Letter

Persistence in active turbulence

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Active fluids such as bacterial swarms, self-propelled colloids, and cell tissues can all display complex spatiotemporal vortices that are reminiscent of inertial turbulence. This emergent behavior, despite the overdamped nature of these systems, is the hallmark of active turbulence. In this Letter, using a generalized hydrodynamic model, we present a study of the persistence problem in active turbulence. We report that the persistence time of passive tracers inside the coherent vortices follows a Weibull probability density whose shape and scale are decided by the strength of activity—contrary to inertial turbulence that displays power-law statistics in this region. In the turbulent background, the persistence time is exponentially distributed that is remindful of inertial turbulence. Finally we show that the driver of persistence inside the coherent vortices is the temporal decorrelation of the topological field, whereas it is the vortex turnover time in the turbulent background.

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Introduction. Persistence in physical systems concerns the probability that a local fluctuating field does not change its sign upto a time t. From Ising spins [1], rough surfaces [2], and disordered media [3], to optimization [4], machine learning [5], and stock markets [6], persistence often contains important information about the evolution history of a complex system. Theoretical investigations have shown that the probability density function of this persistence time $\mathcal{P}(t)$ is nontrivial because the underlying fluctuating field is usually non-Markovian [7]. A related and often useful quantity is the mean first passage time distribution of a particle diffusing in a bounded media. Once computed, such distributions can be profitably exploited to quantify and compare structural correlations and dynamical heterogeneity in complex systems [8,9], directly allowing a characterization of energy landscapes in these complex systems [10]. While there exists a large body of work on persistence and first-passage problems in many-body systems far from equilibrium [11–14], similar investigations in active or self-propelled systems have been very few. Thus, there is a pressing need to explore the persistence problem in active systems, especially with models that allow a general pattern of energy injection, transfer, and dissipation. The outcome of such an investigation could complement recent Lagrangian studies on velocity statistics [15], diffusion [16], effective temperature [17], irreversibility [18], and even anomalous transport [19] leading to superdiffusive first passage distributions at high levels of activity [20]. For a survey of the Eulerian properties of active turbulence, we refer the reader to the recent review in Ref. [21]. In this Letter, we report a careful Lagrangian study of persistence time and its distribution in a model active liquid that can display spatiotemporal vortices that are remindful of classical turbulent flows. To this end, we invoke the Okubo-Weiss criterion [22,23] to perform a topological partitioning of the active flow field into rotation dominated, deformation dominated, and intermediate regions, see Fig. 1 for a visualization. We show that in the rotation dominated regions, $\mathcal{P}(t)$ is given by the Weibull probability density whose shape and scale are decided by the strength of activity. In the deformation dominated regions, $\mathcal{P}(t)$ is exponential. Both these observations are contrary to the case of inertial turbulence that displays power-law statistics in these regions [24,25]. In the intermediate region that forms the turbulent background, $\mathcal{P}(t)$ is exponentially distributed, beautifully remindful of inertial turbulence. Our work is a study of the persistence time distributions in active matter flows that are valid over a wide range of a control parameters, thereby putting a large number of active systems under the purview of work, from elementary forms of life, such as bacterial suspensions to synthetic active matter such as Janus colloids.

Model and simulation. We perform direct numerical simulations of a generalized hydrodynamic model that is known to reproduce the flow field of dense bacterial suspensions in laboratory experiments [26–28]. In two dimensions, the incompressible velocity field of this model is governed by

$$\frac{\partial \boldsymbol{u}}{\partial t} + \lambda_0(\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla P - \Gamma_0\nabla^2\boldsymbol{u} - \Gamma_2\nabla^4\boldsymbol{u} - \mu\boldsymbol{u}$$
$$\nabla\cdot\boldsymbol{u} = 0, \tag{1}$$

where P is the pressure and the nondimensional parameter λ_0 decides the type of bacteria, meaning they are either pusher ($\lambda_0 > 1$) or a puller ($\lambda_0 < 1$) type. Keeping $\Gamma_{0,2} > 0$ mimics energy injection into the active fluid via instabilities. The scalar field $\mu = \alpha + \beta|\boldsymbol{u}|^2$ depends on the local velocity $\boldsymbol{u}$, and was first introduced by Toner and Tu to model the "flocking" behavior in self-propelled rodlike objects [29,30]. The

