PHYSICAL REVIEW E **109**, 044704 (2024)

Features of director reorientation in a thin nematic film under the influence of crossed electric and magnetic fields

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(Received 23 December 2023; accepted 11 April 2024; published 26 April 2024)

The theory based on numerical study of the system of hydrodynamic equations, which includes the director motion, shows that under the influence of crossed electric **E** and magnetic **B** fields, the director reorients in such a way that the transient quasiperiodic patterns may arise in microsized nematic volumes if the corresponding distortion mode has the fastest response and thus suppresses all other modes, including uniform ones. It has been shown that there is a threshold value of the amplitude of the thermal fluctuations of the director over the microsized nematic film which provides the nonuniform rotation mode rather than the uniform one, whereas the lower values of the amplitude dominate the uniform mode.

DOI: 10.1103/PhysRevE.109.044704

I. INTRODUCTION

The main parameter characterizing the quality of a liquid-crystal (LC) display is the time $\tau_{\rm ON} \sim \gamma_1/E^2$ necessary for the reorientation of the director field $\hat{\mathbf{n}}$ under the action of an external electric field $\mathbf{E}$ [1]. If this field is directed across a LC film, then its magnitude should exceed the critical value $\mathrm{E}_{\rm th}$ given by [1]

$$E_{\rm th} = \frac{\pi}{d}\sqrt{\frac{K_1}{\epsilon_0\epsilon_a}}, \tag{1}$$

where γ_1 is the rotational viscosity of the LC phase, d is the film thickness, K_1 is the splay elastic constant, ϵ_0 is the absolute dielectric permittivity of free space, and ϵ_a is the dielectric anisotropy of the LC sample. This form for the critical field is based upon assumption that the director remains strongly anchored (in our case, homogeneously) at the two horizontal surfaces and that the physical properties of the LC are uniform over the entire sample for $\mathrm{E} < \mathrm{E}_{\rm th}$. When the electric field is switched on with a magnitude E greater than $\mathrm{E}_{\rm th}$, the director $\hat{\mathbf{n}}$, in the "splay" geometry, reorients as a simple monodomain [2–5].

In the case $E \gg E_{\rm th}$, when the strong electric field is abruptly applied orthogonally to an initially uniformly and homogeneously aligned nematic (HAN) film, the director reorients in such a way that the transient periodic structures may arise if the corresponding distortion mode has the fastest response and thus suppresses all other modes, including uniform ones [6–10]. A periodic distortion of the LC phase gives rise to a shear viscosity, which reduces the total effective rotational viscosity $\gamma_{\rm eff}$, connected with the reorientation of the director field. Rotating domains arising in this case favor the reduction in the effective viscosity $\gamma_{\rm eff}$, which characterizes the energy dissipation rate and thus creates modes of reorientation of the director field more favorable than a uniform rotation. As a result, this leads to a reduction in the direction field reorientation time $\tau_{\rm ON}$ [4]. It is important, both from an academic and a technological point of view, to investigate the dynamic director reorientation in a thin nematic liquid crystal film confined between two transparent electrodes and subjected to competing constraints.

We report here the results of the quantitative test of the validity of the nonlinear theory for description of the director reorientation in the planar quasi-two-dimensional geometry under the influence of the electric and magnetic fields directed at the angle α to each other. (see Fig. 1). When the electric field is applied to an initially uniformly HAN film, the director reorients in order to minimize the total free energy. If the electric field is much larger than a critical field value, the system is suddenly placed far from the equilibrium. It responds by creating a distortion which maximizes the rate at which the LC lowers its total free energy, and, as a result, the viscous contribution to the total free energy. It will be shown that the periodic response and the form of deformation depend on the value of the angle α between both external fields.

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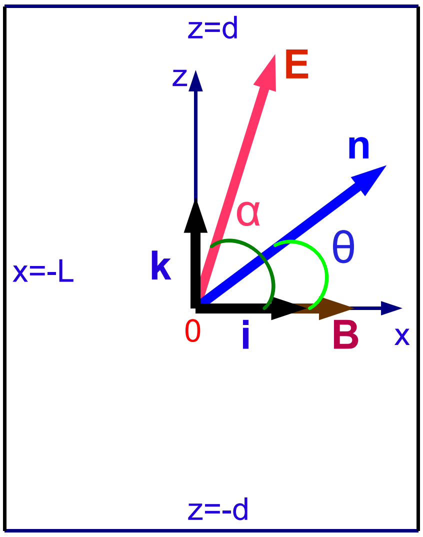

FIG. 1. The geometry used for calculations in the HAN film. The x axis is parallel, while the z axis is orthogonal to the electrodes. The magnetic field, **B**, electric field, **E**, and director, $\hat{\mathbf{n}}$, are aligned in the xz plane. The director makes the angle θ with the x axis, and the electric field makes the angle α with the magnetic field.

Theoretical description of the reorientation of the director field $\hat{\mathbf{n}}$ in the HAN film, described by the angle $\theta(t)$, under the action of crossed electric **E** and magnetic **B** fields will be given within the framework of the classical Ericksen-Leslie theory [11,12].

The outline of this paper is as follows. In the next section we discuss the system of hydrodynamic equations describing director motion in the HAN film, under the influence of crossed electric and magnetic fields. The numerical results for a number of relaxation regimes describing the orientational relaxation of the director field are given in Sec. III. The discussion of these results and our conclusions are summarized in Sec. IV.

