Field-theoretic approach to neutron noise in nuclear reactors

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An operating nuclear reactor is designed to maintain a sustained fission chain reaction in its core, which results from a delicate balance between neutron creations (i.e., fissions) and total absorptions. This balance is associated with random fluctuations that can have two, very different, origins. A distinction must thus be made between low-power noise, whose origin lies in the inherently stochastic nature of neutron interactions with matter, and high-power noise, whose origin lies in the particular thermomechanical constraints associated with the environment in which neutrons propagate. Modeling the behavior of this noisy neutron population with the help of stochastic differential equations, we first show how the Martin-Siggia-Rose-Janssen-De Dominicis (MSRJD) formalism, providing a field theoretical representation of the problem, reveals a convenient and adapted tool for the calculation of observable consequences of neutron noise. In particular, we show how the MSRJD approach is capable of encompassing both types of neutron noises in the same formalism. Emphasizing then on power noise, it is shown how the self-sustained chain reaction developing in a reactor core might be sensitive to noise-induced transitions. Establishing an unprecedented connection between the neutron population evolving in a reactor core and the celebrated Kardar-Parisi-Zhang (KPZ) equation, we indeed find evidence that a noisy reactor core power distribution might be subject to a process analogous to the roughening transition, well-known to occur in systems described by the KPZ equation.

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I. INTRODUCTION

Neutron transport theory, that is, the study of the behavior of the neutron population as it evolves within a fissile material, is the foundation upon which our understanding of nuclear reactor operations is based. The theory assimilates the neutron population to a dilute gas, which can be accurately modeled using a linear Boltzmann equation. As such, nuclear industry has seen the development of a multitude of analytical or numerical methods to solve this equation and obtain the average spatial distribution of the neutrons in a nuclear reactor, i.e., the core power distribution.

Fluctuations around this mean solution has also been the subject of considerable research [1–3], although this "neutron noise" has often been considered—quite legitimately—to be of secondary importance for reactor operations and safety. At low power, fluctuations in the neutron population arise from the inherently stochastic nature of the processes to which neutrons are subjected: interaction probability, precursor decay, variable number of neutrons emitted by fission, etc. In this setting, the neutron population is best described by a stochastic branching process [4] for which the standard Boltzmann transport equation used in reactor physics would correspond to the mean-field equation. Relatively recent experimental results [5], building upon a number of theoretical studies [6–8], have nevertheless sparked new interest in the study of neutron noise by providing an example of a situation where neutron fluctuations need to be taken into account in a (low-power) nuclear reactor. In this experiment, it was shown that nontrivial spatiotemporal correlations of the neutron population might lead to the formation of special patterns in the expected neutron distribution in the reactor's core due to a clustering of the neutron population. This clustering phenomenon is a direct consequence of the stochastic and branching nature of the fission process.

In another direction [9,10], the nature of the neutron branching process has recently been investigated in the framework of statistical field theory, revealing, in particular, an interesting connection between reactor physics and time-directed percolation processes. In the presence of counter-reaction mechanisms (such as the Doppler effect, which, in principle, would require a nonzero power level), criticality was found to be a second-order phase transition that would fall into the directed percolation universality class. In this scenario, small deviations from the mean field equations could be expected. These, in turn, would place constraints on the maximum achievable precision targeted by the latest, high-precision, numerical schemes.

The work carried out in Ref. [10] can be criticized in that the observed deviations and the very existence of the phase transition depend crucially on the coupling of the neutron population to feedback mechanisms, which are generally considered negligible at low power, where the intrinsic stochastic nature of the neutron population is the most important. An interesting refinement would therefore consist of replacing or complementing the low-power noise model used previously with a power noise model, which is the second source of

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neutron fluctuations usually encountered in neutronics [2]. This type of noise is not directly related to the stochastic nature of neutron interactions in matter, but has a purely phenomenological origin, related to random fluctuations in the media in which neutrons propagate. This noise is expressed as the result of various sources of fluctuations or vibrations (pumps, fluid circulation, boiling for BWRs, etc.) that can be found in nuclear reactors operating at a high power.

This paper therefore aims to complement the pioneering work of Ref. [10] by applying the same formalism, that of the Martin-Siggia-Rose-Janssen-De Dominicis (MSRJD) response functional, to the neutron theory of power noise.

In Sec. II, the first model considered is the simplified zero-dimensional kinetic point model, which serves as a reminder of the MSRJD formalism and helps build a bridge to previous work. In particular, we take advantage of this simplified model to highlight the differences and similarities between high and low power noise models. In this context, we show that the response functional formalism allows both types of neutron noise to be treated in a transparent and unified way, something that has been little studied in the literature [11] but might reveal interesting in intermediate power regime, during for instance reactor startup.