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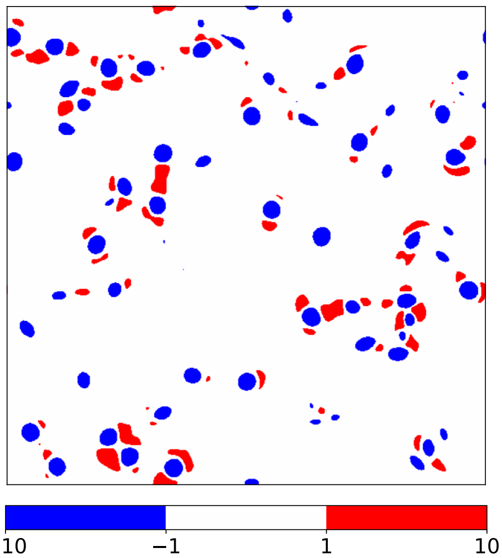

FIG. 1. Okubo-Weiss field of the turbulent fluid normalized to its rms value. The snapshot is taken in the steady state driven at $\alpha = -6$. We can clearly identify three topologically distinct regions, namely rotation dominated ($\mathcal{Q} < -1$), deformation dominated ($\mathcal{Q} > 1$), and intermediate ($-1 \leqslant \mathcal{Q} \leqslant 1$) regions.

parameter α, henceforth referred to as the Ekman friction, acts at all scales and can either lead to a damping of energy when $\alpha > 0$ or an injection of energy when $\alpha < 0$. Former leads the fluid to an isotropic equilibrium and the latter yields a globally ordered polar state with mean velocity $\sqrt{|\alpha|/\beta}$. We normalize all distances to a characteristic length $\sigma_0 = 5\pi\sqrt{2\Gamma_2/\Gamma_0}$ and all times to $t_0 = 5\pi\sqrt{2}\Gamma_2/\Gamma_0^2$. In terms of these reduced units, we fix the values of the model parameters as $\Gamma_0 = (5\pi\sqrt{2})^{-1}$, $\Gamma_2 = (5\pi\sqrt{2})^{-3}$, $\lambda_0 = 3.5$, and $\beta = 0.5$, in order to remain consistent with literature. Equation (1) is then numerically solved using a pseudospectral approach over a square grid of 512^2 points in a doubly periodic box of size 2π. For $\alpha < -6$, we use bigger boxes of size up to 10π and with resolution up to 2048^2 to avoid forming condensates. We overcome the aliasing errors that arise due to the implementation of discrete Fourier transforms by performing 2/3 and 1/2 dealiasing rules, respectively, for the quadratic $[(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}]$ and cubic $[(|\boldsymbol{u}|^2)\boldsymbol{u}]$ terms [31]. Time marching of $\boldsymbol{u}$ is done using the fully implicit Crank-Nicolson scheme with a time step of $\Delta t = 2\times10^{-4}$ that is sufficient to maintain numerical stability in the entire range of parameters explored here. To get Lagrangian statistics, we disperse a distribution of N tracers that follow the dynamics

$$\frac{d\boldsymbol{x}_i(t)}{dt} = \boldsymbol{u}[\boldsymbol{x}_i(t)], \tag{2}$$

where $\boldsymbol{x}_i(t)$ and $\boldsymbol{u}[\boldsymbol{x}_i(t)]$ are, respectively, the tracer location and its velocity. We use a cubic spline interpolation to project Eulerian quantities at any tracer location. As a thumb rule, we disperse these tracers and record their statistics only after the fluid attains a turbulent steady state. To compute the persistence time, we first introduce a fluctuating field that naturally lends a topological characterization of the flow field [24]. We do this by realizing that the velocity gradient of an incompressible fluid is a sum of rotation and deformation

TABLE I. The relative area fractions in topologically distinct regions at various levels of activity α.

α	Rotation	Intermediate	Deformation
−10	0.035	0.935	0.030
−6	0.044	0.935	0.021
0	0.073	0.879	0.048
2	0.079	0.852	0.069
3	0.103	0.814	0.083

tensors

$$\nabla\boldsymbol{u} = \frac{1}{2}\begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} \sigma_n & \sigma_s \\ \sigma_s & -\sigma_n \end{bmatrix} \tag{3}$$