II. TORQUE BALANCE EQUATION

The coordinate system defined by the task assumes that the director $\hat{\mathbf{n}}$ is in the xz plane. The magnetic field is oriented parallel to the electrodes and to the x axis $\mathbf{B} = B_0\hat{\mathbf{i}}$, $\hat{\mathbf{i}}$ is the unit vector along the horizontal axis x, while the unit vector $\hat{\mathbf{k}}$ is directed perpendicular to the bounding electrodes (the z axis) (see Fig. 1). The electric field **E** is switched at the moment $t = 0$ and is directed at the angle α with the magnetic field, and is equal to $\mathbf{E} = \frac{U}{2d}(\cos\alpha\hat{\mathbf{i}} + \sin\alpha\hat{\mathbf{k}})$. Here U is the voltage, and $2d$ is the thickness of the HAN film. The third unit vector of the coordinate system is $\hat{\mathbf{j}} = \hat{\mathbf{i}} \times \hat{\mathbf{k}}$ (axis y). Initial director orientation is parallel to the bounding electrodes $\hat{\mathbf{n}}(t = 0, x, z) \parallel \hat{\mathbf{i}}$, which is defined by the magnetic field. Switching on the electric field induces the deformation of the director field and the new director orientation is determined by the angle θ. Assuming that the electric field **E** varies only in the xz plane, we can suppose that the components of the director, $\hat{\mathbf{n}} = n_x\hat{\mathbf{i}} + n_z\hat{\mathbf{k}} = \cos\theta(x, z, t)\hat{\mathbf{i}} + \sin\theta(x, z, t)\hat{\mathbf{k}}$ (see Fig. 1), as well as other physical quantities, also depend only on the x and z coordinates.

Our previous simulations [9,10], in the framework of the classical Ericksen-Leslie theory [11,12], suggested that to describe the dynamical reorientation of the director correctly, under the influence of a strong electric field, we do not need to include a proper treatment of backflow. This means that in the first approximation, the role of flow becomes negligible in comparison to the electric, magnetic, and elastic forces and only rotations of the director field should be accounted.

In our case the torque balance equation $\mathbf{T}_{\rm elast} + \mathbf{T}_{\rm vis} + \mathbf{T}_{\rm el} + \mathbf{T}_{\rm mg} = 0$ involves the elastic $\mathbf{T}_{\rm el} = \frac{\delta\psi_{el}}{\delta\hat{\mathbf{n}}} \times \hat{\mathbf{n}}$, viscous $\mathbf{T}_{\rm vis} = \frac{\delta\mathcal{R}^{\rm vis}}{\delta\hat{\mathbf{n}}_t} \times \hat{\mathbf{n}}_t$, electric $\mathbf{T}_{\rm elast} = \frac{\delta\psi_{\rm elast}}{\delta\hat{\mathbf{n}}} \times \hat{\mathbf{n}}$, and magnetic $\mathbf{T}_{\rm mg} = \frac{\delta\psi_{\rm mg}}{\delta\hat{\mathbf{n}}} \times \hat{\mathbf{n}}$ torques, where the elastic energy density is equal to $W_{\rm F} = \frac{1}{2}[K_1(\nabla\cdot\hat{\mathbf{n}})^2 + K_3(\hat{\mathbf{n}} \times \nabla \times \hat{\mathbf{n}})^2]$, where K_1 and K_3 are the splay and bend elastic constants, while the viscous dissipation function is equal to $\mathcal{R}^{\rm vis} = \gamma_1\mathbf{N}^2$. Here only rotations of the director field has been accounted, and the vector $\mathbf{N}$ is equal to $\frac{\partial\hat{\mathbf{n}}}{\partial t} = \hat{\mathbf{n}}_{,t}$, where γ_1 is the rotational viscosity coefficient. The electric energy density is equal to $\psi_{el} = -\frac{1}{2}\epsilon_0\epsilon_a(\hat{\mathbf{n}}\cdot\mathbf{E})^2$, while the magnetic energy density is equal to $\psi_{\rm mg} = -\frac{1}{2}\frac{\chi_a}{\mu_0}(\hat{\mathbf{n}}\cdot\mathbf{B})^2$, where μ_0 is the magnetic constant and μ_a is the magnetic anisotropy of the nematic film. In our case these energy densities are $W_{\rm F} = \frac{1}{2}[K_1(n_{\rm x,x} + n_{\rm z,z})^2 + K_3(n_{\rm z,x} - n_{\rm x,z})^2]$, $\psi_{el} = -\frac{1}{2}\epsilon_0\epsilon_a(n_{\rm x}E_{\rm x} + n_zE_z)^2$, and $\psi_{\rm mg} = -\frac{1}{2}\frac{\chi_a}{\mu_0}(n_{\rm x}B_0)^2$, while the viscous dissipation function can be rewritten in the form $\mathcal{R}^{\rm vis} = \gamma_1(n_{{\rm x},t}^2 + n_{{\rm z},t}^2)$. It allows us to rewrite the torque contributions as $\mathbf{T}_{\rm elast} = \{n_z[K_1n_{\rm x,xx} + K_3n_{\rm x,zz} + (K_1 - K_3)n_{\rm z,xz}] - n_x[K_1n_{\rm z,zz} + K_3n_{\rm z,xx} + (K_1 - K_3)n_{\rm x,xz}]\}\hat{\mathbf{j}}$, $\mathbf{T}_{\rm vis} = \gamma_1[n_{\rm x}n_{{\rm z},t} - n_zn_{{\rm x},t}]\hat{\mathbf{j}}$, $\mathbf{T}_{\rm el} = \epsilon_0\epsilon_a(n_{\rm x}E_{\rm x} + n_zE_z)(n_zE_{\rm x} - n_{\rm x}E_z)\hat{\mathbf{j}}$, and $\mathbf{T}_{\rm mg} = \frac{\chi_a}{\mu_0}n_{\rm x}n_zB_0^2\hat{\mathbf{j}}$, respectively, where $n_{\rm x,xx} = \frac{\partial^2 n_{\rm x}}{\partial {\rm x}^2}$. The appropriate angle's forms for the torques are given below as $-T^{\rm y}_{\rm elast} = (K_1\sin^2\theta + K_3\cos^2\theta)\theta_{,\rm xx} + (K_1\cos^2\theta + K_3\sin^2\theta)\theta_{,\rm zz} + (K_3 - K_1)\sin 2\theta\theta_{,\rm xz} + \frac{1}{2}(K_1 - K_3)\sin 2\theta(\theta_{,\rm x}^2 - \theta_{,\rm z}^2) + (K_1 - K_3)\cos 2\theta\theta_{,\rm x}\theta_{,\rm z}$, $T^{\rm y}_{\rm vis} = \gamma_1\theta_t$, $T^{\rm y}_{\rm el} = -\epsilon_0\epsilon_a(\frac{U}{2d})^2\sin 2(\alpha - \theta)$, and $T^{\rm y}_{\rm mg} = \frac{1}{2}\frac{\chi_a}{\mu_0}B_0^2\sin 2\theta$. The dimensionless analog of the torque balance equation takes the form

