary. The current of dislocations is also observed in the elliptic system. The disruption of the crystalline order in the elliptic system upon the expansion of the inner boundary is recorded in a movie presented in the Supplemental Material [52].

Migration of dislocations has also been reported in the transport of interacting vortices in the superconducting film of the Corbino disk geometry [53,54]. The motion of the vortices is driven by an electric current that is injected at the center of the disk and flows radially towards the boundary. With the increase of the current, the vortex array that is initially in triangular lattice experiences a tearing transition from rigid rotation to plastic flow as characterized by the nucleation of dislocations at the center of the disk. Under a stronger current, the dislocations are organized into radial grain boundaries, coherently gliding in the tangential direction and forming a laminar flow over the disk [54].

Here, we compare these two kinds of systems: the vortex array in Corbino disk geometry and the crystalline membrane in annular geometry [53,54]. The vortex array and the electric current in the former system are analogous to the L-J lattice and the mechanical expansion in the latter system, respectively. Both studies focus on the plastic behavior of regularly packed elementary constituents in response to an external driving force of increasing strength. Despite the distinct physical settings, both systems share a common topological scenario in terms of the migration of dislocations in the plasticity phenomena. The topological perspective based

on the dynamics of dislocations offers a unified frame to understand nonequilibrium collective properties of a host of ordered self-assemble structures. Regarding the difference of the two kinds of systems, the Corbino disk system exhibits a second transition to the formation of the coherent laminar flow along the tangential direction under an even stronger current [54]; such a transition is absent in the mechanical membrane system due to the lack of a tangential force on the particles. Furthermore, in the Corbino disk system, the Lorentz force caused by the imposed external current is applied to each vortex, and the direction of the force is azimuthal. In contrast, the external force is applied to a single layer of particles at the inner boundary in the mechanical membrane system, and the force is along the radial direction.

We finally briefly discuss the possible extensions of the current work. First, one may extend the planar geometry, where the 2D crystalline membrane lives, to curved surfaces. The geometric frustration of the crystalline order on various curved surfaces has been well studied in the regime of mechanical equilibrium, and a series of defect motifs has been revealed [37,38]. It is of interest to utilize the stress-focusing effect around the pre-existent topological defects in curved crystals to regulate the patterns of plastic instability upon external mechanical stimuli [27,31,36]. Second, the quasistatic process, on which the current work focuses, may be extended to the dynamical regime by applying temporally varying external stress [30]. The dynamical annular membrane system provides a suitable model to explore the interplay of topological defects and elastic waves, which has potential connections to the control of the dislocation current as revealed in this work and that of the plastic instability process.

IV. CONCLUSION

In summary, we study the plastic instability of the crystalline membrane system in annular geometry. The expansion of the inner boundary leads to the reorganization of the particles, which is analyzed from the perspective of topological defect. We reveal the dynamic pattern of the plastic deformation in the form of the dislocation current. The continuously generated dislocations at the inner boundary collectively migrate to the outer boundary. We also identify the characteristic vortex structure arising from the interplay of topological defects and the displacement field in the process of plastic deformation. These results may find applications in the precise control of structural instabilities in packings of particles (colloids, proteins, etc.) and covalently bonded systems.

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