with the eigenvalues $\gamma^2 = (\sigma_n^2 + \sigma_s^2 - \omega^2)/4 = \mathcal{Q}$. Here $\sigma_n = \partial_x u_x - \partial_y u_y$ and $\sigma_s = \partial_x u_y + \partial_y u_x$ denote the normal and shear strains, respectively, and $\omega = \partial_x u_y - \partial_y u_x$ is the fluid vorticity. Normalized to its root mean-squared value, the Okubo-Weiss field $\mathcal{Q}$ can now be used to partition the fluid into three topologically distinct regions, namely rotation dominated for $\mathcal{Q} < -1$, deformation dominated for $\mathcal{Q} > 1$ and intermediate for $-1 < \mathcal{Q} < 1$. See Fig. 1 for a visualization of this field in the steady state drive at $\alpha = -6$. The intermediate region clearly accounts for majority of the area fraction of the fluid, and this remains true for the range of activity explored in our work, refer to Table I. In the steady

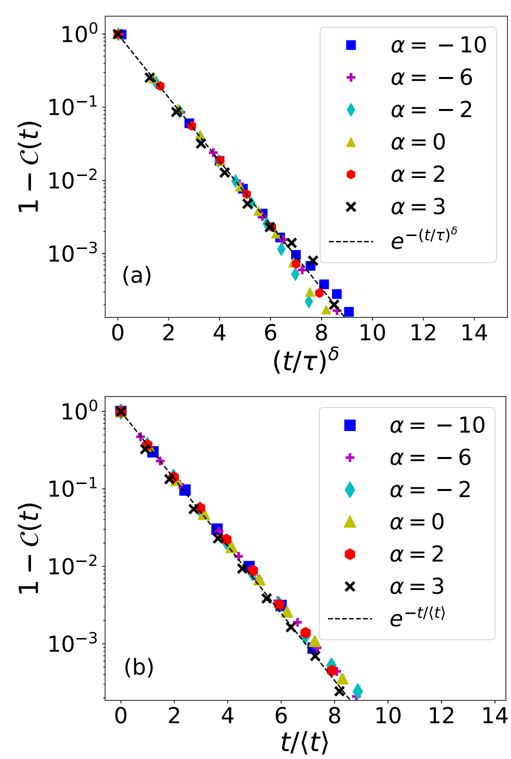

FIG. 2. Cumulative distribution function $\mathcal{C}(t)$ of the persistence time in (a) the rotation dominated region and (b) the turbulent background. Data collapse shows that the persistence probability $\mathcal{P}(t)$ is a Weibull distribution in the former and an exponential in the latter. The deformation dominated region also shows exponential persistence probability (not shown here).

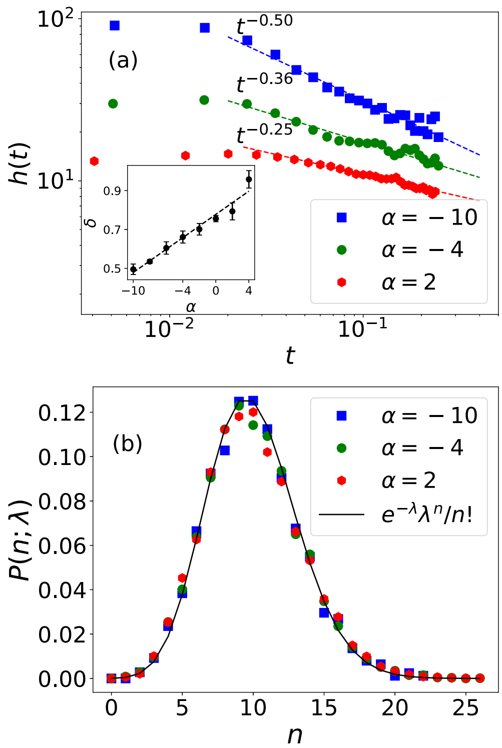

FIG. 3. (a) Hazard function plotted at various activity. The power-law fits verifies the shape exponent δ of the Weibull distribution discussed earlier. Inset suggests that δ is linear in α. The error bars in the estimation of δ denote a unit standard deviation, which is always within 5% of the mean. (b) The probability of n tracers exiting the intermediate region over a time interval of $10\langle t\rangle$. Data for each activity agrees with the Poisson distribution with $\lambda = 10$ (solid line).