$$
\begin{aligned}
\theta_{,\tau} = {} & \delta_1[(\sin^2\theta + K_{31}\cos^2\theta)\theta_{,\rm xx} + (\cos^2\theta + K_{31}\sin^2\theta)\theta_{,\rm zz}] \\
& + \delta_1\left[(K_{31} - 1)\sin 2\theta\theta_{,\rm xz} + \frac{1}{2}(1 - K_{31})\sin 2\theta\left(\theta_{,\rm x}^2 - \theta_{,\rm z}^2\right)\right] \\
& + \delta_1[(K_{31} - 1)\cos 2\theta\theta_{,\rm x}\theta_{,\rm z}] + \frac{\overline{E}(z)^2}{2}\sin 2(\alpha - \theta) - \frac{\delta_2}{2}\sin 2\theta, \qquad (2)
\end{aligned}
$$

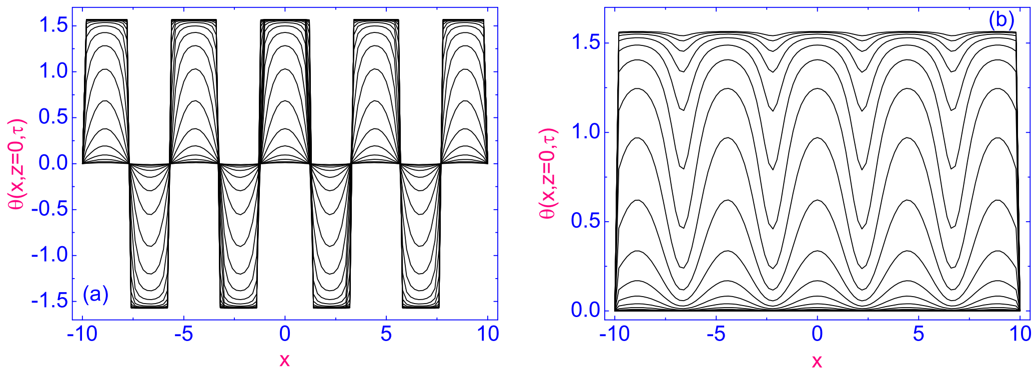

FIG. 2. Evolution of the angle $\theta(\tau, x, z=0)$ along the dimensionless length $-10 \leqslant x \leqslant 10$ of the HAN film to the stationary distribution $\theta^{st}(x, z=0)$ under the effect of both electric and magnetic fields directed at the angle $\alpha = 1.57$ ($\sim 89.96^\circ$) and $q_x \approx 490.5$, for a number of values $\theta_0 = 0.01$ (a) and 0.001 (b), respectively.

where $\overline{E}(z) = 2dE(z)/U$. Here x = x/d and z = z/d are the dimensionless space variables, $\tau = \frac{\epsilon_0\epsilon_a}{\gamma_1}(\frac{U}{2d})^2 t$ is the dimensionless time, and $\delta_1 = \frac{K_1}{\epsilon_0\epsilon_a(\frac{U}{2d})^2}$, $\delta_2 = \frac{\chi_a B_0^2}{\mu_0\epsilon_0\epsilon_a(\frac{U}{2d})^2}$ and $K_{31} = \frac{K_3}{K_1}$ are three parameters of the system.

In our case the voltage applied across the LC film results in a variation of $E(z)$, which can be written as

$$\left[\frac{\overline{E}(z)}{f(\theta(z))}\right]_{,z} = 0, \qquad \int_{-1}^{1} \overline{E}(z)dz = 1, \tag{3}$$

where $f(\theta(z)) = \frac{\epsilon_a}{\epsilon_\perp + \epsilon_a \sin^2\theta(z)}$.

III. SOLUTION OF THE TORQUE BALANCE EQUATION

For the case of the deuterated 4-α, α-d$_2$-pentyl-4′-cyanobiphenyl (5CB-d$_2$) LC sample with a thickness of $2d \approx 194.7$ µm at a temperature of 300 K and a density of 10^3 kg/m^3, the measured data for elastic constants are $K_1 \approx 8.7$ pN and $K_3 \approx 10.2$ pN, whereas the experimental value of dielectric anisotropy is equal to $\epsilon_a \approx 11, 5$. This case is important because NMR spectroscopy methods make it possible to measure the reorientation of the director in a nematic film, where the LC sample is first oriented by a strong magnetic field **B**, and then exposed to a strong electric field applied orthogonally to an initially uniformly aligned nematic film [4,13]. In this case, the director tends to move from the state parallel to the magnetic field (in our case equal to $B_0 = 7.05$ T) to the state parallel to the electric field. Deuterium NMR provides an opportunity to describe the motion of the director field by measuring the quadrupole splitting of $\Delta\nu(\theta) = \Delta\nu_0 P_2(\cos\theta)$, where $\Delta\nu_0$ is splitting when the director is parallel to **B**, and $P_2(\cos\theta)$ is a Legendre polynomial of the second rank. This gives hope for experimental confirmation of the features of the dynamic reorientation of the director in the LC film under the influence of a large electric field directed perpendicular to the magnetic field investigated by numerical methods.