Section III deals with the addition of a space dimension in the diffusion approximation. In this setting, we show that power neutron noise is described by a stochastic heat equation (SHE) with multiplicative noise. This type of model has extensively been studied, notably because of its link to the celebrated Kardar-Parisi-Zhang (KPZ) equation. Through this link, we conclude that there might exist a regime in which the neutron population evolving in a nuclear reactor might see the development of noise-induced transitions on the model of the well-known roughening transition associated with the KPZ equation.

II. POINT KINETIC EQUATIONS

The simplest model in the reactor physicist's toolbox is that of a zero-dimensional reactor in which the neutron population $N(t)$ and the population of so-called precursors of delayed neutrons $M(t)$ are described by the point kinetic equations (assuming here only one family of delayed neutrons),

$$\frac{dN(t)}{dt} = \rho N(t) + \lambda_D M(t) + S$$
$$\frac{dM(t)}{dt} = \beta N(t) - \lambda_D M(t), \tag{1}$$

where ρ is defined as the total reactivity (per unit time), λ_D is the decay constant of the unstable precursors nuclei, S is the neutron production rate from an external neutron source (from spontaneous fission, for instance), and β is the delayed neutron fraction (per unit time), such that βN is the instantaneous production rate of precursors. These equations are believed to encapsulate the main features of the average temporal behavior of the neutron and precursors populations evolving in a fissile medium. As such, they constitute an invaluable tool for understanding the mean temporal behavior of a nuclear reactor. In addition, these equations have been shown to play an important role in elucidating the structure of random fluctuations in the neutron and precursors populations, which can be of two very different origins.

A first source of random fluctuations (zero power noise) has its origin in the stochastic nature of the interactions of the neutrons with the medium in which they propagate. The complete description of the problem can be achieved using the tools of stochastic branching processes [4]. In this setting, the point kinetic equations are but the mean equations associated with an underlying stochastic process [2,3].

It is the other source of randomness, i.e., power noise that will mostly be considered in the present paper. It originates from the mechanical and thermal constraints applying on an operating nuclear reactor. Any source of mechanical or thermal-hydraulic fluctuations in a nuclear reactor can cause random changes in the properties of the medium in which the neutrons propagate. These fluctuations affect the parameters ρ and S of Eq. (1) [12]. These parameters are therefore promoted to random variables [2] and they can be decomposed as the sum of a reference, noiseless, value, and a random fluctuating component: $\rho(t) = \rho + \sigma\xi(t)$ and $S(t) = S + \gamma\zeta(t)$. With these, the point kinetic equations are transformed into a pair of coupled stochastic differential equations (SDEs):

$$\frac{dN(t)}{dt} = \rho N(t) + \lambda_D M(t) + S + \sigma N(t)\xi(t) + \gamma\zeta(t)$$
$$\frac{dM(t)}{dt} = \beta N(t) - \lambda_D M(t). \tag{2}$$

Both noise sources are colored noises, i.e., they possess a nontrivial time autocorrelation function, the system of equation should therefore be understood in the Stratonovich sense [2,13].

Solutions to the system of equations Eq. (2) in different approximation regimes have already been presented in the literature [2]. The approach proposed in this section offers another perspective on these old results: starting from the system of equations Eq. (2), it is proposed to built a statistical field theory of the system. This formalism has the decisive advantage that it comes equipped with all of the powerful machinery of field theory, enabling us to derive all of the main observables of interest for the problem with relative ease. The approach we propose is that of the MSRJD path integral, applied here to power noise. It directly comes as an extension of the original and pioneering results obtained in Ref. [10] in the case of zero power noise. We show how the MSRJD formalism proves to be particularly adapted for neutron noise analysis, concretely allowing for an (almost) unprecedented unified description of both types of neutron noises.

A. Model without precursors

As a first step towards a complete description of the problem, we start in a simplified setting by forgetting about the existence of precursors altogether. Setting $M(t) = 0$ in Eq. (2), one is thus interested in the solution of

$$\frac{dN(t)}{dt} = \rho N(t) + S + \sigma N(t)\xi(t) + \gamma\zeta(t). \tag{3}$$

Any averaged observable $\langle \mathcal{O}[N] \rangle$ can be formally written as the result of a functional integral over all possible realizations of the noise terms:

$$\langle \mathcal{O}[N]\rangle \propto \int \mathcal{D}\xi(t)\mathcal{D}\zeta(t)\mathcal{O}[N]P[\xi]P[\zeta]. \quad (4)$$

Introducing the constraint,

$$\mathcal{C}[N] \equiv \frac{dN(t)}{dt} - \rho N(t) - S - \sigma N(t)\xi(t) - \gamma\zeta(t) = 0, \quad (5)$$

one can insert into Eq. (4) the following resolution of the identity:

$$1 = \int \mathcal{D}N \det\left(\frac{\delta\mathcal{C}[N(t')]}{\delta N(t)}\right)\prod_t \delta(\mathcal{C}[N])$$
$$= \int \mathcal{D}[i\widetilde{N}]\int \mathcal{D}N \det\left(\frac{\delta\mathcal{C}[N(t')]}{\delta N(t)}\right)e^{-\int dt\widetilde{N}\mathcal{C}[N]}. \quad (6)$$

Assuming that both noises follow a Gaussian distribution, the functional integrations over ξ and ζ in Eq. (4) can be explicitly carried out. This approach would lead to a MSRJD field theory, expressed as a path integral over the realizations of the field $N(t)$ and its associated response field $\widetilde{N}(t)$ [14]. One, however, has to pay special attention to the functional determinant appearing in Eq. (6) because here it plays a prominent role: it is usually associated with auxiliary fields in the theory, the Fadeev and Popov ghost fields [14,15].