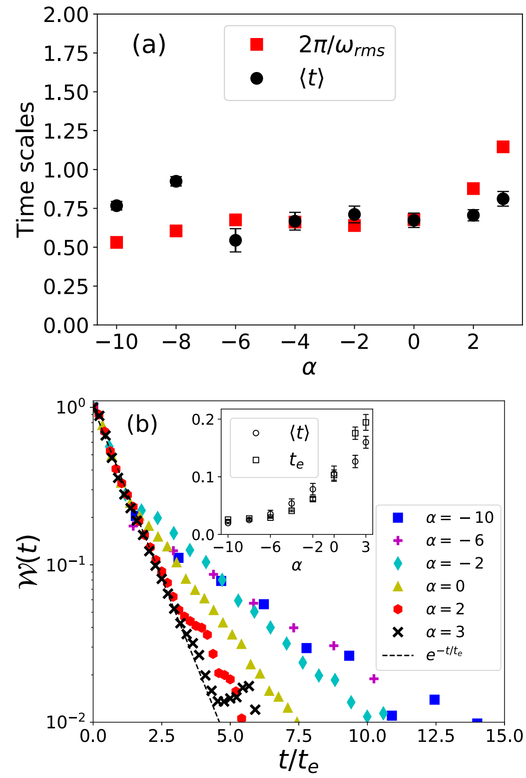

FIG. 4. (a) Comparison of the mean persistence time $\langle t\rangle$ with the vortex turn over time $2\pi/\omega_{\rm rms}$ in the intermediate region. The two time scales agree well over a wide range of activity, clearly indicating that the vortex rotation rate governs the persistence time: faster rotation leads to smaller persistence time. Error bars indicate a unit standard deviation about the mean. (b) Lagrangian autocorrelation of the Okubo-Weiss field computed only from the tracers seeded in the rotation dominated region. The initial decay is clearly exponential as seen from the data collapse. The e-folding timescale of the decay t_e agrees very well with the mean persistence time $\langle t\rangle$ throughout the range of activity explored (inset). Error bars indicate a unit standard deviation about the mean.

state, we release a distribution of passive tracers into the flow and track the time taken by each tracer to exit the topological region where it was initially seeded. This gives us a distribution of Lagrangian persistence times that follows a probability density $\mathcal{P}(t)$. It is a standard practice to obtain $\mathcal{P}(t)$ from its cumulative distribution function $\mathcal{C}(t) = \int_0^t \mathcal{P}(t')dt'$ as the procedure is immune to binning errors. In Fig. 2, we plot this distribution function separately for each topologically distinct region.

We observe that in the region where rotation dominates, $\mathcal{P}(t) = (t/\tau)^{\delta-1}(\delta/\tau)e^{-(t/\tau)^\delta}$ is a Weibull distribution function, quite contrary to the case of inertial turbulence that shows power-law statistics in this region [24,25]. The parameters δ and τ denote, respectively, the shape and scale of this stretched distribution that is often seen in systems displaying extreme value statistics. Note that $\delta < 1$ throughout the range of our simulations, essentially implying that the hazard function (see later) is a monotonically decreasing function of time.

The shape δ and scale τ are further related to the mean and variance of the Weibull distribution as

$$\begin{aligned}\langle t\rangle &= \tau\Gamma(1+1/\delta)\\ \sigma_t^2 &= \tau^2\{\Gamma(1+2/\delta) - [\Gamma(1+1/\delta)]^2\},\end{aligned} \tag{4}$$

where Γ is the Gamma function. The scale τ controls how far the Weibull distribution will extend towards the right. Naturally, any such extension will be accompanied by a decrease in the height of the distribution. To verify our statistics, we note that the Weibull distribution at its core is defined by a simple conditional density, given that the event in question has not occurred yet. Put simply, this is expressed as a hazard function

$$h(t) = -\frac{d}{dt}\ln[1-\mathcal{C}(t)] = \frac{\delta}{\tau}\left(\frac{t}{\tau}\right)^{\delta-1}. \tag{5}$$

In Fig. 3(a), we fit the hazard function with power laws to confirm the shape exponent δ at all levels of activity α. Inset shows that δ-α are in a linear relationship. Notice that at any activity, the hazard function decreases with time indicating a continuously falling failure rate. This happens because the tracers seeded near the edge of the vortex are likely to exit sooner than the ones seeded in the core. This is analogous to population dynamics with significant infant mortality leading to the failure rate decreasing over time as weaker infants are removed from the population.