Thus, the set of δ parameters, which is involved in Eq. (2), takes the values $\delta_1 = 8.6 \times 10^{-6}$ and $\delta_2 = 0.424$. The value of the angle α is varied between 50° and 90°, while L is equal to $10d$. Equation (2) is solved numerically assuming a harmonic dependence along the z axis for the angle $\theta(x, z, \tau)$,

$$\theta(x, z, \tau) = \theta_0(\tau)\cos(q_x x)\cos(q_z z), \tag{4}$$

where $\theta_0(\tau)$ is the amplitude, and $q_x = \frac{\pi}{2}\frac{d}{L}(2k+1)$ ($k = 0, 1, 2, \ldots$) and $q_z = \frac{\pi}{2}(2l+1)$ ($l = 0, 1, 2, \ldots$) are the wavelengths of an individual Fourier modulation component, which will be calculated by minimizing the total energy

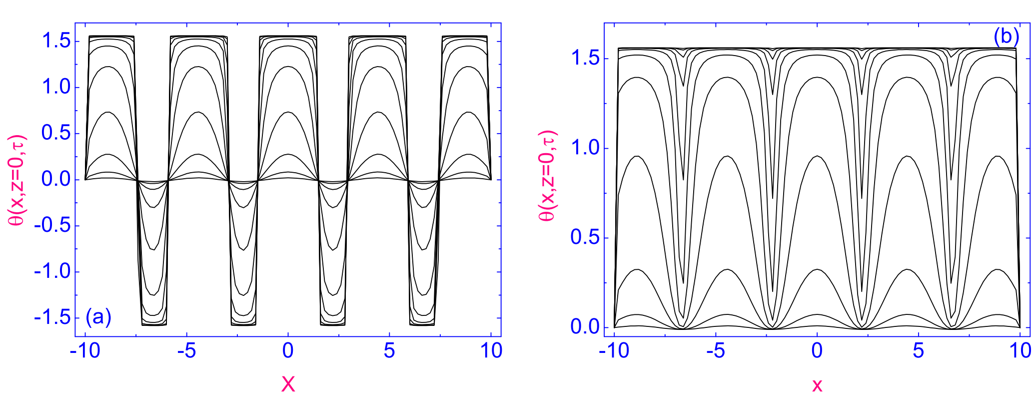

FIG. 3. The same as in Fig. 2, but the angle $\alpha = 1.565$ ($\sim 89.67^\circ$) and $q_x \approx 490.5$, for a number of values of $\theta_0 = 0.01$ (a) and 0.002 (b), respectively.

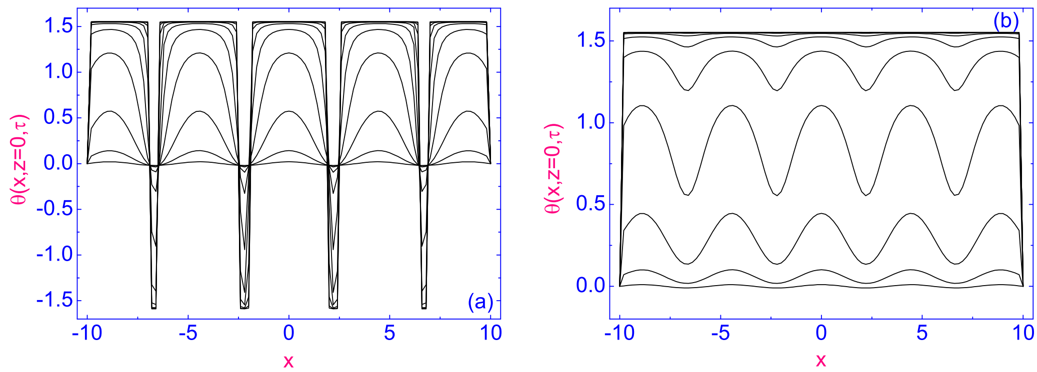

FIG. 4. The same as in Fig. 2, but the angle $\alpha = 1.56\,(\sim 89.38^\circ)$ and $q_x \approx 490.5$ for a number of values of $\theta_0 = 0.01$ (a) and 0.002 (b), respectively.

density $W = W_{\text{elast}} + W_{el} + W_{mg}$ for each value of α. Here

$$W_{\text{elast}} = \frac{\delta_1}{2} \iint dxdz\big[\big(\theta^{st}_{,z}\cos\theta^{st} - \theta^{st}_{,x}\sin\theta^{st}\big)^2 + K_{31}\big(\theta^{st}_{,x}\cos\theta^{st} + \theta^{st}_{,z}\sin\theta^{st}\big)^2\big] \quad (5)$$

and

$$W_{el} + W_{mg} = \frac{1}{2}\iint dxdz[\cos^2(\theta^{st} - \alpha) + \delta_2\cos^2\theta^{st}] \quad (6)$$

are the elastic, electric, and magnetic contributions to the total energy, while $\theta^{st} \equiv \theta^{st}(x, z)$ is the stationary value of the angle θ, respectively.

