Erratum: Bicritical states in temperature-modulated Rayleigh-Bénard convection [Phys. Rev. E 92, 013005 (2015)]

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In the paper there is a shortcoming in Fig. 7, since the numerical calculations were performed by taking four Galerkin terms, that is, $N = 4$, whereas a larger value of N is required to meet the convergence condition. The purpose of the present erratum is to provide the corrected version of Fig. 7 for the parameter space corresponding to which a bicritical state appears in temperature modulated Rayleigh-Bénard convection (TMRBC).

In view of this, Fig. 7 in the present erratum is the correct version, in which the marginal curve $\epsilon = 2.795$ has been replaced by the marginal curve for $\epsilon = 2.989$, since the bicritical state in TMRBC for mercury ($\sigma = 0.015$) is found to occur for the corrected value $\epsilon \approx 2.989$ with the following critical values:

$$\left(k_c^H, k_c^H, \mathrm{Ra}_c\right) \approx (3.449, 6.134, 7093).$$

So, throughout the paper, the value $\epsilon = 2.795$ should be read as "$\epsilon = 2.989$". Further, the nature of this bicritical state is different in the sense that the instability oscillates time periodically with the coexistence of two distinct harmonic wave numbers.

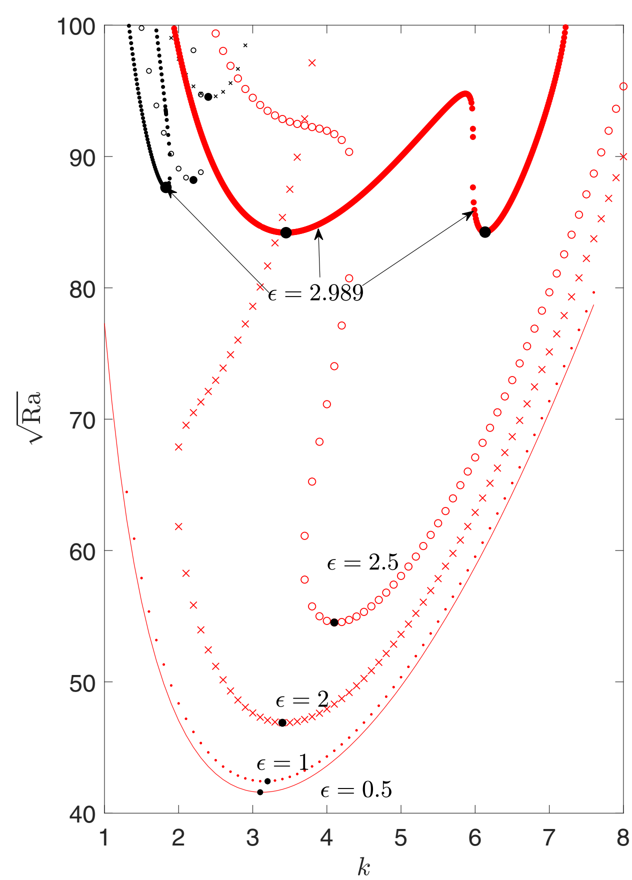

FIG. 7. Marginal curves in the $(k, \sqrt{\mathrm{Ra}})$ plane for $\omega = 5$, $\phi = 0^\circ$, and $\sigma = 0.015$ (mercury). Here, we have taken $N = 16$ and $L = 30$. On each curve corresponding to $\epsilon = 2$, 2.5, and 2.989, the red (larger) and black (smaller) points correspond to harmonic and subharmonic response, respectively. The black mark • at a minimum is the point $(k_c, \sqrt{\mathrm{Ra}_c})$. The bicritical-state corresponds to $(k_c^H, k_c^H, \sqrt{\mathrm{Ra}_c}) \approx (3.449, 6.134, 84.2)$.

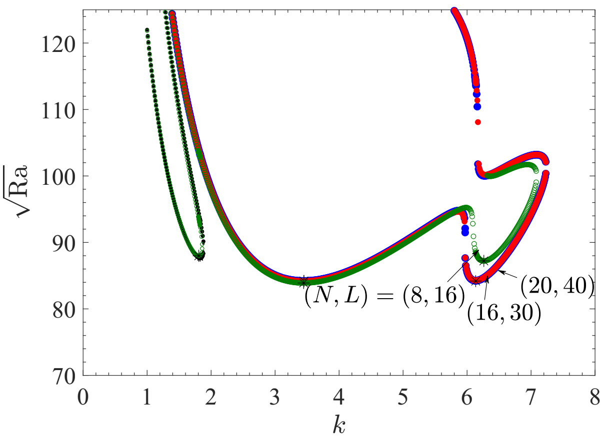

FIG. 7′. Dependence of marginal curves in the $(k, \sqrt{\mathrm{Ra}})$ plane on (N, L) for the fixed parametric values of $\epsilon = 2.989$, $\omega = 5$, $\phi = 0^\circ$, and $\sigma = 0.015$ (mercury).

For completeness, we perform convergence analysis of our numerical method with respect to N and L for the Prandtl number of mercury, that is, $\sigma = 0.015$. We have obtained the neutral instability curves corresponding to the three sets of values of $(N, L) = (8, 16)$, (16,30), and (20,40), which are shown in Fig. 7′ for the fixed parametric values of $\epsilon = 2.989$, $\omega = 5$, $\phi = 0^\circ$, and $\sigma = 0.015$. We observe that the curves corresponding to $(N, L) = (8, 16)$ deviate from those corresponding to the other two sets of values, whereas the curves corresponding to $(N, L) = (16, 30)$ and $(N, L) = (20, 40)$ are practically indistinguishable. This shows that for the Pradtl number of mercury the marginal curves obtained for $N \geqslant 16$ and $L \geqslant 30$ are correct.