It is possible to greatly simplify the handling of these auxiliary fields in the white noise hypothesis (but still sticking to Stratonovich's interpretation). If the noises are independent, and if they both have zero mean and a variance given by

$$\langle\xi(t)\xi(t')\rangle = \delta(t-t') \text{ and } \langle\zeta(t)\zeta(t')\rangle = \delta(t-t'), \quad (7)$$

then it has been shown that the auxiliary fields appearing in the functional determinant can also be directly integrated over [16]. As a result, Eq. (4) can be shown to read

$$\langle\mathcal{O}[N]\rangle = \mathcal{N}\int \mathcal{D}[i\widetilde{N}]\int \mathcal{D}N\,\mathcal{O}[N]\,e^{-\mathcal{S}[N,\widetilde{N}]}, \quad (8)$$

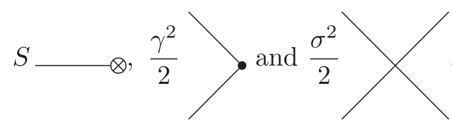
where the so-called response functional has been introduced:

$$\mathcal{S}[N,\widetilde{N}] = \int\left\{\widetilde{N}(t)\left(\frac{d}{dt} - \rho - \frac{\sigma^2}{2}\right)N(t) - \widetilde{N}S - \frac{\sigma^2}{2}\widetilde{N}^2N^2 - \frac{\gamma^2}{2}\widetilde{N}^2\right\}dt. \quad (9)$$

From the response functional, one can directly read off the propagator of the theory [$\mathcal{H}(t'-t)$ being the Heaviside step function],

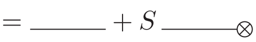
$$\Delta(t'-t) = e^{(\rho+\frac{\sigma^2}{2})(t'-t)}\mathcal{H}(t'-t), \quad (10)$$

as well as the Feynman rules:



(11)

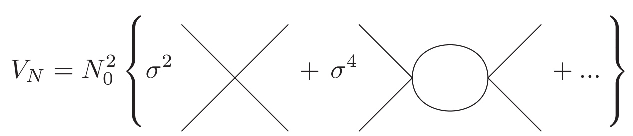
Imposing an initial condition of the form $N(0) = N_0$ [17], one readily obtains

$\langle N(t)\rangle =$ 

$$= N_0 e^{(\rho+\frac{\sigma^2}{2})t} + S\int_0^t e^{(\rho+\frac{\sigma^2}{2})(t-t')}dt'$$
$$= N_0 e^{(\rho+\frac{\sigma^2}{2})t} + \frac{S}{\rho+\sigma^2/2}\left(e^{(\rho+\frac{\sigma^2}{2})t} - 1\right). \quad (12)$$

This expression and the associated diagramatics to which it is associated directly mirror those obtained in Ref. [10] for zero power noise. The difference in the power noise case originates from both the multiplicative noise term and the Stratonovich interpretation associated to Eq. (3). These two ingredients induce a shift in the true reactivity which then becomes $\theta = \rho + \sigma^2/2$. This particularity is well-known and has already been discussed in the context of neutronics in, for instance, Ref. [2].

The calculation of the variance is much more interesting, as the structure of the fluctuations is of a radically different nature than that associated with zero power noise, which is dominated by the branching structure of the fission process. The multiplicative noise term, in particular, profoundly alters the dynamics of temporal fluctuations. By allowing couplings of the type $\sigma^2\widetilde{N}^2N^2$, the diagrammatics of the problem is indeed greatly enriched. The variance of the neutron population can be decomposed as the sum $\mathrm{Var}(N) = \langle N^2\rangle - \langle N\rangle^2 = V_\gamma + V_N + V_S + V_{NS}$, with



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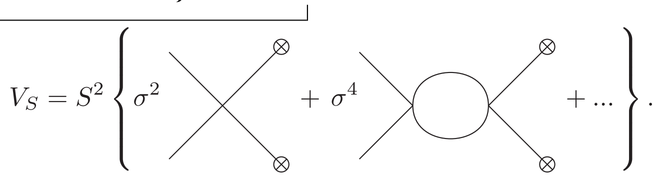