This form for the angle $\theta(x, z, \tau)$ is satisfied for the strong anchoring conditions

$$\theta(-10 < x < 10, z = \pm 1) = \theta(x = \pm 10, -1 < z < 1) = 0. \quad (7)$$

The Galerkin's method [14] is applied for the solution of the balance Eq. (2). The basic function of the method is $\varphi = \cos(\frac{\pi}{2}z)\cos(q_x x)$, which has the normalization property as $(\varphi, \varphi) = 1$, where (φ, φ) is the scalar product $((\varphi, \varphi) = \int_{-1}^{1}\varphi^2 dxdz = 1)$. The scalar product taken for both sides of the balance equation gives the ordinary differential equation for the amplitude $\theta_0(\tau)$ in the form

$$\frac{d\theta_0(\tau)}{d\tau} = F(\theta_0(\tau), q_x, \alpha), \quad (8)$$

where α and q_x are the external variables, and $F = (\theta_{,\tau}, \varphi)$. The number of initial conditions $\theta_0 = 0.001$, 0.01, 0.02,...,0.05 for chosen value of the angle α has been taken. The amplitude Eq. (8) has been solved by the fourth order Runge-Kutte method [14]. The convergence criterion $\delta = (\theta_0^{i+1} - \theta_0^i)/\theta_0^i$ for the above iterative procedures was chosen to be equal to 10^{-4}, where i is the iteration number. The numerical procedure was then carried out, during the time τ_R, until a prescribed accuracy was achieved. Later it will be shown that when using any initial amplitude values θ_0 less than 0.01 and for values $\alpha < 1.55\,(\sim 88.8^\circ)$, we cannot obtain a stationary periodic structure

$$\theta^{\text{st}}(\alpha, q_x) = \theta_0^{\text{st}}(\alpha, q_x)\cos\left(\frac{\pi}{2}z\right)\cos(q_x x) \quad (9)$$

in the nematic phase under the effect of the crossed abovementioned electric and magnetic fields. Using these functions one can calculate the maximal value of the total energy density corresponding to the optimal value of q_x.

An example of numerical integration of Eq. (8), showing the formation of the periodic evolution of $\theta(x, z = 0, \tau)$ along the dimensionless x axis to the stationary value $\theta^{\text{st}}(x, z = 0)$, under the influence of both electric and magnetic fields directed at the angle $\alpha = 1.57\,(\sim 89.96^\circ)$ and $q_x \approx 490.5$ for a number of values $\theta_0 = 0.01$ [see Fig. 2(a)] and 0.001 [see Fig. 2(b)], is shown in Fig. 2. Our calculations show that the periodic structure may appear in the HAN film under the above mentioned conditions, when the electric field is directed at the angle $\alpha = 1.57\,(\sim 89.96^\circ)$ and $q_x \approx 490.5$ for a number of values of θ_0, greater than 0.01 [see Fig. 2(a)]. When the value of θ_0 is less than 0.01, for instance 0.001, one can see that the periodic structure does not develop under the abovementioned conditions [see Fig. 2(b)]. The main feature of the numerical solutions is that the further decreasing of angle α leads to destruction of the periodic structure (see Figs. 3–6). In these cases the destroyed periodic structure (quasiperiodic structure) appears only in the case of the bigger values of θ_0. Physically, this means that the further decreasing of the angle α, from 1.57 ($\sim 89.96^\circ$) to 1.565 ($\sim 89.67^\circ$) (see Fig. 3), 1.56 ($\sim 89.38^\circ$) (see Fig. 4), or 1.55 ($\sim 88.8^\circ$)

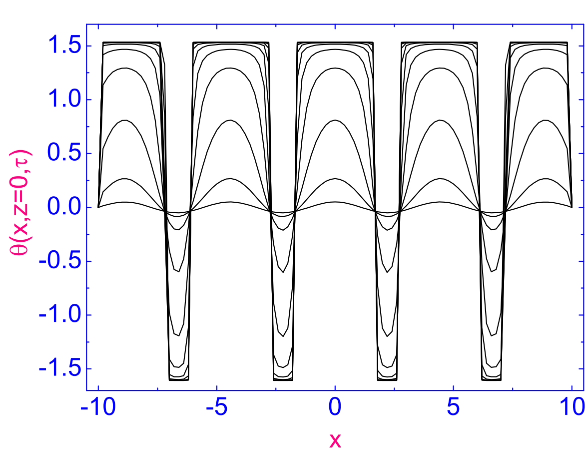

FIG. 5. The same as in Fig. 2, but the angle $\alpha = 1.55\,(\sim 88.8^\circ)$ and $q_x \approx 490.5$ for the value of θ_0 to be equal to 0.01.

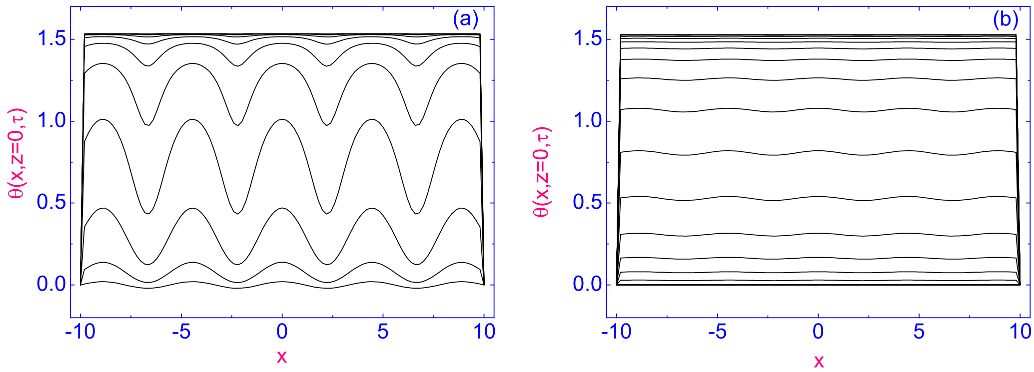

FIG. 6. The same as in Fig. 2, but the angle $\alpha = 1.55\,(\sim 88.8^\circ)$ and $q_x \approx 490.5$ for a number of values of $\theta_0 = 0.005$ (a) and 0.002 (b), respectively.