Further, Table I shows variation of k and $\sqrt{\mathrm{Ra}}$ with respect to N and L for the fixed parametric values of $\sigma = 0.015$, $\epsilon = 2.988$, $\omega = 5$, and $\phi = 0^\circ$. Clearly the present numerical scheme converges for at least $(N, L) = (16, 30)$.

The numerical method used in the paper requires caution when dealing with the limiting cases (e.g., $\omega \to 0$, $\sigma \to 0$, or a combination of these two limits). The scaling used in the paper as well as that by Or and Kelly [1] are well suited for numerical calculations for $\sigma > 1$ but may have difficulties for the case when $\sigma \to 0$ (see [2]).

Figure 8 was obtained for larger values of ϵ but for $\sigma = 7$ and larger values of ω. Although our numerical scheme converges faster for larger values of ω and $\sigma > 1$, we recompute Fig. 8 as Fig. 8′ here. The neutral curves are computed for three sets of values of $(N, L) = (8, 16)$, (16,30), (20,40) for $(\epsilon, \omega) = (3.3, 51.82)$, and $(N, L) = (8, 20)$, (16,30), (20,40) for $(\epsilon, \omega) = (10, 40.5)$; the method does not converge for $(\epsilon, N, L) = (10, 8, 16)$. For the neutral curves for $\epsilon = 3.3$, we observe that the neutral curves for $(N, L) = (8, 16)$ deviate a little from those which correspond to the other two sets of values of (N, L), whereas the minima for harmonic and subharmonic parts of the neutral curves coincide for all the three sets of the values of (N, L). We conclude that the neutral curve for $(\epsilon, \omega) = (3.3, 51.82)$ in the original Fig. 8 should be replaced by the present Fig. 8′ for $(N, L) = (16, 30)$. On the other hand, the neutral curves corresponding to $\epsilon = 10$ in Fig. 8′ coincide for all the three sets of values of (N, L), and match with those obtained in the original Fig. 8. So, the corresponding results for the critical values of the control parameter and hence the bicritical states as in Fig. 8 remain unchanged.

We have rechecked by taking $N \geqslant 16$ and $L \geqslant 30$ for the remaining numerical results mentioned in the paper, and are all found to be correct.

TABLE I. Dependence of $(k, \sqrt{\mathrm{Ra}})$ on N and L for the fixed parametric values of $\sigma = 0.015$, $\epsilon = 2.989$, $\omega = 5$, and $\phi = 0^\circ$.

N	L	$(k_c^{SH}, \sqrt{\mathrm{Ra}})$	$(k^H, \sqrt{\mathrm{Ra}})$	$(k^H, \sqrt{\mathrm{Ra}})$
8	16	(1.807, 87.984)	(6.262, 87.219)	(3.446, 83.900)
16	30	(1.826, 87.719)	(6.135, 84.289)	(3.449, 84.181)
20	40	(1.829, 87.648)	(6.134, 84.245)	(3.449, 84.200)
24	48	(1.830, 87.622)	(6.133, 84.231)	(3.449, 84.206)
28	56	(1.831, 87.611)	(6.133, 84.226)	(3.449, 84.208)
32	64	(1.831, 87.606)	(6.133, 84.223)	(3.449, 84.209)

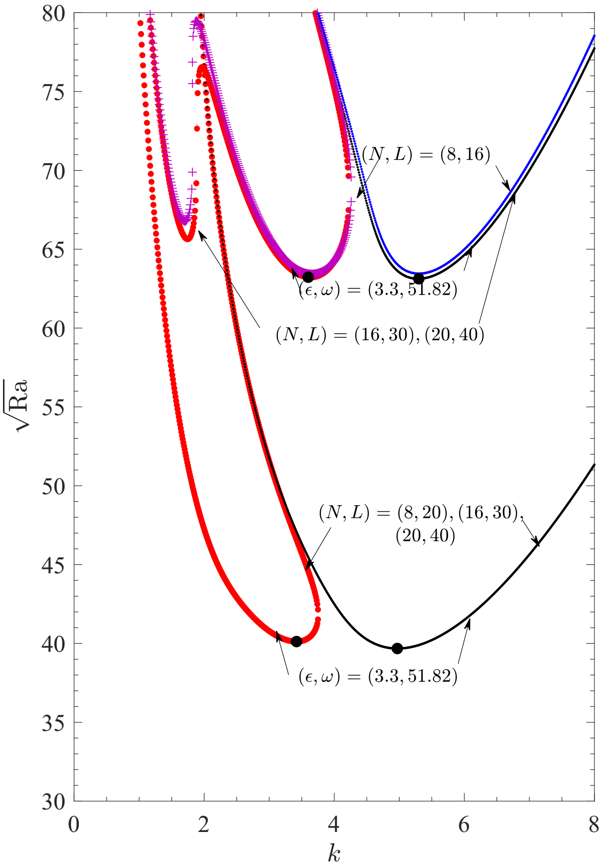

FIG. 8′. Dependence of marginal curves in the $(k, \sqrt{\mathrm{Ra}})$ plane on (N, L) for the fixed parametric values of $(\epsilon, \omega) = (3.3, 51.82)$, $(10, 40.5)$, $\phi = 0^\circ$, and $\sigma = 7$ (water).

We thank Dr. M. Riahi for bringing to our notice the numerical convergence issues in Fig. 7. We are also indebted to the referees for several suggestions in improving the present erratum.

[1] A. C. Or and R. E. Kelly, Time-modulated convection with zero mean temperature gradient, Phys. Rev. E **60**, 1741 (1999).
[2] E. A. Spiegel, Thermal turbulence at very small Prandtl number, J. Geophys. Res. **67**, 3063 (1962).