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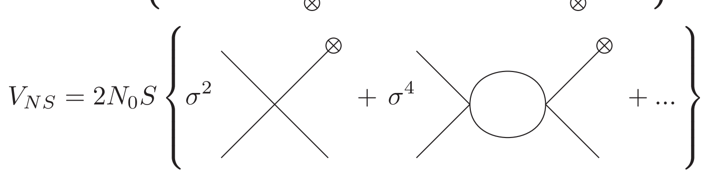
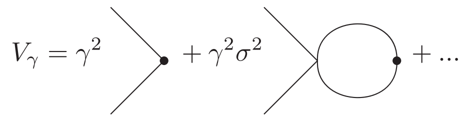


(15)

These three terms correspond to the contribution of pure multiplicative noise, i.e., the total contribution associated to fluctuations affecting the reactivity of the reactor. The last term corresponds to the stochastic part of the external neutron source,



(16)

In each case, one can observe that infinite series are generated by a feedback mechanism associated with the multiplicative noise term that tend to amplify any preexisting fluctuation, whatever its origin. The first series V_N can quite trivially be summed over and yields (recall that we set $\theta = \rho + \sigma^2/2$)

$$V_N = N_0^2 e^{2\theta t}(e^{\sigma^2 t} - 1), \tag{17}$$

in agreement with the literature [2]. The other series can be shown to involve the function

$$\mathcal{I}_n(\alpha, t) = \left(\frac{\sigma^2}{\alpha}\right)^n \left(\sum_{\ell=n}^{\infty} \frac{(\alpha t)^\ell}{\ell!}\right) \tag{18}$$

$$= \left(\frac{\sigma^2}{\alpha}\right)^n e^{\alpha t}\left(1 - \frac{\Gamma(n, \alpha t)}{\Gamma(n)}\right), \tag{19}$$

where $\Gamma(n)$ and $\Gamma(n, \alpha t)$ are, respectively, the gamma and upper incomplete gamma functions. One then finds

$$V_S = \left(\frac{S}{\theta}\right)^2 e^{2\theta t} \sum_{n=1}^{\infty} \left\{\mathcal{I}_n(-2\theta, t) - 2\mathcal{I}_n(-\theta, t) + \frac{(\sigma^2 t)^n}{n!}\right\}, \tag{20}$$

$$V_{NS} = \frac{2N_0 S}{\sigma^2} e^{2\theta t} \sum_{n=2}^{\infty} \mathcal{I}_n(-\theta, t), \tag{21}$$

and

$$V_\gamma = \left(\frac{\gamma}{\sigma}\right)^2 e^{2\theta t} \sum_{n=1}^{\infty} \mathcal{I}_n(-2\theta, t). \tag{22}$$

These theoretical predictions are in good agreement with the results of a numerical integration of Eq. (3), performed using the Milstein method, as described in Ref. [18] and shown in Fig. 1. Note, in particular, the rapid convergence of the variance, which although being built upon infinite sums over a parameter n, seems to converge quite rapidly after $n = 3$ or so. Equally remarkable is the role played by the terms V_S and V_{NS}, which dominate, in the present case, the total variance. This is due to the cumulative role played by the steady external source, which introduces S new neutrons per second in the system and thus dominates the overall dynamic. A crossover is expected to occur with the term V_N, which will eventually dominate the long-time behavior of the variance.

B. Inclusion of zero power noise

The zero-power noise model used in Ref. [10] is based on the use of a SDE whose noise term possesses an amplitude proportional to $\sqrt{N}$ (ignoring the presence of external sources). In the previous section, it was argued that fluctuations of the neutron population occurring in a power nuclear reactor can be interpreted as multiplicative noise, i.e., a noise term whose amplitude is proportional to N (again, ignoring the presence of external sources).

At high power, $N \gg 1$, which, in turn, implies $N \gg \sqrt{N}$: intrinsic fluctuations of the neutron population (i.e., zero power noise) are indeed negligible compared to the external sources of fluctuations (i.e., power noise). *A contrario*, in the low-power regime, these external sources of fluctuation are nonexistent because the thermomechanical vibrations from which they originate cease to exist as soon as the heat generated by the power level drops below a certain threshold.

This is the origin of the separation between the two sources of neutron noise: there exist only few situations where a unified treatment of the two distinct sources would be necessary. One notable exception would be reactor startup: there, the reactor power transiently and continuously passes through all possible values between zero and the nominal operating power.