We now turn our attention to the intermediate region characterized by $-1 \leqslant \mathcal{Q} \leqslant 1$. This is a turbulent background where the density $\mathcal{P}(t) = e^{-t/\langle t\rangle}/\langle t\rangle$ is an exponential distribution [Fig. 2(b)], that is remindful of the inertial turbulence. To verify the exponential statistics, we realize that when the waiting time is exponentially distributed with a mean $\langle t\rangle$, the probability of n tracers exiting the intermediate region over a time interval $\lambda\langle t\rangle$ must be the Poisson probability distribution

$$P(n;\lambda) = e^{-\lambda}\frac{\lambda^n}{n!}. \tag{6}$$

Indeed our data on event probability fits nicely to the Poisson distribution, see Fig. 3(b). We are thus drawn to the fact that the exit of tracers from the intermediate region is a memoryless stochastic point process—a similarity with inertial turbulence that is worth noting. The density $\mathcal{P}(t)$ in the deformation dominated region is also exponential (not shown here) in contrast to the inertial turbulence that exhibits power-law scaling [24]. The curious reader might wonder if this persistence is driven by an intrinsic timescale of the turbulent fluid. This is discussed next.

In the intermediate region, the root mean-squared vorticity $\omega_{\rm rms}$ can be used to compute a characteristic turnover time as $2\pi/\omega_{\rm rms}$. We find that this timescale agrees well with the mean persistence time throughout the range of activity explored in our work, see Fig. 4(a). It is therefore plausible to think of vorticity wandering as the driver of persistence in this region. In the rotation dominated region, we turn our attention to the autocorrelation of the Lagrangian Okubo-Weiss field $\mathcal{W}(t) = \langle\mathcal{Q}(t)\mathcal{Q}(0)\rangle/\langle\mathcal{Q}(0)^2\rangle$ computed only from tracers seeded in this region. Here $\langle\cdots\rangle$ indicates a joint average over tracers as well as initial times. This is plotted in Fig. 4(b) for various levels of activity. Clearly there is an initial exponential decay that allows a data collapse upto a timescale of the order t_e. This is followed by a stretched exponential at late times $t \gg t_e$ that decays faster with increasing α resulting in shorter trapping times. The e-folding timescale t_e agrees very well with the mean persistence time, which progressively increases with α, see inset Fig. 4(b). We also note that $\mathcal{W}(t)$ clearly retains memory in contradistinction to the turbulent background where it is memoryless. We thus conclude here that the driver of persistence in the rotation dominated region is the temporal relaxation of the Okubo-Weiss field.

Conclusions. This Letter constitutes a study of the persistence in dense swarms of active matter. We do this by a topological partitioning of the flow field that lends a natural characterization of the flow in terms of regions dominated by rotation, deformation, and background turbulence. By observing passive tracers that just go with the flow, we find that their persistence time inside the coherent vortices follows a Weibull distribution whose shape and scale are decided by the level of activity. In the turbulent background outside of these vortices, persistence time is exponentially distributed, beautifully remindful of inertial turbulence. We also show that driver of persistence inside the coherent vortices is the temporal decorrelation of the topological field, whereas it is the vortex turnover time in the turbulent background. We believe our findings could be relevant to experiments targeting dense bacterial swarms.