(see Figs. 5 and 6), leads to destruction of the periodic structure in the HAN film. So, according to our calculations, the periodic structure may appear, under the above mentioned conditions, only when the angle α is close to the right angle ($\sim\frac{\pi}{2}$). With the aim to investigate the effect of the angle α on the formation of the spatially periodic structure, at the fixed value of $\theta_0 = 0.01$, a number of values of α, from $\alpha = 1.40\,(\sim 80.0^\circ)$ [see Fig. 7(a)] to 1.222 ($\sim 70.0^\circ$) [see Fig. 7(b)], and from $\alpha = 1.047\,(\sim 60.0^\circ)$ [see Fig. 8(a)] to $0.82\,(\sim 50.0^\circ)$ [see Fig. 8(b)], has been considered. Our calculations showed that under the above conditions ($U = 200$ V, $2d = 194.7$ μm, and $B_0 = 7.05$ T) only the initial value of $\theta_0 = 0.01$, at angles $\alpha = 1.57\,(\sim 89.96^\circ)$, provides the formation of the periodic structure in the HAN film. Indeed, when the values of α are less than 1.55, the initial value of $\theta_0 = 0.01$ does not provide the formation of even the quasiperiodic structure in the HAN film. In all cases shown in Figs. 7 and 8, the initially symmetric distribution of the angle $\theta(x, z = 0, \tau_1)$ along the length $-10 \leqslant x \leqslant 10$ of the LC cell under the effect of horizontally directed forces, formed by an electric field component directed along the x axis and by the magnetic field, is transformed only into quasiperiodic structures characterized by large negative deviations of the director field. Calculated interval values $\Delta = [\theta^{st}_{max}; \theta^{st}_{min}]$ between the maximum θ^{st}_{max} and the minimum θ^{st}_{min} with the values of the stationary angle θ^{st} and the maximum values of the difference $\Delta_{max} = \theta^{st}_{max} - \theta^{st}_{min}$ for different values of α are collected in Table I. Physically, this means that decreasing the angle α, between the magnetic **B** and electric **E** fields, from α being equal to $1.57\,(\sim 89.96^\circ)$ [see Fig. 2(a)] to $1.40\,(\sim 80.0^\circ)$ [see Fig. 7(a)] and $1.222\,(\sim 70.0^\circ)$ [see Fig. 7(b)], and further from $1.047\,(\sim 60.0^\circ)$ [see Fig. 8(a)] to $0.82\,(\sim 50.0^\circ)$ [see Fig. 8(b)], leads to destruction of the periodic structure in the HAN film.

The stationary distribution of the angle $\theta(x, z = 0, \tau = \tau_R) = \theta^{st}(x, z = 0)$ along the length $-10 \leqslant x \leqslant 10$ of the HAN film for the number of values of α—$1.57\,(\sim 90.0^\circ)$ [see Fig. 9(a), curve 1], $1.40\,(\sim 80.0^\circ)$ [see Fig. 9(a), curve 2], $1.222\,(\sim 70.0^\circ)$ [see Fig. 9(a), curve 3], $1.047\,(\sim 60.0^\circ)$ [see Fig. 9(a), curve 4], and $0.82\,(\sim 50.0^\circ)$ [see Fig. 9(a), curve 5]–are shown in Fig. 9(a). Our calculations show that the periodic structure in the HAN film may appear, under the abovementioned conditions ($U = 200$ V, $2d = 194.7\,\mu m$ and $B_0 = 7.05$ T), only when the angle α is close to 1.57 ($\sim 89.96^\circ$). Even a small decrease in the magnitude of the angle α, from 1.57 ($\sim 89.96^\circ$) to 1.55 ($\sim 88.8^\circ$), leads to the destruction of periodicity, and we are dealing with a quasiperiodic structure [see Fig. 9(b), curves 1 and 2]. Moreover, our calculations showed that as the magnitude of the angle α increases, the relaxation time $\tau_R(\alpha)$ of the director field to its stationary distribution over the HAN film increases near the value of $\alpha = 1.55\,(\sim 88.8^\circ)$. Indeed, the value of the relaxation time $\tau_R(\alpha = i)\,(i = 0.82; 1.047; 1.222; 1.4) \sim 11\,(14\,\text{ms})$, while in the case of $\alpha = 1.55$, the value of

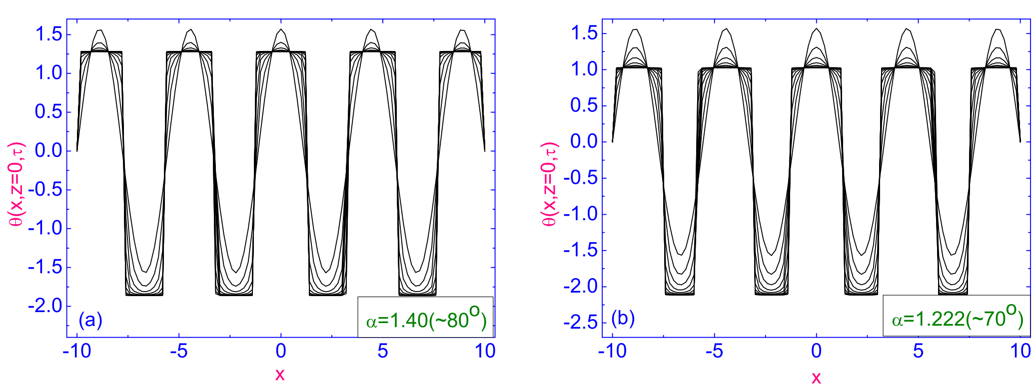

FIG. 7. The same as in Fig. 2, but the values of the angle α are equal to $1.40\,(\sim 80.0^\circ)$ (a) and $1.222\,(\sim 70.0^\circ)$(b), respectively. Here q_x is equal to 490.5, while the value of θ_0 is equal to be 0.01, respectively.