In any case, it would appear quite satisfactory, if only from an academic point of view, to dispose of a truly unified description of both types of neutron noise (on the model of Ref. [11]). The SDE approach—and by extension the MSRJD response functional formalism—seems to be a good

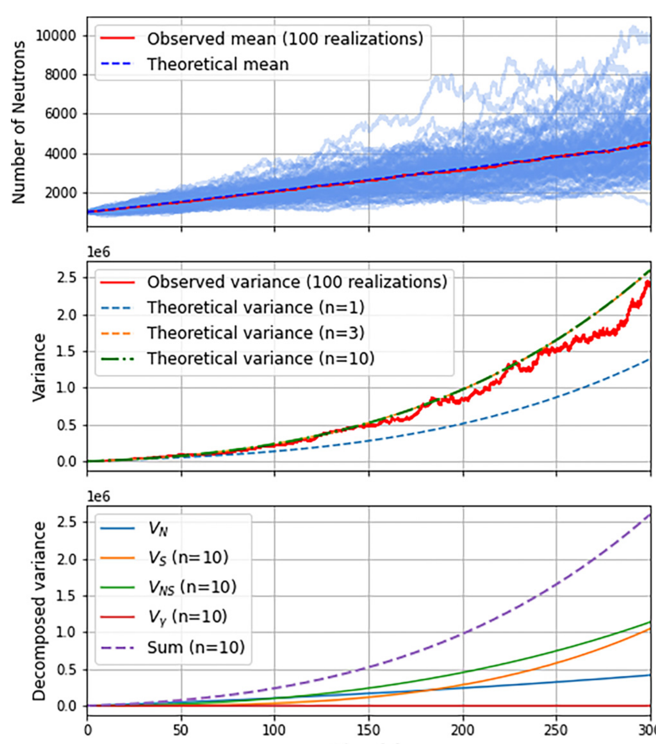


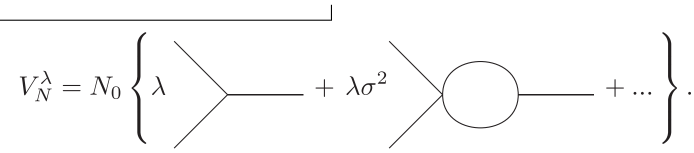
FIG. 1. Comparison of the theoretical results obtained for both the mean number of neutrons and its variance with a numerical integration of Eq. (3). The calculations are performed with the following set of parameters: $N_0 = 1000$, $S = 10$, $\rho = -4.10^{-5}$, $\sigma = 0.03$ (so $\theta = 49.10^{-5}$), and $\gamma = 0.02$.

candidate in this direction, but one must first solve an interpretation problem associated with these equations. Indeed, the two types of noise can both be described by SDEs, but they formally follow different prescriptions as for their interpretations:

(1) Zero power noise finds its origin in a microscopic jump process. To derive a stochastic differential equation from this stochastic, discrete process requires the use of a dedicated expansion method as, for instance, the Van Kampen's system size expansion technique [19]. The resulting equation is to be formally interpreted in the Itô sense.

(2) The power noise used in the present paper is a white noise idealization of a colored noise. As such, its associated SDE must be interpreted in the sense of Stratonovich.

A simple way of understanding the impact of differing interpretations would be to explore, for example, what happens to zero-power theory in Stratonovich's interpretation. Applying again the method proposed in Ref. [16] to the response functional found in Ref. [10], then one can easily show that the only effect brought by this prescription is a term proportional to $\widetilde{N}^2$. This term can be harmlessly absorbed by a redefinition of the constant γ in the response functional of Eq. (9). Apart from this innocuous reinterpretation of the parameter γ, including a square root term into the functional Eq. (9) is a straightforward operation. Another Feynman rule of the type $\lambda\widetilde{N}^2N$ must be added. This rule adds, for instance, two supplementary terms in the calculation of the variance of the neutron population,



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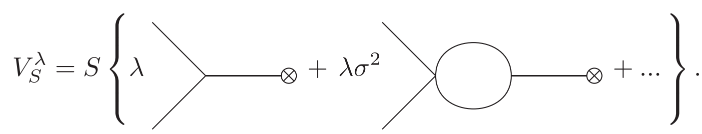
and



(24)

The two terms appear, as before, in the form of two infinite series of diagrams. The first infinite series yields

$$V_N^\lambda = \frac{\lambda N_0}{\sigma^2} e^{2\theta t} \sum_{n=1}^{\infty} \mathcal{I}_n(-\theta, t), \tag{25}$$

which has exactly the same form as the term V_{NS} derived earlier, modulo the substitution $2S \to \lambda$. The parameter λ being associated to the variance of the number of neutrons emitted by fission, it is expected that this constant is much lower than the intensity S of the steady source term. It is therefore expected that, in general, this term is much lower than V_{NS}. The second series of diagrams gives