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[1] B. Derrida, V. Hakim, and V. Pasquier, Exact first-passage exponents of 1d domain growth: Relation to a reaction-diffusion model, Phys. Rev. Lett. **75**, 751 (1995).
[2] J. Krug, H. Kallabis, S. N. Majumdar, S. J. Cornell, A. J. Bray, and C. Sire, Persistence exponents for fluctuating interfaces, Phys. Rev. E **56**, 2702 (1997).
[3] C. M. Newman and D. L. Stein, Blocking and persistence in the zero-temperature dynamics of homogeneous and disordered ising models, Phys. Rev. Lett. **82**, 3944 (1999).
[4] G. G. Brown, R. F. Dell, and R. K. Wood, Optimization and persistence, Interfaces **27**, 15 (1997).
[5] G. Widmer and M. Kubat, Learning in the presence of concept drift and hidden contexts, Mach. Learn. **23**, 69 (1996).
[6] M. Constantin and S. Das Sarma, Volatility, persistence, and survival in financial markets, Phys. Rev. E **72**, 051106 (2005).
[7] S. N. Majumdar, Persistence in nonequilibrium systems, Curr. Sci. **77**, 370 (1999).
[8] A. Bassolas and V. Nicosia, First-passage times to quantify and compare structural correlations and heterogeneity in complex systems, Commun. Phys. **4**, 76 (2021).
[9] X. Li and A. B. Kolomeisky, Mechanisms and topology determination of complex chemical and biological network systems from first-passage theoretical approach, J. Chem. Phys. **139**, 144106 (2013).
[10] R. Bebon and U. S. Schwarz, First-passage times in complex energy landscapes: A case study with nonmuscle myosin II assembly, New J. Phys. **24**, 063034 (2022).
[11] A. J. Bray, S. N. Majumdar, and G. Schehr, Persistence and first-passage properties in nonequilibrium systems, Adv. Phys. **62**, 225 (2013).
[12] J. L. Lancaster and J. P. Godoy, Persistence of power-law correlations in nonequilibrium steady states of gapped quantum spin chains, Phys. Rev. Res. **1**, 033104 (2019).
[13] S. Jose, First passage statistics of active random walks on one and two dimensional lattices, J. Stat. Mech. (2022) 113208.
[14] S. Salcedo-Sanz, D. Casillas-Pérez, J. Del Ser, C. Casanova-Mateo, L. Cuadra, M. Piles, and G. Camps-Valls, Persistence in complex systems, Phys. Rep. **957**, 1 (2022).
[15] M. James and M. Wilczek, Vortex dynamics and lagrangian statistics in a model for active turbulence, Europhys. J. E **41**, 21 (2018).
[16] S. CP and A. Joy, Friction scaling laws for transport in active turbulence, Phys. Rev. Fluids **5**, 024302 (2020).
[17] S. CP and A. Joy, Effective temperature and Einstein relation for particles in active matter flows, Phys. Rev. E **105**, 065114 (2022).
[18] K. V. Kiran, A. Gupta, A. K. Verma, and R. Pandit, Irreversibility in bacterial turbulence: Insights from the

mean-bacterial-velocity model, Phys. Rev. Fluids **8**, 023102 (2023).
[19] S. Mukherjee, R. K. Singh, M. James, and S. S. Ray, Anomalous diffusion and lévy walks distinguish active from inertial turbulence, Phys. Rev. Lett. **127**, 118001 (2021).
[20] R. K. Singh, S. Mukherjee, and S. S. Ray, Lagrangian manifestation of anomalies in active turbulence, Phys. Rev. Fluids **7**, 033101 (2022).
[21] R. Alert, J. Casademunt, and J.-F. Joanny, Active turbulence, Annu. Rev. Condens. Matter Phys. **13**, 143 (2022).
[22] A. Okubo, Horizontal dispersion of floatable particles in the vicinity of velocity singularities such as convergences, in *Deep sea Research and Oceanographic Abstracts* (Elsevier, Amsterdam, 1970), Vol. 17, pp. 445–454.
[23] J. Weiss, The dynamics of enstrophy transfer in two-dimensional hydrodynamics, Physica D **48**, 273 (1991).
[24] B. Kadoch, D. del-Castillo-Negrete, W. J. T. Bos, and K. Schneider, Lagrangian statistics and flow topology in forced two-dimensional turbulence, Phys. Rev. E **83**, 036314 (2011).
[25] P. Perlekar, S. S. Ray, D. Mitra, and R. Pandit, Persistence problem in two-dimensional fluid turbulence, Phys. Rev. Lett. **106**, 054501 (2011).
[26] H. H. Wensink, J. Dunkel, S. Heidenreich, K. Drescher, R. E. Goldstein, H. Löwen, and J. M. Yeomans, Meso-scale turbulence in living fluids, Proc. Natl. Acad. Sci. USA **109**, 14308 (2012).
[27] J. Dunkel, S. Heidenreich, M. Bär, and R. E. Goldstein, Minimal continuum theories of structure formation in dense active fluids, New J. Phys. **15**, 045016 (2013).
[28] J. Dunkel, S. Heidenreich, K. Drescher, H. H. Wensink, M. Bär, and R. E. Goldstein, Fluid dynamics of bacterial turbulence, Phys. Rev. Lett. **110**, 228102 (2013).
[29] J. Toner, Y. Tu, and S. Ramaswamy, Hydrodynamics and phases of flocks, Ann. Phys. (NY) **318**, 170 (2005).
[30] J. Toner and Y. Tu, Flocks, herds, and schools: A quantitative theory of flocking, Phys. Rev. E **58**, 4828 (1998).
[31] C. Canuto, M. Y. Hussaini, A. Quarteroni, A. Thomas Jr. *et al.*, *Spectral Methods in Fluid Dynamics* (Springer Science & Business Media, Berlin, 2012).