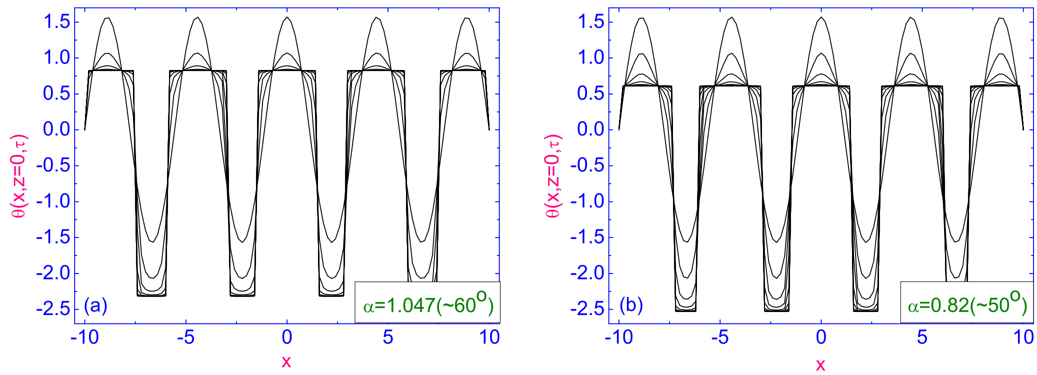

FIG. 8. The same as in Fig. 7, but the values of the angle α is equal to 1.047 (∼60.0°) (a) and 0.82 (∼50.0°) (b), respectively.

$\tau_R(\alpha = 1.55)$ is equal to 18 (∼23 ms). These data for relaxation times $\tau_R(\alpha)$ are also collected in Table I. This means that the contribution of forces acting along the x axis, both the components of the electric field, increasing with decreasing angle α, and the magnetic field, prevents the formation of a quasiperiodic structure in the volume of the HAN film at an angle value α less than 1.55 (∼88.8°). At the same time, the periodic structure in the HAN film may appear, under the abovementioned conditions ($U = 200$ V, $2d = 194.7$ μm and $B_0 = 7.05$ T), only when the angle α is close to 1.57 (∼89.96°).

IV. DISCUSSION AND CONCLUSIONS

When the strong electric field is abruptly applied orthogonally (or approximately orthogonally) to an initially uniformly and homogeneously aligned nematic film, the director can reorient in three ways: first, as a monodomain, when the mode of uniform director reorientation is dominated [5,13]; second, when the director can reorient in such a way that the transient periodic structures may arise if the corresponding distortion mode has the fastest response and thus suppresses all other modes, including uniform ones [6–10,15]; and third, when, under the effect of externally applied electric field, the kink or double π forms of the distortion wave propagating along the normal to the bounding surfaces may occur in the microsized nematic volume [15–17]. It was shown that periodic distortion of the LC phase reduces the total effective rotational viscosity γ_{eff}, connected with the reorientation of the director field [4]. Rotating domains arising in this case favor the reduction in the effective viscosity γ_{eff}, which characterizes the energy dissipation rate and thus creates modes of reorientation of the director field more favorable than a uniform rotation. As a result, this leads to a reduction of the direction field reorientation time τ_{ON} [4,15]. It is important, both from an academic and a technological point of view, to investigate the director reorientation in a thin LC film confined between two transparent electrodes and subjected to competing constraints.

We have numerically investigated the peculiarities of the director relaxation during the turn-on aligning processes in the microsized HAN film, when the director field is reoriented by strong crossed electric **E** and magnetic **B** fields. The analysis of numerical results for the reorientation process indicates the appearance of spatially periodic structures in a confined nematic film, with the thickness of $2d = 194.7$ μm only in response to a suddenly applied strong electric field ($U = 200$ V) directed at the angle of $\alpha = 1.57$ (∼89.96°) to the magnetic field ($B_0 = 7.05$ T). This case is important because the methods of NMR spectroscopy make it possible to measure the director reorientation in the nematic film [4,13] and compare it with the calculated data.

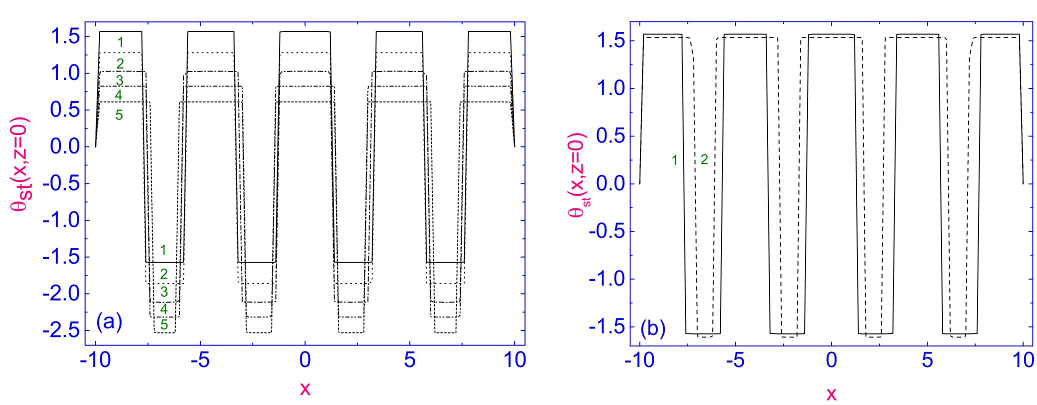

FIG. 9. (a) Stationary distribution of the angle $\theta^{\text{st}}(x, z = 0)$ along the length $-10 \leqslant x \leqslant 10$ of the HAN film, for the number of values of α: 1.57 (∼89.96°) (curve 1), 1.40 (∼80.0°) (curve 2), 1.222 (∼70.0°) (curve 3), 1.047 (∼60.0°) (curve 4), and 0.82 (∼50.0°) (curve 5), respectively. (b) Same as in (a), but the values of α are 1.57 (∼89.96°) (curve 1) and 1.55 (∼88.8°) (curve 2), respectively.