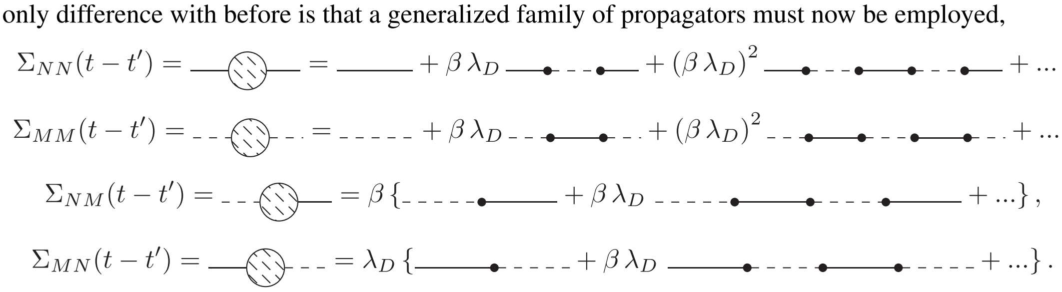
$$V_S^\lambda = \frac{\lambda S}{\theta\sigma^2} e^{2\theta t} \left\{ \sum_{n=1}^{\infty} \mathcal{I}_n(-\theta, t) - \sum_{n=1}^{\infty} \mathcal{I}_n(-2\theta, t) \right\}. \tag{26}$$

Likewise, this term will only give an appreciable contribution to the variance if λ can be compared with either the steady source term S or the initial neutron population N_0.

The main conclusion to be drawn from the above analysis is that a unified description of both zero power and power noise seems to be achievable through the MSRJD path integral formalism, at least in the present simplified setting where the power is modeled as a white noise but is still interpreted in the Stratonovich interpretation. In a more realistic scenario, i.e., taking into account for the noises' temporal autocorrelation functions, the mixed interpretations associated with both types of noises could complicate the overall diagrammatics (via couplings with the ghosts fields) and would thus require a dedicated analysis. Such a detailed investigation will not be pursued any further in the present paper.

C. With precursors

One can now set $M(t) \neq 0$ in Eq. (2) and write a field theory associated to this pair of equations. The general procedure associated to the system has already been described in Ref. [10]. Because the precursor population is not directly associated with any kind of random fluctuations (in the power noise case), the Feynman rules of the theory are the same as in the previous section. The only difference with before is that a generalized family of propagators must now be employed,



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for which analytical expressions have been derived in Ref. [10].

Equipped with this set of propagators, the calculation of the physical observables can be straightforwardly adapted. The mean number of neutrons and precursors [with initial condition $(N(0), M(0)) = (N_0, 0)$], for instance, reads

$$\langle N \rangle = N_0 \Sigma_{NN}(t) + S \int_0^t \Sigma_{NN}(u) du, \tag{31}$$

$$\langle M \rangle = N_0 \Sigma_{NM}(t) + S \int_0^t \Sigma_{NM}(u) du. \tag{32}$$

These expressions are compared, like before, to a numerical integration of Eq. (2) in Fig. 2. An excellent agreement is found for both the neutron and the precursor populations.

III. STOCHASTIC DIFFUSION EQUATION

Refining upon the point kinetic model, the simplest route towards a modeling that includes a spatial dimension is to resort to the diffusion approximation. Neglecting the precursor population and the eventual existence of an external source term S, the equation describing the evolution of the neutron population takes the form of a SHE to which a multiplicative noise term is supplemented:

$$\frac{\partial}{\partial t} N(\vec{x}, t) = \rho N(\vec{x}, t) + D \Delta N(\vec{x}, t) + \sigma N(\vec{x}, t) \xi(\vec{x}, t). \tag{33}$$

Introducing an arbitrary momentum scale κ and measuring time in terms of distance squared (i.e., scaling time with the diffusion coefficient), one can write (in d dimension)

$$[x] = \kappa^{-1}, \quad [t] = \kappa^{-2}, \quad [N] = \kappa^{d}. \tag{34}$$

From this, we conclude that $[\sigma] = \kappa^{2-d}$, so the critical dimension of the problem is $d_c = 2$. In dimensions $d > 2$, no deviations from the mean-field behavior are therefore to be expected. The perturbative calculation of the vertex function of the problem can be readily adapted from a similar calculation performed in Ref. [20]. The renormalized vertex function indeed presents no departure from mean field, at least in the present perturbative setting.

It is, however, well-known that there exists a nontrivial connection between the SHE and the celebrated KPZ equation (in this case, with an additive term C):

$$\frac{\partial}{\partial t} h(\vec{x}, t) = \nu \Delta h + \frac{\lambda}{2} (\vec{\nabla} h)^2 + \eta \xi(\vec{x}, t) + C. \tag{35}$$

Performing the so-called Cole-Hopf transformation $N(\vec{x}, t) = \exp[\frac{\lambda}{2\nu} h(\vec{x}, t)]$, one can indeed easily show that N is a solution of Eq. (33), provided the identification (see Appendix)

$$\begin{aligned} \rho &= \frac{\lambda C}{2\nu}, \\ D &= \nu, \\ \sigma &= \frac{\lambda}{2\nu} \eta. \end{aligned} \tag{36}$$

Equation (33) therefore corresponds to a whole one-parameter family of KPZ equations, parameterized by either the parameter λ or C. This finding establishes an interesting link between the dynamical behavior of the neutron population in a fissile medium and the dynamics of surface growth, which is the most common application of the KPZ equation.