TABLE I. Calculated values of $\Delta = [\theta^{st}_{max}; \theta^{st}_{min}]$ between the maximum θ^{st}_{max} and the minimum θ^{st}_{min} of the stationary angle θ^{st}, the maximum values of the difference $\Delta_{max} = \theta^{st}_{max} - \theta^{st}_{min}$, and the values of the relaxation times $\tau_R(\alpha)$ for different values of α.

α	0.82 (∼50.0°)	1.047 (∼60.0°)	1.222 (∼70.0°)	1.40 (∼80.0°)	1.55 (∼88.8°)	1.57 (∼90.0°)
Δ	[0.60; −2.50]	[0.826; −2.3]	[1.02; −2.10]	[1.27; −1.87]	[1.53; −1.61]	[1.57; −1.57]
Δ_{max}	3.10 (∼178°)	3.12 (∼178.5°)	3.13 (∼179°)	3.14 (∼180°)	3.14 (∼180°)	3.14 (∼180°)
$\tau_R(\alpha)$	11 (∼14 ms)	11 (∼14 ms)	11 (∼14 ms)	11 (∼14 ms)	18 (∼23 ms)	21 (∼26.6 ms)

The direct comparison of the calculated data and those obtained using NMR spectroscopic technique of relaxation time $\tau^{cal}_R(\alpha \sim 88.8^\circ)$∼23 ms and $\tau^{exp}_R(\alpha \sim 88.7^\circ)$∼21 ms (see Fig. 5(a) [4,13]) shows a good agreement between these values. Analysis of these results shows that in deuterated $5CB$-d_2 at the temperature of 300 K and the density of 10^3 kg/m^3, the application of the large electric (∼1.03 V/μm) field applied at the angle close to the right angle to the magnetic field (∼7.05 T) leads to the values $\tau_R(\alpha) \sim 21 \div 23$ ms, for angle $\alpha \sim 88.7^\circ$) and higher. Calculations of the reorientation of the director field in a microsized nematic volume imposed by crossed electric and magnetic fields in the form of a simple monodomain show that the scaled relaxation time is 30, which corresponds to a relaxation time of 38.1 ms [5].

Thus, this analysis of the numerical results, based on the predictions of the hydrodynamic theory, including the director reorientation and the strong anchoring conditions, provides evidence for the appearance of spatially periodic patterns in response to the large electric field applied at an angle to the magnetic field. In turn, the large-amplitude periodic distortions modulated in the microsized nematic volume parallel to the horizontal restricted surfaces lead to the increase of the elastic energy of the conservative LC system, and as a result, it causes the decrease of the viscous contribution W_{vis} to the total energy of the nematic system. In turn, the decrease of W_{vis} leads to a lower values of the rotational viscosity coefficient γ_1(eff), and should lead to the faster relaxation time $\tau_R(\alpha)$, as observed experimentally [4,13].

We believe that the present investigation has shed some light on the problem of the reorientation processes in nematic films confined between two electrodes, induced by both electric and magnetic fields.

ACKNOWLEDGMENT

A.V.Z. acknowledges financial support from the Ministry of Science and Higher Education of the Russian Federation for IPMash RAS (FFNF-2024-0009).

[1] P. G. de Gennes and J. Prost, *The Physics of Liquid Crystals*, 2nd Ed. (Oxford University Press, Oxford, 1995).
[2] G. R. Luckhurst, T. Miyamoto, A. Sugimura, and B. A. Timimi, J. Chem. Phys. **117**, 5899 (2002).
[3] A. F. Martins and A. Veron, Liq. Cryst. **37**, 747 (2010).
[4] A. Sugimura and A. V. Zakharov, Phys. Rev. E **84**, 021703 (2011).
[5] A. Sugimura, A. A. Vakulenko, and A. V. Zakharov, Phys. Procedia **14**, 102 (2011).
[6] E. Guyon, R. Meyers, and J. Salan, Mol. Cryst. Liq. Cryst. **54**, 261 (1979).
[7] F. Lonberg, S. Fraden, A. J. Hurd, and R. E. Meyer, Phys. Rev. Lett. **52**, 1903 (1984).
[8] G. Srajer, S. Fraden, and R. B. Meyer, Phys. Rev. A **39**, 4828 (1989).
[9] A. V. Zakharov and A. A. Vakulenko, J. Chem. Phys. **139**, 244904 (2013).
[10] A. A. Vakulenko and A. V. Zakharov, Phys. Rev. E **88**, 022505 (2013).
[11] J. L. Ericksen, Arch. Ration. Mech. Anal. **4**, 231 (1959).
[12] F. M. Leslie, Arch. Ration. Mech. Anal. **28**, 265 (1968).
[13] A. Sugimura and G. R. Luckhurst, Prog. Nucl. Magn. Reson. Spectrosc. **94-95**, 37 (2016).
[14] I. S. Berezin and N. P. Zhidkov, *Computing Methods*, 4th ed. (Clarendon, Oxford, **1965**).
[15] I. Sliwa, P. V. Maslennikov, and A. V. Zakharov, Phys. Rev. E **105**, 064702 (2022).
[16] I. Sliwa, P. V. Maslennikov, and A. V. Zakharov, J. Mol. Liq. **331**, 115818 (2021).
[17] A. V. Zakharov, P. V. Maslennikov, and S. V. Pasechnik, Phys. Rev. E **103**, 012702 (2021).