The most spectacular consequence of this nontrivial bridge between the two domains is most certainly to be found in the existence of a so-called *roughening transition* associated to the KPZ equation: for a sufficiently high value of the noise amplitude η in Eq. (35), the solution $h(\vec{x}, t)$ might transition to a state associated to substantial deviations from the mean field behavior.

Extensive studies relying on nonperturbative renormalization group methods [21] have indeed shown the existence of two fixed points in the phase diagram associated to the KPZ equation, which depend crucially on the noise intensity:

(1) The weak, low noise, fixed point, is Gaussian and corresponds to a regime where no deviations from the mean-field behavior is expected.

(2) The strong coupling fixed point associated to substantial deviations from the mean field equation.

Since it is directly related to the KPZ equation, via the Cole-Hopf transformation explicited above, it is expected that the SHE describing power neutron noise behaves similarly. A good way to hint at this peculiar behavior is to reconsider the calculation of Eq. (13) performed in the context of the zero-dimensional, point kinetic model. The infinite series of loop diagrams appearing in the calculation can be absorbed

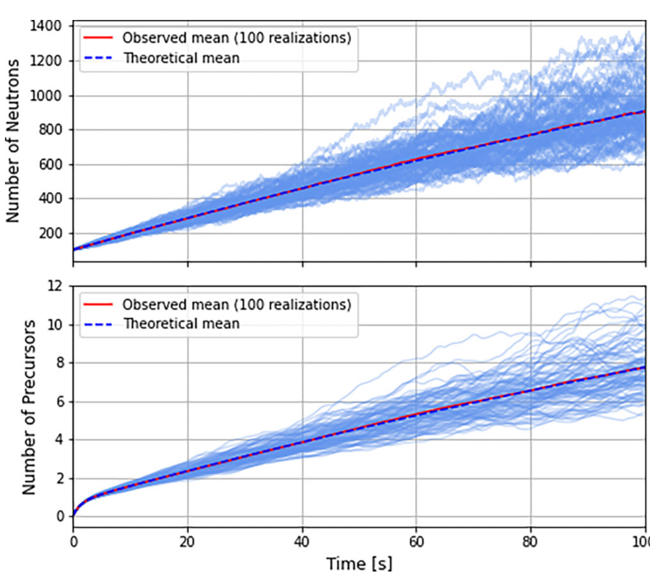
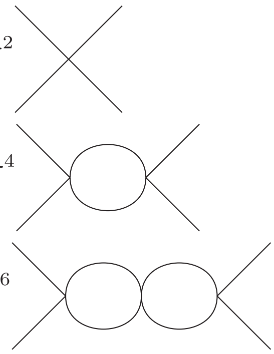


FIG. 2. Comparison of the theoretical mean numbers of neutron and precursor derived with the MSRJD approach with a direct numerical integration of Eq. (2) using the Milstein technique. The calculation is performed using the following set of parameters: $N_0 = 1000$, $S = 10$, $\rho = -400.10^{-5}$, $\beta = 600.10^{-5}$, $\sigma = 0.03$, $\gamma = 0.02$, and $\lambda_D = \ln 2$.

into the definition of a *renormalized* noise parameter σ_R:

$$\sigma_R = \sigma^2 [\text{tree}] + \sigma^4 [\text{one loop}] + \sigma^6 [\text{two loops}] + \dots \qquad (37)$$



$$= \frac{\sigma^2}{1-\sigma^2\mathcal{I}}.$$

$\mathcal{I}$ is the integral of the one loop diagram, evaluated at zero external frequency and momenta. In dimension $d > 2$, $\mathcal{I}$ is finite. Then, when σ is such that $\sigma^2\mathcal{I} > 1$, the perturbative expansion ceases to be meaningful [22]. It has been shown [23] that in this case, the value of σ_R flows towards a nonperturbative fixed point, whose critical exponents necessarily share similarities with those of the KPZ strong noise fixed point.

The noise value at which a change of behavior can be expected is controlled by the value of the parameter σ_R, which ultimately corresponds to the typical variations in reactivities observed in a reactor [recall that we defined $\rho(t) = \rho + \delta\rho(t)$, with $\delta\rho = \sigma\xi(t)$]. The value at which the system might transition would require detailed numerical investigations on the model on the analysis carried out, for instance, in Ref. [24], and remains to be precisely done in the specific context on reactor physics.

A. A pinning (depinning) phase transition?

A complete picture of the problem, as applied to reactor physics, must include the addition of counter-reaction mechanisms, which are a key element of any nuclear reactor modeling. The most prominent feedback mechanism occurring in an operating reactor is the Doppler broadening of the heavy nuclei cross sections. Any increase in the overall neutron population implies an increase in temperature of the surrounding media (simply because the number of fissions tends to increase), which, in turn, tends to enhance neutron absorptions by heavy nuclei, such as ^{238}U. Such a feedback mechanism can generically be captured by the inclusion of a restoring force of the type $-\alpha N(N - N_{\text{ref.}})$ [1,25,26]. With this supplementary term, the equation of the problem can be written (absorbing the $\alpha N_{\text{ref.}} N$ term in a redefinition of ρ)

$$\frac{\partial}{\partial t} N = \rho N + D\Delta N - \alpha N^2 + \sigma N \xi(\vec{x}, t). \qquad (38)$$

This equation can be mapped to a KPZ-type equation through the same Cole-Hopf transformation as earlier: Writing $N(\vec{x}, t) = \exp[\frac{\lambda}{2\nu} h(\vec{x}, t)]$, one can readily show that the function h is solution of the equation

$$\frac{\partial}{\partial t} h(\vec{x}, t) = \nu\Delta h + \frac{\lambda}{2}(\vec{\nabla} h)^2 + \eta\,\xi(\vec{x}, t) + C - \alpha e^{\frac{\lambda}{\nu} h}. \qquad (39)$$

This equation is known in the literature as the KPZ equation *with an upper wall*. Such type of model has been extensively studied in the literature [22,24]. When ρ varies, it is found that the system presents a dynamical phase transition from an inactive ($N = 0$) to an active ($N \neq 0$) state. Through the link between Eq. (38) and the KPZ equation with wall, the transition would fall in the class of the pinning (depinning) phase transition.

A complete characterization of the consequences of that transition would far exceed the scope of the present paper. This would in particular require dedicated and careful calculations of observables near criticality. For applications in reactor physics, it is still unclear how one could find evidence of the appearance of such a transition in an operating nuclear reactor. A good hope might lie in a careful derivation of the power's temporal autocorrelation function (or equivalently the power spectral density [2]). This would require a much more refined modeling, including in particular to move from the idealized white noise hypothesis (see, for instance, Ref. [27]) and the inclusion of precursors.

IV. CONCLUSION

A field theoretic formulation, along the lines of the MSRJD response functional formalism, has been successfully derived. Building upon the groundwork laid for zero-power noise in Ref. [10], it has been shown that the field theoretical approach enables a unified treatment of both power and zero-power noise, allowing us, in particular, to deal with the different integration prescriptions of the SDEs associated with each type of noise (which is unprecedented in the realm of neutron noise analysis, to the author's knowledge). The versatility of the MSRJD approach, and the powerful tools associated with field theory, allow for a straightforward and easy calculation of the most relevant observables in reactor physics.

The decisive advantage of an approach based on statistical field theory lies in its unique relevance to the physics of phase transitions (see, for instance, Refs. [28,29]). Along this line, evidence has been found, suggesting the possible existence of noise-induced transitions associated to the realm of reactor physics. Relying on well-established results, we were able to relate reactor physics and the rich phenomenology of the KPZ equation, thus suggesting the possible existence of deviations from the expected behavior of a reactor core in high noise regimes.

The precise characterization of these deviations, their dependence on the noise intensity, and their tangible manifestation in a reactor still remain to be properly studied. This endeavor will require the consideration of more refined theoretical and numerical models than those presented in this paper. Possible refinements include incorporating zero-power noise, external neutron sources, precursors, colored noise sources, multigroup diffusion (i.e., taking into account the energy dependence of the neutron field), etc. These refinements are largely unexplored in the literature and could fundamentally alter the phenomenology of the problem. Further exploration of these avenues is essential for a complete and comprehensive understanding of noise-induced transitions in reactor physics.

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APPENDIX

To emphasize its importance, the connection relating Eqs. (33) and (35) is explicitly derived in the present Appendix. The relation is most easily seen starting from the inverse Cole-Hopf transformation: $h(\vec{x},t) = \frac{2\nu}{\lambda}\ln N(\vec{x},t)$. Denoting the time derivative by a dot and the spatial derivative with a prime, one has

(1) $\dot{h} = \frac{2\nu}{\lambda}\frac{\dot{N}}{N}$,

(2) $h' = \frac{2\nu}{\lambda}\frac{N'}{N}$,

(3) $h'' = \frac{2\nu}{\lambda}(\frac{NN''-(N')^2}{N^2})$.

Equation (35) then reads

$$\frac{2\nu}{\lambda}\frac{\dot{N}}{N} = \frac{2\nu^2}{\lambda}\left(\frac{NN''-(N')^2}{N^2}\right) + \frac{\lambda}{2}\left(\frac{2\nu}{\lambda}\frac{N'}{N}\right)^2 + \eta\xi + C \tag{A1}$$

or

$$\frac{2\nu}{\lambda}\dot{N} = \frac{2\nu^2}{\lambda}N'' + \eta N\xi + CN. \tag{A2}$$

From this, we conclude that $N(\vec{x},t)$ is the solution of Eq. (33), provided that relations Eq. (36) are satisfied.

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